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A Non-Linear Conjugate Gradient (CG) Method to Solve for Vp, Vs and Density from Seismic Elastic Impedance

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Summary

The concept of Elastic Impedance (EI) was firstly introduced by Connolly (1999) and has been applied for fluid and lithology predictions in petroleum industry. Because of a strong dependency to incident-angle, this makes a practical use of Connolly's EI formula difficult. Several alternatives or revisions of the original Connolly's EI formula have been proposed in recent years. For examples, Whitcombe (2002) derived a dimensionless version of the EI formula, which will normalize the EI dataset with different angle. Lu and McMechan (2004) presented two algorithms to estimate P-impedance and S-impedance from the EI data. It is still difficult to use all available angle-dependent EIs for lithology recognition and hydrocarbon interpretation. In this paper, we presented a non-linear Conjugate Gradient (CG) method to solve the EI equation from all available EI gathers to estimate compressional velocity, shear velocity and density, and then reconstruct other rock properties, such as v_p/v_s ratio and Poisson's ratio, which allows a better use of EI information in oil and gas industry.

Method

According to Connolly, EI depends on incident angle (θ)

$$EI(\theta) = v_p^{(1+tan^2\theta)} \, v_s^{(-8ksin^2\theta)} \, \rho^{(1-4ksin^2\theta)}$$

where v_p is compressional velocity, v_s is shear velocity and ρ is density and k is an empirically calibrated constant. Because the EI depends strongly on incident angle, this makes the practical application difficult, though Connolly (1999) found that EI(30) is similar to the acoustic impedance. However, the relationship among the compressional velocity, shear velocity and density described in Connolly's EI equation can be reformulated to estimate from all available EI angle gathers.

Taking the logarithm of EI equation and rearranging the terms, we have:

$$\mathrm{ei}_t(\theta) = \ln\big(EI(\theta)\big) = (1 + tan^2\theta)\ln(v_p) - 8ksin^2\theta\ln(v_s) + (1 - 4ksin^2\theta)\ln(\rho)$$

We define a cost function of the EI equation as follows:

$$\mathrm{f}(v_p, v_s, \rho) = \sum_{i=N_1}^{N_2} \frac{\|ei_t(i) - ei_s(i)\|^2}{\sigma_i^2}$$

subject to $v_p^l \leq v_p \leq v_p^u$, $v_s^l \leq v_s \leq v_s^u$ and $\rho^l \leq \rho \leq \rho^u$

where i is incident angle from $N_1$to N_2, σ_i is EI variance of each angle, which can be calculated from EI inversion results. ei_t is true EI estimated from EI formula and ei_s is elastic inversion result from angle seismic gathers. v_p^l compressional velocity lower bound, v_p^u compressional velocity upper bound . v_s^l shear velocity lower bound, v_s^u shear velocity upper bound. ρ^l density lower bound, ρ^u density upper bound.

The basic problem of EI we consider is:

minimize f(x)

subject to $a \leq c(x) \leq b$

Where $f(x)$ and $c(x)$ are assumed to be twice continuously differentiable and the problem of EI is convex if the f is convex and the c is concave. The x is the vector of v_p, v_s, ρ, which need to be estimated from EI gathers. a and b are the lower and upper bound of x.

The gradient of function $f(x)$ at point x_k is denoted by $g(x_k)$ or g_k for the sake of simplicity. The iterative formula of nonlinear conjugate gradient method is given by:

$$x_{k+1} = x_k + \alpha_k d_k$$

Where α_k is a step-length which can be estimated using the golden section search method, and d_k is a search direction which is determined by:

$$d_k = -g_k + \beta_k d_{k-1}$$

where β_k is a scalar. There have been many well known algorithms for the scalar, β_k, such as Fletcher-Reeves method:

$$\beta_k = \frac{\|g_k\|^2}{\|g_{k+1}\|^2}$$

The non-linear CG algorithm for solving EI equations as follows:

Step1: given initial v_{p0}, v_{s0}, ρ_0 and estimate g_0 and $d_0 = -g_0$

Step2: if $\|g_{k+1}\| \leq \epsilon$ then stop

Step3: golden section search to estimate α_{k+1}

Step4: estimate x_{k+1} using Elastic Impedance formula

Step5: calculate the gradient g_{k+1} using Elastic Impedance formula

Step6: estimate β_{k+1}

Step7: calculate the search direction d_{k+1}

Step8: repeat step 2 to step 8 until stop criterion meets

Example

Figure 1 is two synthetic EI gathers which are generated using Connolly's EI formula from two well logs. The angle range is from 0 to 45 degree on each colormap, in which the far left trace is zero degree and right is 45 degree. The color represents the angle-dependent Elastic Impedance from 0 to 45 degree.

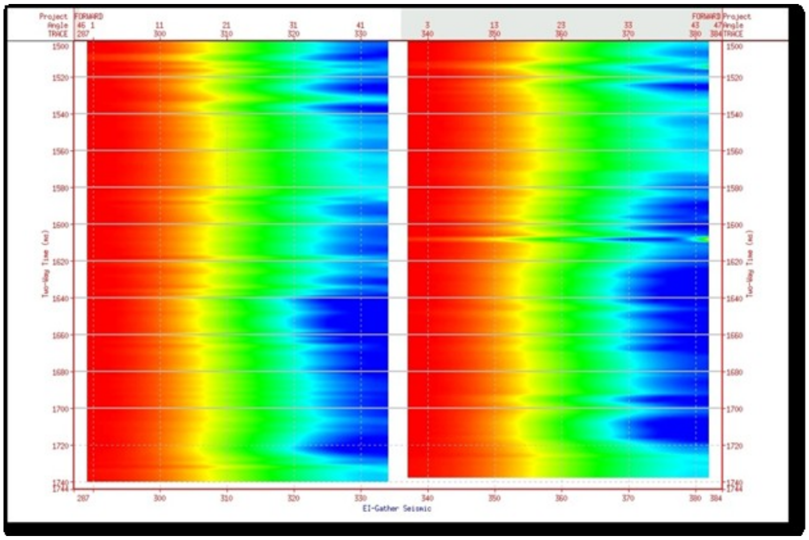

Figure 1: two EI gathers generated from two wells.

The initial model of compressional velocity, shear velocity and density for Conjugate Gradient (CG) optimization can be constructed using available low frequency well logs. In the examples, the optimization solutions (v_p, v_s, ρ) using the non-linear CG method match very well with the real dataset. Our future work may include a real dataset for testing the method to estimate the compressional velocity, shear velocity and density from EI gathers.

Conclusions

In this paper we presented a non-linear CG method to optimize compressional velocity, shear velocity and density from EI gathers, which will overcome the disadvantage of the angle-dependent EI gathers. Because our method can use all available EI gathers, robust and realistic rock properties can be estimated. Although more real dataset are needed to be tested in the near future, this opens a door for the more convenient use of the Elastic Impedance for lithology recognition and hydrocarbon studies in future.

References

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