A phase shift plus interpolation extrapolation for two-way wave equation migration

Jianhua Pan
Arcis Corporation, Calgary, Alberta
jpan@arcis.com

Summary

Most conventional approaches to migration by downward continuation use one-way wave equation which allows wavefield to propagate only downward or upward. However one-way wave equation migration (WEM), although reliable, may not give satisfactory images when the structures have steep dips and the velocity model is so complex that multiple reflections exist. This paper proposes a downward continuation approach with two-way wave equation which propagates downgoing wave and upgoing wave simultaneously. The mechanism is similar to phase shift plus interpolation (PSPI) with one-way wave equation except that in two-way PSPI signs of phase-shifts applied to downgoing wave and upgoing wave are different and the derivative of wavefield to depth needs to be calculated. A synthetic example presented demonstrates that this two-way PSPI migration allows steep dip to be imaged.

Introduction

Seismic migration is usually conducted these days using either the Kirchhoff integral method or the conventional (one-way) wavefield extrapolation. Although effective for many seismic datasets, both methods have limitations in terms of imaging complex structures.

The standard shot profile one-way WEM images the subsurface by continuing the source and receiver wavefields for each shot downward in the depth. The image is formed by applying an imaging condition to combine these two wavefields at subsurface locations to produce images at those locations. Summation of all shot-images forms the final image. The main limitation for this method is that one assumes source and receiver wavefields only travel in one direction, i.e. forward for the source wavefield and backward for the receiver wavefield. In practice, both wavefields travel up and down when the velocity model is complex. This generates turning rays and multiples. In addition, the one-way WEM usually limits dips to less than 85 degrees and thus the steep dip and turning rays are imaged through Kirchhoff techniques, which fail to handle multiple arrivals. As a result, the two-way WEM is needed.

The two-way WEM is based on solving the two-way wave equation and it does not have the limitations listed above for the one-way WEM and the Kirchhoff migration. The suggested method for the two-way WEM is the reverse-time migration, which solves two-way wave equation in time-space domain by finite difference approximation.

This paper introduces an alternative approach, the downward continuation technique, to solve the two-way wave equation. In this approach, both downgoing and upgoing waves are downward continued simulteniously at each depth level and then imaging condition is applied. Unlike earlier two-way downward continuation techniques, (Kosloff and Baysal, 1983; Wapenaar, et al., 1987; Sandberg and Beylkin, 2009), which are in frequency-space domain and some of them needs solving linear system at each depth level, this approach performs downward continuation in the frequency-wavenumber domain through phase shift plus interpolation. The difference from one-way PSPI is that at each depth two-way PSPI applies positive phase-shift to downgoing wave and negative phase-shift to upgoing wave respectively (in the practical implementation it is one-step calculation). The downgoing and upgoing waves are formulated in terms of both full wavefield and its derivative with respect to depth. Thus the computational cost is twice as much as the one way PSPI since derivative of the full wavefield needs to be calculated. The data example shows that steep dips are successfully migrated.

Algorithm

For the sake of simplicity, one only considers the 2-D full acoustic wave equation with a constant density

$$P_{xx} + P_{zz} = \frac{1}{v^2} P_{tt}, \tag{1}$$

where P is the acoustic wavefield and v is the velocity. Assume the velocity is constant in the interval (z_0, z) and then after Fourier transformation of time and the horizontal space variables, the partial differential equation (1) becomes an ordinary differential equation

$$P_{zz}(\omega, k_x, z) = -k_z^2 P(\omega, k_x, z), \tag{2}$$

where ω is the angular frequency, k_x is the horizontal wave number and k_z given as $k_z^2 = \frac{\omega^2}{v^2} - k_x^2$ is the vertical wave number. Without confusion, one still uses P to denote the wavefield in frequency-wavenumber domain. The negative k_z^2 indicates that the wave is evanescent and it will be suppressed. Thus one can see k_z^2 as nonnegative in the following discussion. The equation (2) has an unlimited number of solutions, of which, there are two special ones $\exp(ik_z\Delta z)$ and $\exp(-ik_z\Delta z)$ with $\Delta z = z - z_0$, downward extrapolator and upward extrapolator respectively. From the theory of differential equations, any solution of the equation (2) is in the form

$$P(z_0 + \Delta z) = A^+ \exp(ik_z\Delta z) + A^- \exp(-ik_z\Delta z), \tag{3}$$

where A^+ and A^- could be any constants independent of z and $P(z) = P(\omega, k_x, z)$. When $A^+ = P(z_0)$ and $A^- = 0$, $P(z) = P(z_0)\exp(ik_z\Delta z)$, which is the one-way downgoing phase-shift extrapolation. Similarly if $A^+ = 0$ and $A^- = P(z_0)$, then $P(z) = P(z_0)\exp(-ik_z\Delta z)$, which is the one-way upgoing phase-shift extrapolation. With the knowledge of $P(z_0)$ as well as $P_z(z_0)$, two constants in the equation (3) can be solved for as

$$A^+ = \frac{1}{2}[P(z_0) + \frac{1}{ik_z} P_z(z_0)] \quad \text{and} \quad A^- = \frac{1}{2}[P(z_0) - \frac{1}{ik_z} P_z(z_0)],$$

which are downgoing and upgoing wavefields at the depth z_0. Hence equation (3) shows that applying positive and negative phase-shift to downging and upgoing waves at the depth z_0 generates new downgoing and upgoing waves and the summation of these two new one-way wavefields produces full wavefields at the depth $z_0 + \Delta z$. The equation (3) can be simplified as

$$P(z_0+\Delta z)=\cos(k_z\Delta z)P(z_0)+\frac{\sin(k_z\Delta z)}{k_z}P_z(z_0). \quad (4)$$

The formulation of $P_z(z_0+\Delta z)$ is derived from the derivative of the equation (4) with respect to Δz, i.e.

$$P_z(z_0+\Delta z)=-k_z\sin(k_z\Delta z)P(z_0)+\cos(k_z\Delta z)P_z(z_0). \quad (5)$$

The equation (4) coupled with the equation (5) is the downward continuation of the full wavefield and its derivative for the velocity without lateral variations. For the constant velocity in the interval $(z_0, z_0+n\Delta z)$, it is easy to check that

$$P(z_0+n\Delta z)=\cos(nk_z\Delta z)P(z_0)+\frac{\sin(nk_z\Delta z)}{k_z}P_z(z_0), \quad (6)$$

and thus the extrapolation is absolutely stable in this situation. The derivative of the wavefield on the top boundary $P_z(0)$ can be obtained from the absorbing boundary condition (Clayton and Engquist, 1977). Suppressing evanescent waves is analogous to one-way WEM, i.e.

$$P(z_0+\Delta z)=P(z_0)\exp(-\sqrt{-k_z^2}\Delta z)$$

and

$$P_z(z_0+\Delta z)=P_z(z_0)\exp(-\sqrt{-k_z^2}\Delta z).$$

For velocity with lateral variations, full wavefields with several reference velocities are calculated at each depth and are then Fourier-transformed to the spatial domain. The full wavefield at any specific location is the interpolation between two calculated reference full wavefields. The interpolation is guided by the velocity value. The whole procedure is called two-way PSPI, which is slightly different from one-way PSPI. Since the derivatives of full wavefields have to be computed, the computational cost is thus twice as much as the one-way WEM.

Data Examples

The data examples shown is the model representing a cross section through the foothills of the Canadian Rockies, with the rugged surface. Images migrated through one-way WEM (Figure 1) and two-way WEM (Figure 2) are presented. It can be seen that the one-way WEM image is cleaner and has fewer artifacts than the two-way WEM image. This is quite reasonable since two-way WEM brings not only primary reflections but also multiples. However, the one-way WEM loses steep dipping events, as opposed to the two-way WEM.

Conclusions

A two-way PSPI extrapolation technique is introduced for seismic migration. The mechanism is quite similar to the one-way PSPI. In two-way PSPI the derivative of the full wavefield needs to be calculated. As a two-way WEM, the computational cost is cheap. The data example shows that the steep dipping structure nearly 90 degrees is migrated.

Acknowledgements
The author would like to thank Arcis Corporation for the permission to present this paper.

References

Clayton, R.W., and Engquist, B., 1977, Absorging boundary conditions for acoustic and elastic wave equations. Bulletin of the Seismological Society of America, 67, 1529-1540.

Kosloff, D. and Baysal, E., 1983, Migration with the full acoustic equation, Geophysics, 48, 677-687.

Sandberg, K.,and Beylkin, G., 2010, Full-wave-equation depth extrapolation for migration, Geophysics, 74, WCA121-WCA128.

Wapenaar, C.P.A., Kinneging, N. A., and Berkhout, A. J.,1987, Principle of prestack migration based on the full elastic two-way wave equation. Geophysics, 52, 151-173.

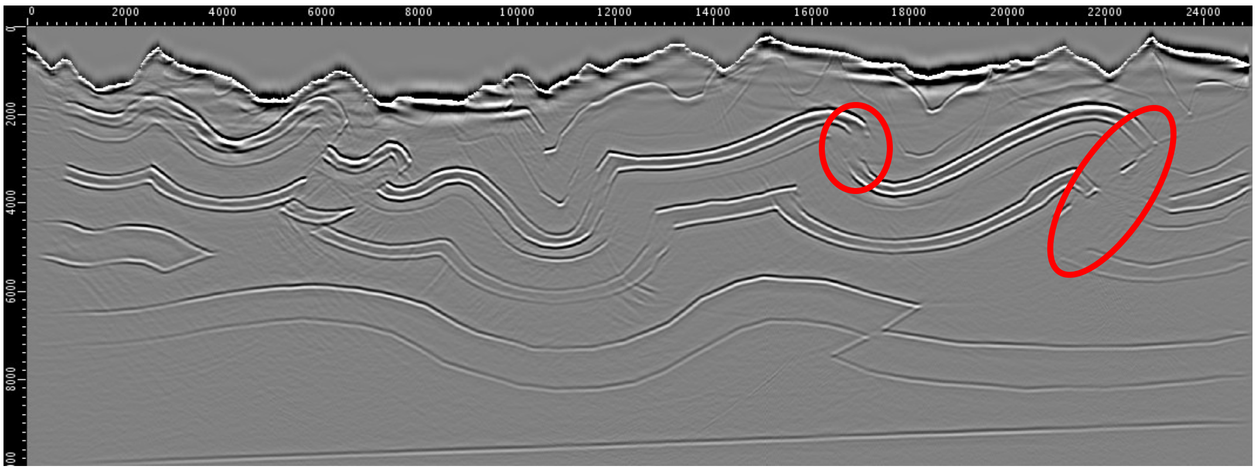

Figure 1: Foothills of Canadian Rockies, one- way WEM image.

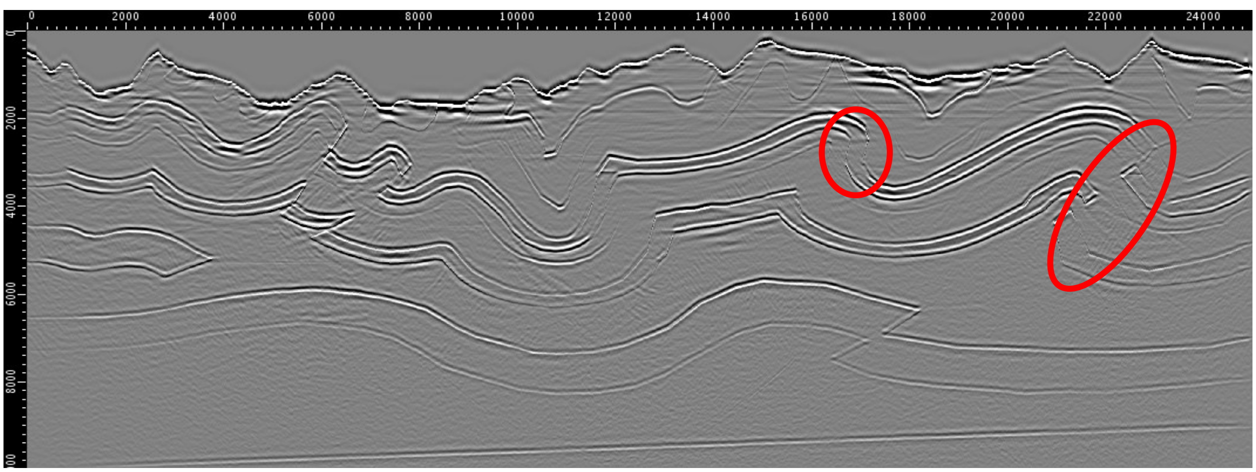

Figure 2: Foothills of Canadian Rockies, two-way WEM image.