n-*th* root entropy functions for blind deconvolution

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Summary

Minimum Entropy Deconvolution (MED) seeks to estimate a reflectivity series that consists of a small number of large spikes that can honor the seismic trace. We investigate a method to recover the reflectivity series based on n*th*-root entropy functions. This approach recovers comparatively small normalized reflectivity values and therefore, it attempts to diminish one of the shortcoming of the MED method. Synthetic data are used to test this family of entropy functions.

Introduction

MED is a deconvolution method proposed by Wiggins in 1978. The MED method belongs to the category of blind deconvolution methods as it attempts to recover simultaneously the seismic source wavelet and the seismic reflectivity. MED was an important attempt to bypass the classical minimum phase assumption made by spiking and predictive deconvolution techniques (Robinson and Treitel, 1980). Unfortunately, the MED technique tends to retrieve reflectivity sequences that are too sparse and therefore, it produces results with an unrealistic seismic character.

One can propose a MED algorithm by maximizing a generalized entropy norm (Sacchi et al., 1994). A particular solution of the generalized entropy norm that depends on a particular choice of the entropy function was proposed by Wiggins (1978). However, there are different ways to define entropy norms by using different entropy functions. For example, logarithmic, quadratic and cubic entropy functions can be defined. The main problem with the entropy norms proposed so far is that they do not recover small amplitudes and they distort the relative amplitude of the reflection coefficients. In this work we present an n*th*-root family of entropy norms to attempt to overcome these limitations.

The convolution model

The seismic signal can be represented as the convolution of the reflectivity y and the wavelet ω. The seismogram can be represented as follows

$$x = \omega * y + n \tag{1}$$

The purpose of the deconvolution is to recover the reflectivity y from the signal x without amplifying the noise n. For that purpose, we need to estimate a linear operator f such that $f * \omega = \delta$. We must stress that we will only estimate an approximation $\hat{f}$ to the desired filter, *i.e.*, $\hat{f} * \omega = a$ where a, the residual wavelet, should be resemble a delta function. Applying $\hat{f}$ to both sides of equation 1 leads to the following expression

$$\begin{aligned}\hat{y} &= a * y + \hat{f} * n \\ &= y + (a - \delta) * y + \hat{f} * n\end{aligned} \tag{2}$$

Equation 2 shows that the linear operator must approximate a to δ . In addition, the operator should not amplify the noise. The estimated reflectivity resembles the true reflectivity when the two aforementioned goals are attained.

Entropy norms

At the core of equation 2 is the estimation of the operator f. Methods to estimate the deconvolution operator often assume a white reflectivity series and a minimum phase wavelet (Robinson and Treitel, 1980). In this article, we investigate the estimation of f via the MED method. Minimum entropy methods operate by defining and maximizing an entropy norm $V(y)$ that measures the simplicity of a time series (reflectivity) $y = (y_1, y_2, \ldots, y_N)$. In this work, a general form to measure simplicity is presented (Sacchi et al., 1994). The entropy norm is given by

$$V(y) = \frac{1}{NF(N)} \sum_{i=1}^{N} q_i F(q_i) \qquad (3)$$

with normalized amplitude variable q is given by

$$q_i = \frac{y_i^2}{\sum y_i^2 / N} \qquad (4)$$

In equation (3), $F(q_i)$ is called the entropy function (De Vries and Berkhout, 1984). The latter must be a monotonic function of q to guarantee that V is a measure of simplicity or sparsity. Maximizing V with respect to the operator coefficients leads

$$\begin{gathered} \frac{\partial V(y)}{\partial f_k} = 0 \qquad k = 1,2,\ldots,LF \\ \frac{\partial V(y)}{\partial (f_k)} = \frac{1}{N ln(N)} \sum_i \left(F(q_i) + q_i \frac{\partial F(q_i)}{\partial q_i} \right) \frac{\partial q_i}{\partial f_k} \end{gathered} \qquad (5)$$

where LF is the length of the filter. After a few algebraic steps, equation (5) reduces to the following expression

$$\sum_l f_l \sum_n x_{n-k} x_{n-l} = \sum_i b_i \, x_{i-k}. \qquad (6)$$

Now expression (6) represents a shaping filter (Robinson and Treitel, 1980) that attempts to convert the seismic trace into a sparser version of itself (b_i). The “deformation” of the seismic trace is given by

$$\begin{gathered} b_i = \frac{G(q_i) y_i}{\frac{1}{N} \sum_j G(q_i) q_j} \\ G(q_i) = F(q_i) + q_i \frac{\partial F(q_i)}{\partial (q_i)}. \end{gathered} \qquad (7)$$

The original formulation of the MED proposed by Wiggin’s (1978) uses a varimax norm that entails choosing $F(q)=q$. In this case, the function G is proportional to the square of y. To sum it all up, the original MED tries to shape the trace into a cubic version of itself. Other norms are possible as suggested in Sacchi et al. (1994).

Entropy functional and sparseness

The function $F(q_i) = q_i$ corresponds to Wiggins' entropy function. Logarithmic entropy function, on the other hand, is defined by $F(q_i) = ln\, q_i$. One can use norms that accentuate sparsity via the following functions $F(q_i) = q_i^2$ and $F(q_i) = q_i^3$ (quadratic and cubic entropy functions). However, the latter can lead to solutions where the relative reflection amplitude is not preserved. We investigate entropy functions (*n-root* functions) given by: $F(q_i) = q_i^{\frac{1}{2}}$, $F(q_i) = q_i^{\frac{1}{3}}$, $F(q_i) = q_i^{\frac{1}{4}}$ and $F(q_i) = q_i^{\frac{1}{5}}$. The iterative solution of equation (6) (Wiggins, 1978) leads to the sparse sequence y that should resemble the seismic reflectivity. Figure 1 portrays the non-linearity that is imposed upon the seismic trace by MED (equation 7) for different entropy functions. The horizontal axis indicates all possible values of the reflection coefficients versus b for different entropy functions. Figure 2 illustrates the non-linearity for different n-th root entropy functions. Figure 1 shows that for values of $y < 0.2$, b is close to zero for Wiggins’ entropy function and, for the quadratic and cubic entropy functions. This means that the algorithm will not be able recover small reflection coefficients. On the other hand, the logarithmic entropy function introduces an amplitude deformation including a polarity reversal. An ideal functional should have a linear behavior in order to do not introduce distortions in the relative strength of the reflection coefficients. However, such a function will not serve our purposes because the deformation is needed to “create” sparsity and therefore, to transform the trace into a reflectivity series. The n-*th root* functions in Figure 2 show a close to linear behavior and consequently, one expects less amplitude distortion in the recovered reflectivity.

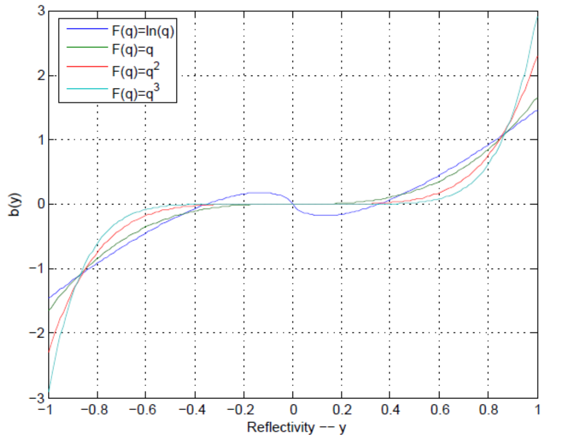

Figure 1. Amplitude deformation imposed by the MED method for logarithmic and polynomial entropy functions.

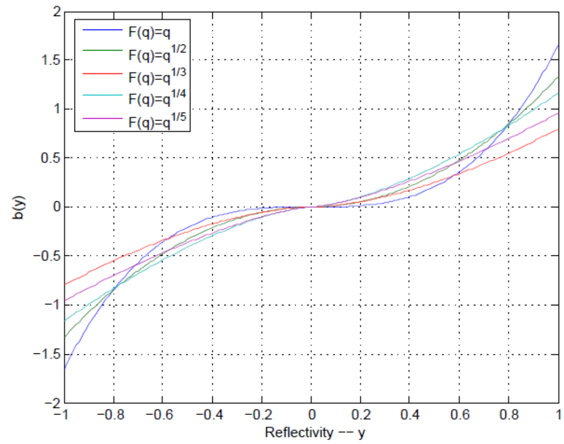

Figure 2. Amplitude deformation imposed by the MED method for different *n-th* root entropy functions.

In figure 3 we illustrate a synthetic example where we compare deconvolution outputs obtained with different entropy functions. In every simulation the filter length and initial filter was kept unchanged in order to examine the behavior of the deconvolution exclusively with respect to the entropy function. The goal is to find a resolution compromise: reducing sparsity and the generation of high frequencies leads to better preservation of the relative strength of the reflection coefficients.

The reflection strength is better preserved for the logarithm norm and for the *n-th root* function than for the polynomial functional. The visual differences are minimal; a careful examination, however, shows important differences. For instance, the distortion for the reflector at 0.2 seconds was computed by the evaluating the true estimated reflection coefficient amplitude ratio. The latter for the logarithmic norm and for the *n-th root* norm is about 80% whereas for Wiggins' norm and for the cubic norm is about 50%.

Discussion and summary

We have investigated the influence of the entropy functional on the estimation of reflection coefficients via MED. This study confirms that entropy norms that are too sensitive to sparsity might not be ideal for seismic deconvolution, as they tend to introduce heavy distortions in the estimators of the reflection coefficients. On the other hand, entropy functions that are better adapted to estimate seismic reflectivity could be obtained by looking at the behavior of the non-linear transformation that maps data into high frequency data in the MED algorithm (equation 7). It is clear that this analysis can be used to find optimal norms to estimate the seismic reflectivity. The latter is the current focus of our research.

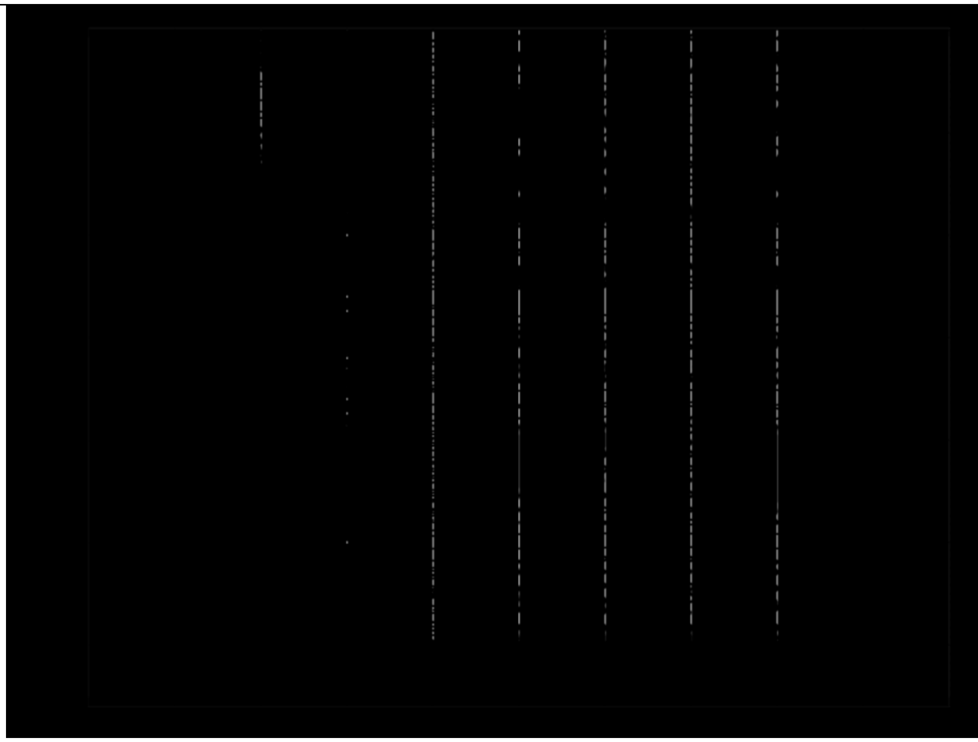

Figure 3. A simulation demonstrating the influence of the entropy function on the final estimation of the reflectivity. The reflectivity is indicated by r, the seismogram by s. The symbol ln indicates the MED method with a logarithmic norm. The symbol $n=1$ corresponds to Wiggins entropy function. The cubic entropy function is displayed with the symbol $n=3$. The n-th root solution is given by $n=1/3$. In short, $f(q)=ln(q)$ and $f(q)=q^n$ were tested.

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