Stable high-resolution three-term AVO inversion

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Summary

Three-term AVO inversion can be used to estimate subsurface information about P-wave and S-wave velocities and, in addition, density. The density term, however, exhibits little sensitivity to amplitude variations and therefore, its inversion is unstable. One way to stabilize the density term is by including a scale matrix that provides correlation information between the 3 unknown AVO parameters. In this article, we investigate a Bayesian procedure to promote sparsity in the estimation of highly correlated AVO model parameters. To this end, we model the prior distribution of the AVO parameters via a Trivariate Cauchy distribution. We present an iterative algorithm to solve the Bayesian inversion and, in addition, we provide comparisons with the classical inversion approach that uses a Multivariate Gaussian prior distribution.

Introduction

AVO inversion is one of the techniques that are being used to estimate subsurface physical parameters such as P-wave velocity, S-wave velocity, and density from seismic reflection data. The AVO inverse problem is an ill-conditioned problem. In other words, small perturbations in the data result in large perturbations in the estimated parameters. The latter makes AVO inversion unstable, particularly, when one attempts to estimate the density term (Downton and Lines, 2004; Li, 2005). One way to stabilize the inversion is by including prior information. To this end, we propose to use the classical Bayesian framework and prescribe a prior distribution that promotes sparse and correlated AVO solutions (Alemie, 2010). For this purpose, we investigate the application of a Trivariate Cauchy probability distribution to model the prior distribution of AVO attributes. The Trivariate Cauchy distribution is a special case of the Multivariate *t* distribution with three variables and one degree of freedom (Chauanhai, 1994; Johnson and Kotz, 1972). This prior distribution allows incorporating the correlation information via a scale matrix. In addition, the Trivariate Cauchy distribution has long tails and therefore, it is a prior that promotes the formation of sparse solutions.

Theory

The forward physical model is the three-term Aki and Richards's approximation to the Zoeppritz equations (Aki and Richards, 1980). It consists of three model parameters namely P-wave reflectivity ($\mathbf{A}$), S-wave reflectivity ($\mathbf{B}$) and density reflectivity ($\mathbf{C}$). These parameters or attributes are represented in vector form via $\mathbf{m}=[\mathbf{A}\ \mathbf{B}\ \mathbf{C}]^T$. The relationship between the observed data, $\mathbf{d}$, and the model parameters, $\mathbf{m}$, can be expressed as follows

$$\mathbf{d} = \mathbf{Lm} + \mathbf{n}, \tag{1}$$

where $\mathbf{L}$ is a linear operator that includes a wavelet convolution matrix and the offset dependent amplitudes associated to the Aki and Richards AVO approximation (Buland and Omre, 2003). The operator is built using a smooth background model and by considering a zero phase offset dependent wavelet estimated from the data (Alemie, 2010). The last term in equation (1) represents the noise that considers observational, processing and operator errors. Equation (1) is valid for a single AVO gather and can be regarded as a multi-dimensional convolution operator that transforms the 3 AVO traces, $\mathbf{A}(t)$, $\mathbf{B}(t)$ and $\mathbf{C}(t)$, into a multi-channel seismic signal (the AVO gather). In this regard, it is clear that AVO inversion can be interpreted as a process analogous to multi-channel deconvolution (Downton and Lines, 2004). The three-term AVO solution is found by minimizing the following objective function

$$J = \|\mathbf{Lm} - \mathbf{d}\|_2^2 + R(\mathbf{m}). \qquad (2)$$

The last equation is derived using the Bayesian framework by considering Gaussian errors and a priori distribution that induces the regularization term $R(\mathbf{m})$. Alemie (2010) proposed to use a Trivariate Cauchy distribution as prior for the AVO inverse problem. In this case, one can show that the regularization term induced by the Trivariate Cauchy distribution is given by

$$R(\mathrm{m}) = 2\sum_{\mathrm{i}=1}^{\mathrm{N}} \ln(1 + \mathbf{m}^{\mathrm{T}} \Phi^{i} \mathbf{m}), \qquad (3)$$

where

$$\Phi^{\mathrm{i}} = \mathbf{D}_{\mathrm{i}}^{\mathrm{T}} \Psi^{-1} \mathbf{D}_{i}. \qquad (4)$$

The matrix $\boldsymbol{\Psi}$ is the scale matrix of the problem. This matrix plays the role of the correlation matrix in Gaussian statistics and therefore, contains entries with correlation among the AVO parameters. The matrix $\mathbf{D}$ is defined as

$$[\mathbf{D}_{nl}^{i}] = \begin{cases} 1, & if \quad n = 1 \; and \quad l = i \\ 1, & if \quad n = 2 \; and \quad l = i + 1 \\ 1, & if \quad n = 3 \; and \quad l = i + 2N \\ 0, & otherwise \end{cases}, \qquad (5)$$

where N is the number of time samples. This matrix is needed to accommodate the fact that $\mathbf{m}$ contains the 3 AVO time series in a single column vector. Minimizing the Bayesian cost function leads to the following solution

$$(\mathbf{L}^{T} \Gamma^{T} \Gamma \mathbf{L} + \mu \mathbf{Q})\mathbf{m} = \mathbf{L} \Gamma^{T} \Gamma \mathbf{d}, \qquad (6)$$

where

$$[\mathbf{Q}_{kn}] = \sum_{i=1}^{N} \frac{2\Phi_{kn}^{i}}{1 + \mathbf{m}^{T} \Phi^{i} \mathbf{m}} \qquad k, n = 1,2,3,\ldots,3N. \qquad (7)$$

Note that the matrix $\mathbf{Q}$ is a non-diagonal matrix, which contains the scale matrix of the problem. In real scenarios the scale matrix can be computed by measuring correlation between parameters from borehole-derived AVO attributes (Downton and Lines, 2004). The matrix Γ is a muting operator that is included to avoid fitting data in the regions were mutes were applied to control excessive stretch or to control the fact that the AVO model is no longer valid after a maximum offset or angle. The parameter μ is the hyper-parameter that is determined to yield solution with optimal degree of fitting. The latter can be achieved by two methods: L-curve approach or by the Chi-square test (Tarantola, 1987). The system in equation (6) is non-linear and therefore, an iterative approach is used to find the AVO solution. This is equivalent to use the well-known iterative reweighed Least-squares algorithm to solve sparsity-constrained problems (Sacchi and Ulrych, 1995). In general, about 3-4 iterations are required to reach a sparse solution for the three-term AVO parameter problem.

Examples

In this section, synthetic and real data examples are used to demonstrate the regularization strategy outlined above. Using the velocity and density models provided in Figure 1 and a 35Hz Ricker wavelet, synthetic data are generated (Figure 2). The inversion is run in a selected time window 0.2 s to 0.636 s with an overlap of a length of the wavelet. The stability and quality of the inversion was accessed using a Monte Carlo simulation. In Figures 3a-3i**,** each plot depicts the root mean square errors versus signal to noise ratio (S/N) for the three model parameters: root mean square errors for P-wave reflectivity ($RMSE_A$), S-wave reflectivity ($RMSE_B$) and density reflectivity ($RMSE_C$). The columns in Figure 3 show the type of prior distribution used: Univariate Cauchy, Multivariate Gaussian and Trivariate Cauchy. Note that the error portrays the deviation of inverted parameters from their true values. The error bars represent the corresponding standard deviations. It is easy to observe that increasing the noise level increases the uncertainty in the inverted result. The errors are quite large in the case of Univariate Cauchy prior. In addition, the Univariate Cauchy prior lacks of stability. In other words, sparsity is achieved but the 3 AVO terms are not showing a good degree of correlation. This is simply because the Univariate Cauchy treats the parameters as if they were uncorrelated. The Multivariate Gaussian gives better results than the Univariate Cauchy as it allows including the correlation. In addition to stabilizing the inversion via the scale matrix, the Trivariate Cauchy prior plays a role in making the solution sparse and hence enhanced resolution.

The real data set consists of 61 NMO corrected CDP gathers. The offset ranges from 240 m - 3210 m. The time ranges from 0 to 1.502 s with sampling interval of 2 ms. The wavelet is estimated from each trace and an average wavelet was used for each CDP. Well-log data was used to incorporate the correlation information matrix in the inversion. Understanding the ineffectiveness of the Univariate Cauchy prior distribution, the real data inversion was done only for the other two prior distributions: Multivariate Gaussian and Trivariate Cauchy. Figures 4a, 4b, and 4c are the results of the inversion using the Multivariate Gaussian as prior distribution. Figures 5a, 5b, and 5c are results of the inversion using Trivariate Cauchy as prior distribution. The same trend is observed using both the Gaussian and Trivariate Cauchy priors that reflects they do a similar job in incorporating the well-log information thereby stabilizing the inversion. As expected, the result has better resolution in the case of Trivariate Cauchy.

Conclusions

Both synthetic and real data examples demonstrated that including correlation information in the AVO inverse problem helps to add stability to the inversion. This is particularly important if one wants to invert the density term. This in turn increases reliability of the estimated parameters. Long tail prior distributions like the Trivariate Cauchy distribution do help to introduce sparsity in the solution and thereby, they lead to results with enhanced resolution.

Acknowledgements

We would like to acknowledge the sponsors of Signal Analysis and Imaging Group at the University of Alberta for supporting this research. We would also like to thank Juan Soldo (Petrobras) for providing the real data.

References

Aki, K. and Richards, P. G., 1980, Quantitative seismology: theory and methods: W. H. Freeman, San Francisco.

Alemie, W.M., 2010, Regularization of the AVO inverse problem by means of a multivariate Cauchy probability distribution, MSc thesis, University of Alberta (www-geo.phys.ualberta.ca/saig/theses.php).

Buland, A. and Omre, H., 2003, Bayesian linearized AVO inversion: Geophysics, 68, 185–198.

Chauanhai, L., 1994, Statistical analysis using the Multivariate t distribution, Phd thesis, Harvard University.

Downton, J.E. and Lines, L.R., 2004, Three term AVO waveform inversion: SEG Technical Program Expanded Abstracts, 23,
215–218.

Johnson, N.L. and Kotz, S., 1972, Distribution in statistics: continuous multivariate distribution: New York, John Willey and Sons.
Li, Y., 2005, A study on applicability of density inversion in defining reservoirs: SEG Technical Program Expanded Abstracts, 24, 1646–1649.
Sacchi, M.D. and Ulrych, T.J., 1995, High-resolution velocity gathers and offset space reconstruction: Geophysics, 60, 1169–117.
Tarantola, A., 1987, Inverse problem theory, method for da fitting and model parameter estimation, Elsevier Science Publishers.

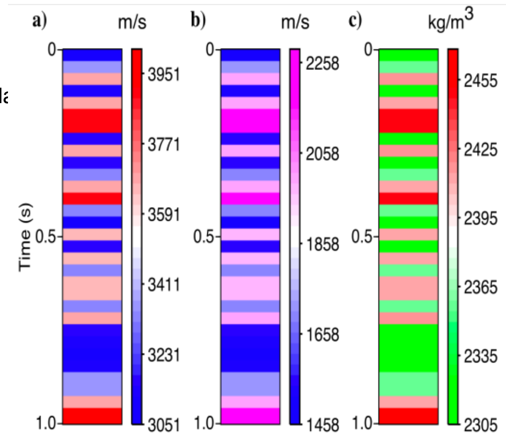

Figure 1: (**a**) P-wave velocity (**b**) S-wave velocity and (**c**) density.

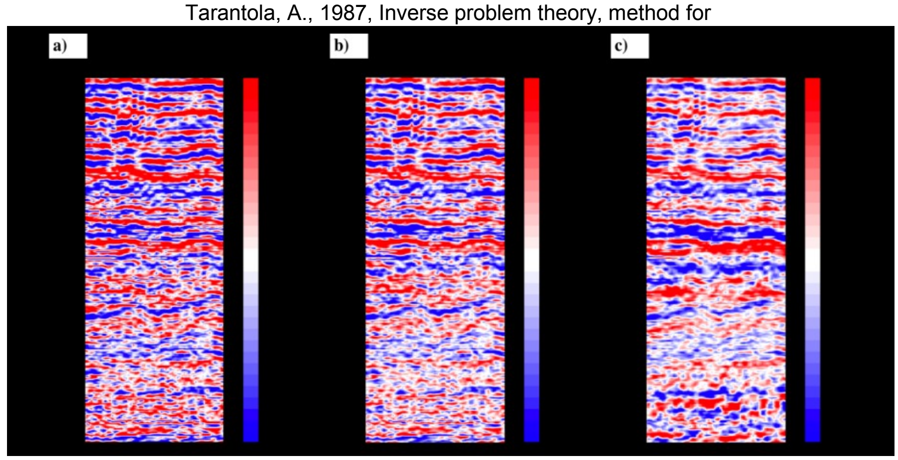

Figure 4: (**a**) P-wave reflectivity (**b**) S-wave reflectivity, and (**c**) density reflectivity using a Multivariate Gaussian prior distribution.

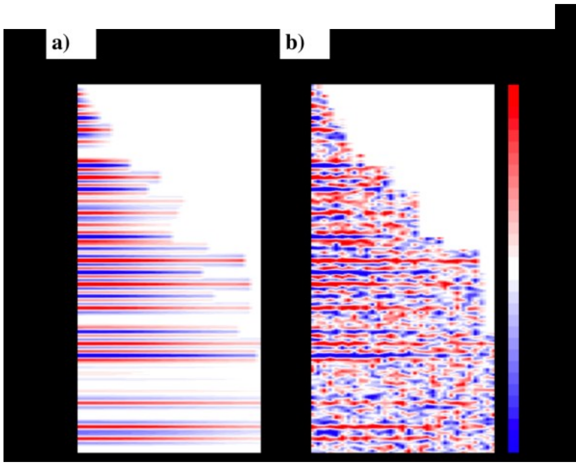

Figure 2: Synthetic data (**a**) without noise (**b**) with noise S/N=4.

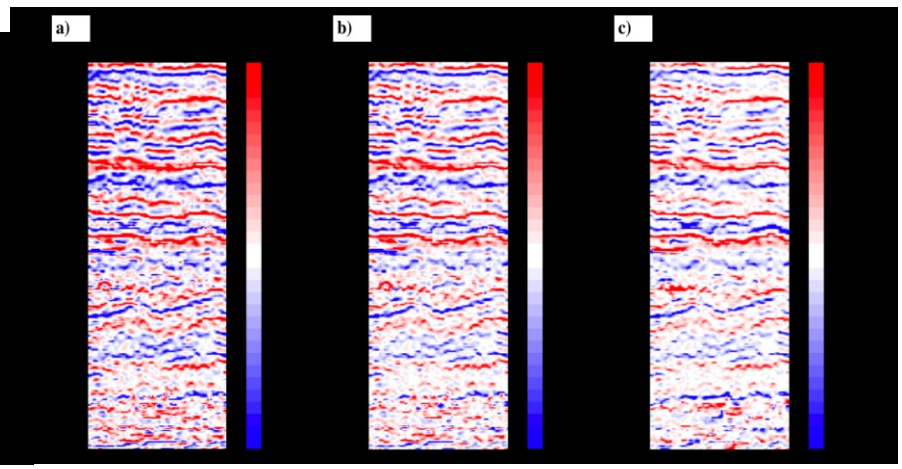

Figure 5: (**a**) P-wave reflectivity (**b**) S-wave reflectivity, and (**c**) density reflectivity using a Trivariate Cauchy prior distribution.

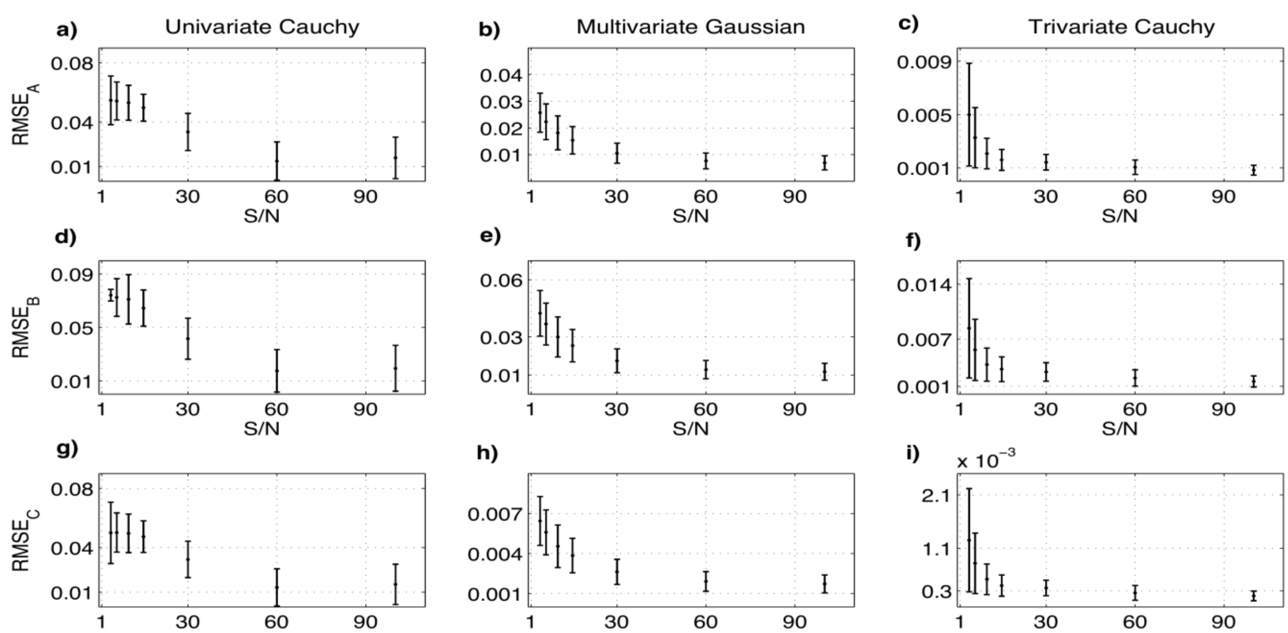

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Figure 3: Error analysis obtained via 20 realizations for each S/R.