Efficency and Accuracy Enhancement for One-way Wave Equation Migration Through Improved Mapping Function in the Time-shift Imaging Condition

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Summary

I introduce two interrelated ideas for improving efficiency in wave equation migration (WEM). First, I propose a new mapping function which improves the implementation of the recently-introduced ‘time-shift’ imaging condition (Ng, 2007) by providing a more accurate estimate of the infilled image in between relatively coarsely sampled wavefield extrapolation depth steps (thereby leading to a 2 to 4 times speed-up for 2D and 3D prestack WEM). The second idea is aimed at suppressing artifacts wherever such coarse wavefield extrapolation depth steps are employed. Specifically, I incorporate a simple split-step Fourier statics correction term to compensate for a certain migration error which is introduced when the wavefield extrapolation depth level transitions through undulating surface topography.

Introduction

Last year I presented a simple way to speed up WEM via the time-shift imaging condition (Ng, 2007) that permits the use of a relatively coarse downward extrapolation depth step by generating infilled images at all intermediate fine depth steps (e.g., with vertical sampling grid dz equal to 10 m) which fall in between the actual migrated images (at a coarse extrapolation depth step Δz, say, 20 to 40 m). At that time I introduced a simple mapping function which calculates different imaging time lags in order to extract an image from the cross-correlation of up-going receiver wavefield and down-going source wavefield without actual migration. This improved the WEM efficiency by a significant amount – approximately equal to the ratio of $\Delta z/dz$ (i.e., 2 to 4 times). However, this simple mapping function does not address the diffraction focusing mechanism of the wavefield extrapolation as the depth varies, and consequently it may not be able to track the wavefield change when the actual migration depth step becomes excessively coarse. Here, I suggest a new mapping function to better approximate the diffraction focusing term of the migration and therefore provide a better interpolation of the infilled image without much loss of accuracy. In practice, the size of coarse depth step is limited by the complexity of the velocity, which topic is beyond the scope of this paper.

After implementing this new mapping function, I found that increasing the depth step size Δz beyond values used in common practice (e.g., $\Delta z > 20$ m) exposes a problem when migrating from surface using the industry-standard ‘zero-velocity layer’ technique (Beasley and Lynn, 1992). Specifically, when the downward extrapolation depth level is transitioning across the surface topography (i.e., when portions of the topographical surface lie in between two depth steps), a migration error is introduced due to the ambiguity in placement of the zero-velocity layer. This error manifests in the form of ‘statics’ in the output common-image gathers (CIGs) on the order of $2\Delta z$. While the use of variable-sized depth step extrapolators could solve this problem (Margrave and Yao, 2000; Al-Saleh, et. al, 2006), such techniques are either slow or complicated. Instead I suggest installing a fast and simple split-step Fourier statics correction as a 'plug-in' to the widely used zero-velocity approach for any downward extrapolator.

Theory and/or Method

The common shot WEM imaging condition used in this paper is based on the scaled temporal cross-correlation between the upward propagating receiver data wavefield *U* and the downward propagating source wavefield *D*. The wavefield extrapolator can be of any kind, but here a phase-shift plus interpolation (PSPI) is used. At depth z_1 and lateral position x, the frequency domain expression of the cross-correlation for 2D (Kelly and Ren, 2003) is

$$R(x, z_1, \omega) = U(x, z_1, \omega) \cdot D^*(x, z_1, \omega) / (D \cdot D^* + \varepsilon), \qquad (1)$$

where ε is a small stabilizing pre-whitening scalar, and * denotes the complex conjugate. Instead of implementing the conventional imaging condition by computing the zero time lag cross-correlation (i.e., by summing $R(x,z_1,\omega)$ along all frequencies ω in order to estimate an image at every depth step), I (Ng, 2007) solve the image $i(x,z)$ via a full inverse Fourier transform of (1) to obtain the cross-correlated wavefield $r(x,\tau(z))$ at all time lags τ:

$$r(x, \tau(z)) = \frac{1}{2\pi} \int_{\omega} R(x, z_1, \omega) e^{j\omega\tau(z)} d\omega \qquad (2)$$
$$= i(x, z) \ .$$

Note from (2) that output infilled image $i(x,z)$ at arbitrary depth z in the vicinity of z_1 can be extracted from $r(x,\tau(z))$ between actual depth steps using a suitable time lag mapping function. Last year, I proposed to use

$$\tau(z) = \int_{z_1}^{z} \frac{2\beta}{v(x,z)} dz \ = 2\beta(\mathrm{z} - \mathrm{z}_1) / v(x,z), \quad | z - z_1 | < \Delta \mathrm{z} \, , \qquad (3)$$

where v is interval velocity, and β is an empirical constant between 0.5 to 1 which is tuned to optimize a range of dips from steeply dipping to flat respectively. Although equation (3) tries to emulate different coincidence times between the receiver and source wavefield that would be observed at different depths, it does not contain the diffraction collapsing focusing term of the complete downward extrapolator, and it may break down for steeply dipping data and for extremely coarse depth steps. In order to address that, here I propose a new cross-correlation time lag mapping function that is based on computing the difference in the direct arrival times taken from the shot-to-image point at depth z and shot-to-image point at depth z_1,

$$\tau(x, z) = 2(\sqrt{x^2 + z^2} - \sqrt{x^2 + z_1^2}) / v(x, z) \, . \qquad (4)$$

When equation (4) is used in conjunction with equation (2), this will allow an even larger depth step interpolation as it considers the source focusing term change.

As noted earlier, in order to make the above fast coarse depth step migration scheme work satisfactorily in the presence of surface topography, there is a critical issue that conventional (fine depth step) migration need not address. The issue arises when the coarse depth step is making its transition through the surface, and the surface lies in between two depth steps. Right at this transitioning, the placement of the zero-velocity layer is ambiguous and needs to be modified. I suggest a handy 'plug-in' $C(x,z_{surf})$ term in the form of a split-step Fourier statics correction to be applied to both the *U* and *D* fields immediately after each downward extrapolation step during the transitioning:

$$U_{corrected}(x, z_1 + \Delta z) = U(x, z_1 + \Delta z) C(x, z_{surf}) \quad \text{and}$$
$$D_{corrected}(x, z_1 + \Delta z) = D(x, z_1 + \Delta z) C^*(x, z_{surf}) \, , \qquad (5)$$

where $z_{surf}(x)$ is the surface topography, and the split-step Fourier statics correction term is given as

$$C(x, z_{surf}) = \exp[j\omega \frac{z_{surf}(x) - z_1}{v(x,z)}], \qquad (6)$$

for all x such that $z_1 < z_{surf}(x) \le z_1 + \Delta z$, and C is set to unity otherwise. This is illustrated in figure 1. Although the plug-in C term is a statics correction only, it gives most of the required correction.

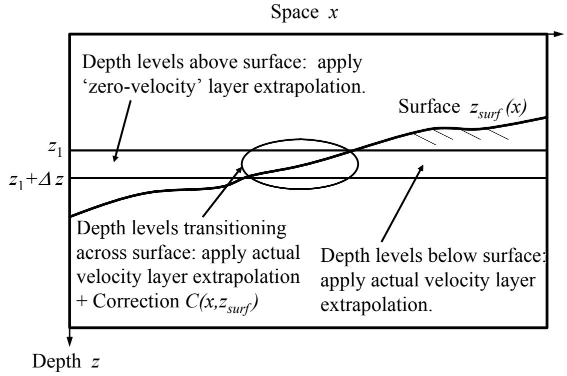

Figure 1: Migration depth level transitioning through a surface causes placement ambiguity of zero-velocity layer.

Example 1: Impulse response test (figure 2)

The objective of this example is to show the quality of the impulse responses from three migration interpolation methods under a very coarse downward extrapolation depth step Δz of 100 m, which is ten times the depth sampling interval dz of 10 m. Thus, nine out of ten output depth samples are infill interpolated with 1000% speed gain. The depth varying velocity increases evenly from 4100 m/s at the surface to 6000 m/s at 2 km depth, and the CDP interval dx is 15 m. The input impulse maximum frequency is 75 Hz. Figure 2 (a) is the shot migration result after interpolation using my original time lag mapping function equation (3) with β =.8 in the time-shift imaging condition. The results are generally good except at the steep dips where they begin to break down. Figure 2 (b) is the shot migration result after interpolation using the new proposed mapping function (equation (4)) in the time-shift imaging condition; the results are very good essentially matching those migrated at a fine depth step without interpolation, even at this very coarse depth step.

Example 2: Real data set test (figure 3)

This example illustrates the action of coarse depth step migration the above infill interpolation technique in the presence of surface topography, both with and without the split-step Fourier statics correction. This land data set is taken from the Western Canadian foothills, and the surface topography varies by about 100 m over a 3 km split spread cable length of a shot. The data are migrated from topography using the zero-velocity layer approach together with the new mapping function (equation (4)) in the time-shift imaging condition. At a certain location where the structures show a dominant dip of 30 degrees, the CIGs are shown in figure 3. In figures 3 (a) and (b), the migration depth step is three times as large as the depth samples in order to gain three times ($\Delta z = 3dz = 30$ m) in speed. Figure 3 (a) is the shot migration result without the split-step Fourier statics correction as stated in equation (6). The results reveal huge migration errors in the form of 'statics' as large as twice the depth step size ($2\Delta z$, 60 m) due to the surface topography 'lingering' within the coarse depth step. Figure 3 (b) is the result with the newly proposed correction C. Note the events within the CIG are properly flattened without statics.

Conclusions

As the new proposed time lag mapping function approximates the diffraction focusing of the source wavefield, it is found to be an improved infill interpolator in between coarse extrapolation depth steps. Furthermore, in order to make coarse depth step extrapolation perform well in the presence of surface topography, a fast 'plug-in' split-step Fourier statics correction is suggested which operates in conjunction with the classic zero-velocity layer correction. The tests illustrate that when both innovations are implemented in conjunction with the time-shift imaging condition, they produce accurate images and great cost savings.

References

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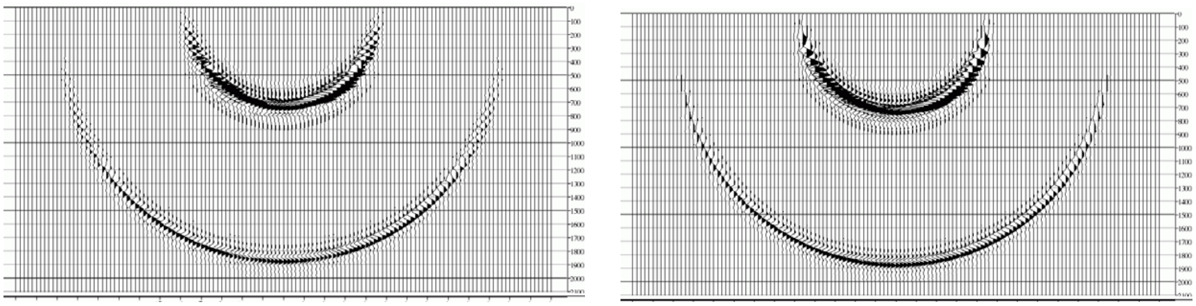
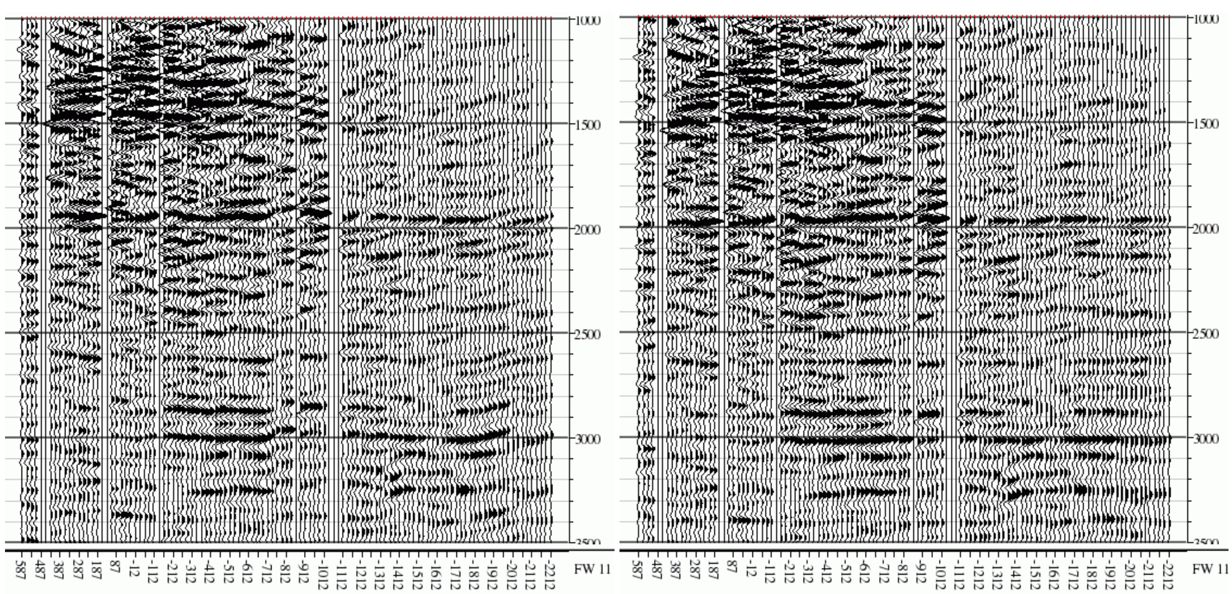


(a) $\Delta z = 10dz = 100$ m, interpolation by eqt. (3). Gained 10X in speed. Steep dips are degraded.

(b) $\Delta z = 10dz = 100$ m, interpolation by eqt. (4). Gained 10X in speed. Results are very good.

Figure 2: Shot migration impulse response comparisons in a depth varying velocity environment. Coarse depth step migration is used for 1000% speed gain. (Labels are in meters)



(a) $\Delta z = 3dz = 30$ m, interpolation by eqt. (4) gaining 3X in speed. Reflectors reveal large migration 'statics' errors due to surface topography.

(b) $\Delta z = 3dz = 30$ m, interpolation by eqt. (4) gaining 3X in speed. Split-step Fourier statics correction term eqt. (6) is applied, and structural reflectors are well imaged flat.

Figure 3: Real data set: CIGs after shot migration in the presence of surface topography showing the effects of the split-step Fourier statics correction with coarse depth step migrations which gained 300% in speed. The CIG location is selected from a structured portion of the data set in which most reflectors exhibit a 30 degrees dip. (Labels are in meters)