Determining Elastic Constants of an Orthorhombic Material by Physical Seismic Modeling

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Summary

Analysis of group velocity measurements taken from a model made of laminate phenolic LE material, shown to possess orthorhombic symmetry, provides the characterization of the material through the determining of its elastic constants. Elastic constants are most often found from phase velocity measurements which for anisotropic media can be quite difficult. The P- and S-waves group velocity in different directions are measured from surface traveltimes picked from the scaled physical model data acquired over specific acquisition configurations. A simplified expression between the P-wave group velocity and elastic constants, has allowed for the estimation of off-diagonal elastic constants. Examining the P-, S_V-, and S_H-wave group velocity surfaces, we detected a mildly deformed type of elliptical anisotropy for the laminate phenolic material.

Introduction

An orthorhombic model has three mutually orthogonal planes of mirror symmetry, with three distinct directions. Vertical fractures in horizontal fine layering forms an equivalent orthorhombic medium (Schoenberg and Helbig, 1997); also, the transverse isotropy is just a degenerate case of orthorhombic symmetry with two distinctive directions. Elastic wave propagation in an orthorhombic medium can be characterized by its elastic constants, C_{ij}. Describing an orthorhombic medium, nine independent density normalized elastic constants ($A_{ij}= C_{ij}/\rho$) are required. For an orthorhombic solid, taking the symmetry axes of the sample as principal axes, three P-wave velocities along principal axes determine the A_{ii} ($i=1:3$); three S-wave velocities also along the principal axes determine A_{ii} ($i=4:6$). The off-diagonal elastic constants (A_{23}, A_{13}, A_{12}) cannot be determined from the P-wave or S-waves velocity measurements independently.

The velocity measurements have be done using the pulse through transmission technique on sets of cylindrical cores cut at particular angles of 0^o, 90^o, and 45^o to the layering by many researchers. In this method, transducers are attached to the flat ends of the cores and knowing the dimensions of sample cores, the firstbreak traveltimes provide the P- and S-wave phase velocity, wavefront velocity, with the source and receiver transducer width comparable to the source-receiver distance. Mah and Schmitt (2001) employing the τ-p transform, measured the phase velocities in a variety of directions in an orthorhombic material to determine the nine elastic constants. In the above mentioned experiments, the essential assumption is that the recorded firstbreaks represented phase velocities. However, the phase velocity is sometimes difficult to measure experimentally; for rock samples with dimensions of few times larger than transducer size, the measured velocity is energy propagation velocity which is the group velocity.

We present a method to determine all nine elastic constants of the phenolic LE material using a set of group velocity measurements. The group velocity measurements have been utilized by a physical modeling experiment carried out within the CREWES project at the University of Calgary.

Theory

The linear expression for P phase velocity in geophysics discipline comes from the work of Backus (1965) as: $\rho v^2(\mathbf{n}) = c_{ijkl} n_i n_j n_k n_l$ or equivalently $v^2(\mathbf{n}) = a_{ijkl} n_i n_j n_k n_l$, where ρ is density, $\mathbf{n} = (n_1, n_2, n_3)$ is a unit vector in the phase direction, and $a_{ijkl} = c_{ijkl}/\rho$. Backus's equation for an orthorhombic medium reduces to

$$v^2(\mathbf{n}) = A_{11} n_1^4 + A_{22} n_2^4 + A_{33} n_3^4 + 2(A_{12} + 2A_{66}) n_1^2 n_2^2 + 2(A_{13} + 2A_{55}) n_1^2 n_3^2 + 2(A_{23} + 2A_{44}) n_2^2 n_3^2. \quad (1)$$

Daley and Krebes (2004) reorganized equation (1) as:

$$v^2(\mathbf{n}) = A_{11} n_1^2 + A_{22} n_2^2 + A_{33} n_3^2 + E_{12} n_1^2 n_2^2 + E_{13} n_1^2 n_3^2 + E_{23} n_2^2 n_3^2, \quad (2)$$

where the quantities E_{ij} are defined as:

$$\begin{aligned} E_{12} &= 2(A_{12} + 2A_{66}) - (A_{11} + A_{22}), \\ E_{13} &= 2(A_{13} + 2A_{55}) - (A_{11} + A_{22}), \\ E_{23} &= 2(A_{23} + 2A_{44}) - (A_{22} + A_{33}). \end{aligned} \quad (3)$$

The E_{12}, E_{13}, and E_{23} are linearized versions of anellipsoidal deviations terms in the (x_1, x_2), (x_1, x_3) and (x_2, x_3) planes. To clarify the meaning of the anellipsoidal deviation terms, ellipsoidal anisotropy must be defined. Ellipsoidal anisotropy refers to the assumption regarding approximating the non-spherical wavefront in anisotropic media to an ellipsoid. The relation between the phase velocity and elastic constants, as in equation (1) has been used by many researchers to invert for off-diagonal elastic constants. However, the major problem is the fact that the measured velocity from traveltime data is more likely the group velocity and not phase velocity. Unfortunately, group velocity direction is not, in general, the same as the phase velocity direction; only for extreme directions (including vertical or horizontal propagation) does group velocity equals phase velocity.

For an orthorhombic medium, Daley and Krebes (2006) obtained a reasonably accurate approximation for P-wave group velocity from the eikonal equation. They obtained an expression for P-wave group velocity in an orthorhombic medium as

$$\frac{1}{V^2(\mathbf{N})} \cong \frac{N_1^2}{A_{11}} + \frac{N_2^2}{A_{22}} + \frac{N_3^2}{A_{33}} + \frac{E_{12} N_1^2 N_2^2}{A_{11} A_{22}} + \frac{E_{13} N_1^2 N_3^2}{A_{11} A_{33}} + \frac{E_{23} N_2^2 N_3^2}{A_{22} A_{33}}. \quad (4)$$

The measurements of P- and S-wave group velocities in principal directions determine the diagonal elastic constants A_{ii}. Determining the off-diagonal elastic constants A_{ij}, the group velocity expression (equation (4)) given the P-wave group velocity measurements in arbitrary directions will be least-squares inverted for anellipsoidal deviation parameters (E_{12}, E_{13}, E_{23}). Finally, using the relationships between anellipsoidal deviation parameters and the A_{ij}'s (equation (3)), the off-diagonal A_{ij} constants will be calculated.

Physical modeling experiment

The physical modeling system is designed to carry out simulated seismic surveys on scaled earth-models. Our physical modeling experiment has a scale (1:10000) for distance, and scale of (10000:1) for frequency. Flat-faced piezoelectric cylindrical transducers are used as both source and receivers. The physical modeling experiment has been done on a model of LE-grade phenolic material. Phenolic LE material is composed of laminated sheets of linen fabric, with alternating fabric sheets bonded with phenolic resin. Our phenolic model has an area of 5740 m×5740 m and a thickness of 701 m, see Figure (1).

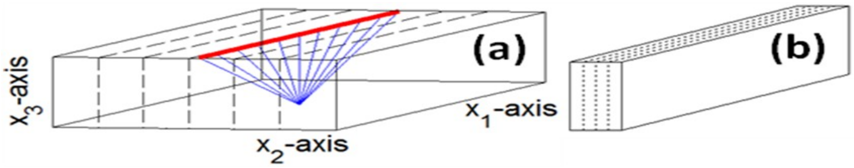

Figure 1: (a) Our model consisted of six slabs of phenolic glued together. The transmission profile with wave propagation in the symmetry plane of (x_1,x_3) is shown as an example. (b) A slab of phenolic material with dash lines displaying the linen planes.

Group velocity measurements

Several transmission shot gathers have been acquired to facilitate the P- and S-wave group velocity measurements. In the transmission shot gathers the source and receivers are located on opposite sides of the phenolic model or the source and receivers are at the same side of the model with the source in a vertical distance from the receiver profile. In these shot gathers the traveltime of the direct arrival, from the source to each individual receiver gives the group velocity in the direction of the source-receiver raypath. Considering our model as a homogeneous layer, the source-receiver raypath is a straight line connecting them; knowing the coordinates of source and receivers, the path length and angle of incidence are determined using basic trigonometry. The direct arrival from the source to receivers is represented by firstbreak event; for every trace, the group velocity is the source-receiver path length divided by firstbreak pick time. Note, the effective path length is, in fact, shorter than the nominal distance between transducer centers. The piezoelectric transducer generates the seismic waves along its entire element width; hence elective path length equals to the distance between nearest edges of transducer.

In the transmission experiment, receiver profiles initially located along the x_1- and x_2-axis at the top face of model, with source at the bottom face gives group velocities in different directions on the principal (x_1,x_3), see Figure (1), and (x_2,x_3) planes. The group velocities of the (x_1,x_2) plane are measured from receiver profiles along the x_1- and x_2-axis at the top surface with the source also at the top surface with the source-receiver vertical distance of 1000 m. Additionally, the receiver profiles along $\pm 45^o$ azimuth lines provide group velocities in $\pm 45^o$ azimuth planes. The velocity measurements is done using the vertical component data to provide the P-wave group velocity, followed by radial and transverse component data to obtain the S-waves group velocity. See Figure (2) for three-component data acquired over the receiver profile shown in Figure (1).

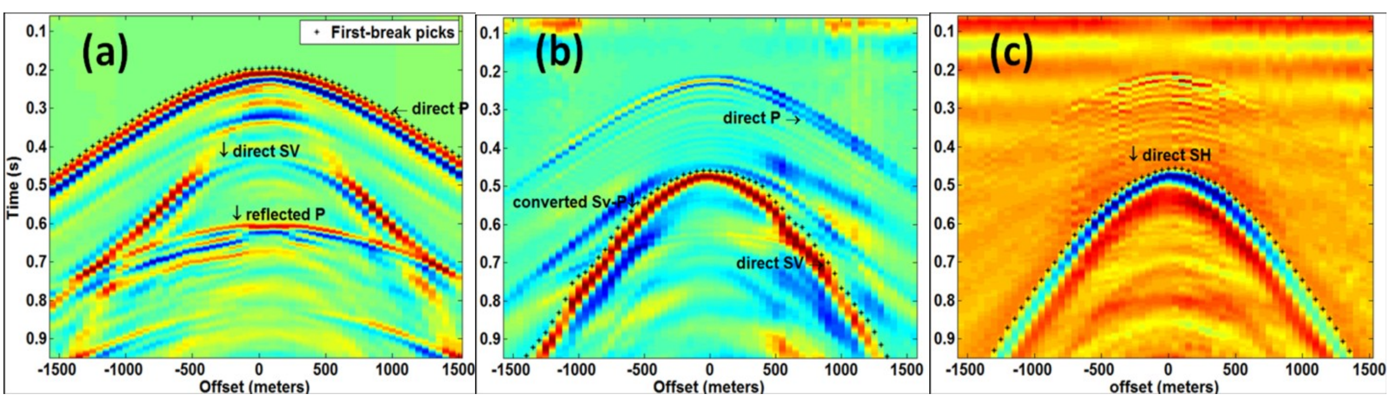

Figure 2: (x_1,x_3) plane of propagation . (a) Vertical-component data. (b) Radial-component data. (c) Transverse-component data.

The polar graph of the measured group velocity versus propagation angle, **group velocity surface,** for the symmetry planes of (x_1,x_3), (x_2,x_3), (x_1,x_2), and also in $\pm 45^o$ azimuth planes are examined for all three modes. The group velocity surface is seismic wavefront at unit time. Figure (3) shows the P-, S_V-, S_H-wave velocity surfaces for the symmetry plane of (x_1,x_3). To examine how elliptical anisotropy assumptions can describe our phenolic material, an elliptical wavefront was plotted for each group velocity surface. For each plane the measured velocity in the zero and 90° directions, are considered as the major and minor axes of the ellipse. For all of the examined planes of propagation the group velocity surface almost follows the ellipse with negligible deviation, indicating the adequation of elliptical anisotropy in describing the phenolic material anisotropy.

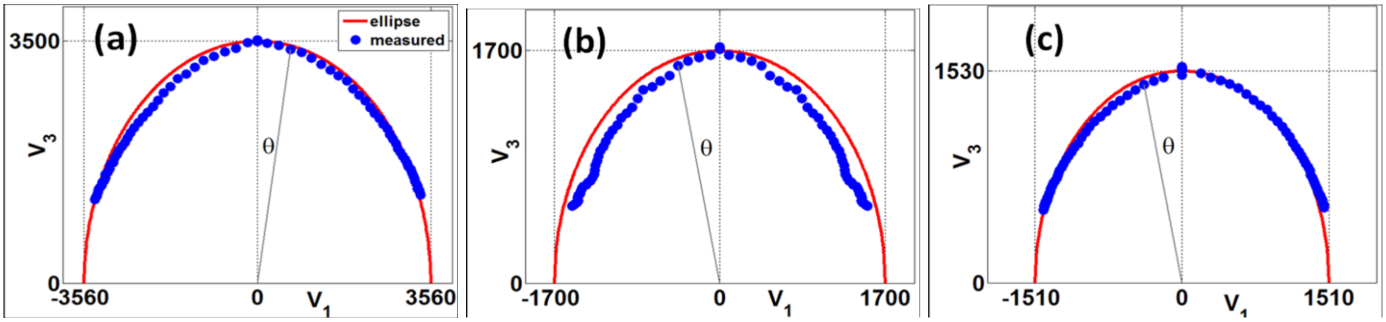

Figure (3): (x_1,x_3) plane of propagation. (a) P-wave group surface. (b) S_V-wave group surface. (c) S_H-wave group surface.

The measurements of P-wave velocity becomes the data vector in a least-squares inversion to estimate the (E_{23},E_{13},E_{12}). Finally using equation (3), the off-diagonal density normalized elastic constants were calculated. Table (1) shows all nine density normalized elastic constants of the phenolic LE model. The Thomsen parameters (ε, δ, γ) are calculated from elastic constants and presented in Table (2).

Table (1): Density normalized elastic constants of the phenolic LE model. The A_{ij} have the units of kg/s.

12.67±0.006	6.13±0.003	6.68±0.003	0	0	0
	8.70±0.006	5.79±0.003	0	0	0
		12.25±0.006	0	0	0
			2.34±0.001	0	0
				2.89±0.001	0
					2.28±0.001

Table (2): Thomsen anisotropy parameters for the phenolic LE model.

	ε	γ	δ
(x_1,x_3) plane	0.0173	-0.0130	0.0175
(x_2,x_3) plane	-0.1448	-0.1055	-0.1318
(x_1,x_2) plane	-0.1567	0.1173	-0.1417

Conclusions

The simplified P-wave group velocity expression in terms of elastic constants by Daley and Krebes (2006) is very useful because its simplicity of form aids in a robust calculation of elastic constants in an orthorhombic material. This method can be applied in the ultrasonic laboratory experiments, and maybe applicable to seismic data in a borehole environment to calculate elastic constants. Examining the P- and S-wave group velocity surfaces revealed that the laminate phenolic material has elliptical anisotropy. The Thomsen parameters values calculated from elastic constants showed that the phenolic LE model has week anisotropy with very close values for the ε and δ parameters.

Acknowledgements

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