De-aliased Cadzow Reconstruction

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Summary

We introduce a strategy for beyond-alias interpolation of seismic data using Cadzow method. First, in the frequency-space (*f*-*x*) domain, a Hankel matrix is built from the spatial samples of the low frequencies. To perform interpolation at a given frequency, the spatial samples are interlaced with zero samples and another Hankel matrix is built form the zero-interlaced data. Next, the rank-reduced eigenstate of the Hankel matrix at low frequencies is used for beyond-alias conditioning of the Hankel matrix at a given frequency. Finally, anti-diagonal averaging of the conditioned Hankel matrix gives the final interpolated data. Synthetic and real data examples are provided to examine the performance of the proposed interpolation method.

Introduction

Interpolation of seismic data has become a key step for seismic data processing. Interpolated data can boost the resolution and signal to noise ratio of the final subsurface image. One determining factor in the success of various seismic reconstruction methods is the sampling function. Irregular sampling scenarios are often favored by most of the signal processing based reconstruction techniques since they do not require to obtain a finer underlying grid. On the other hand, regular sampling functions aim to up-sample the data and decrease the grid size of data. The latter becomes impossible for the signals that are not band-limited. However, it is possible to achieve a beyond-alias spatial up-sampling of seismic data if one can deploy a multi-frequency strategy.

The solution for beyond-alias interpolation of seismic records first proposed by Spitz (1991) in the frequency-space (*f*-*x*) domain. In order to interpolate the spatial samples at a given frequency Spitz used the estimated prediction filter at the correspondent half frequency. This approach is valid if the events in the time-space (*t*-*x*) domain are linear and the amplitude of the wavelet remains constant laterally. Gulunay (2003) utilize the same principle and proposed a beyond-alias interpolation method in the frequency-wavenumber (*f*-*k*) domain. Later, Naghizadeh and Sacchi (2007) extended Spitz' method to irregularly sampled data by introducing multi-step autoregressive (MSAR) algorithm.

Recently, a new category of seismic data reconstruction using rank-reduction techniques has been proposed by Trickett et al. (2010), and Oropeza and Sacchi (2011). These methods are known as Cadzow or singular spectrum analysis (SSA). The Hankel matrix of a signal composed of few harmonics are deemed to be low rank. Therefore, by a rank-reduced singular value decomposition of the Hankel matrix and anti-diagonal averaging of its elements one can recover the randomly missing samples of a signal. The singular values created by random sampling are small and can be easily distinguished from the ones that belong to original data. However, for regularly missing samples the rank-reduction technique can not distinguish between the singular values associated with the original and decimated signals. In this article we aim to alleviate this problem by extending the de-aliasing technique from Spitz (1991) interpolation method to rank-reduction methods.

The article is organized as follow. First we review the principles of the Cadzow/SSA interpolation method. Next, we develop the theory for alias-free Cadzow interpolation of seismic records. Finally, we use the synthetic and real data examples to show the effectiveness of the proposed de-aliasing technique.

Theory

Let's $\mathbf{d}(f)$ represent the N spatial samples of the frequency f in *f-x* domain. A Hankel matrix $\mathbf{M}_r$ built from the data vector $\mathbf{d}(f)$ can be represented as

$$\{\mathbf{M}_r|\mathbf{d}(f)\} = \begin{pmatrix} d_1(f) & d_2(f) & d_3(f) & \cdots & d_{N-r+1}(f) \\ d_2(f) & d_3(f) & d_4(f) & \cdots & d_{N-r+2}(f) \\ d_3(f) & d_4(f) & d_5(f) & \cdots & d_{N-r+3}(f) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ d_r(f) & d_{r+1}(f) & d_{r+2}(f) & \cdots & d_N(f) \end{pmatrix}. \quad (1)$$

where r represents the number of rows in Hankel matrix. It can be proved that for a seismic section composed of k linear events with distinct dips the Hankel matrix of each frequency component in the *f-x* domain is rank k. This property of Hankel matrices allows deploying rank reduction techniques for de-noising and interpolation of seismic data (Trickett, 2003; Oropeza and Sacchi, 2011). However, rank-reduction interpolation only works with randomly sampled data. Spitz (1991) proposed a strategy to extract prediction filters from low frequencies for beyond-alias interpolation of high frequencies. Here, we propose a similar rationale for removing alias using rank-reduction algorithms.

In order to decrease the spatial sampling interval by a factor of α, namely moving from Δx to $\frac{\Delta x}{\alpha}$, we first apply singular value decomposition to the Hankel matrix of data at frequency f/α

$$\{\mathbf{M}_r|\mathbf{d}(f/\alpha)\} = \mathbf{U}_r^\alpha \Sigma_r^\alpha \mathbf{V}_r^\alpha. \quad (2)$$

Next, we define an up-sampling matrix $\mathbf{G}$ such that

$$\mathbf{G} = \begin{pmatrix} \Delta & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \Delta & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \Delta & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \Delta \end{pmatrix}_{\alpha N \times N}, \quad (3)$$

where $\mathbf{0} = [0,0,0,\cdots,0]^T_{1\times\alpha}$, $\Delta = [1,0,0,\cdots,0]^T_{1\times\alpha}$, and superscript T represents the transpose of vector. The k rank-reduced Hankel matrix of data vector at frequency f is obtained by

$$\mathbf{Q}_{r,k}(f) = \mathbf{U}_{r,k}^\alpha \mathbf{U}_{r,k}^{\alpha\ H}\{\mathbf{M}_r|\mathbf{G}\mathbf{d}(f)\} \quad (4)$$

where $\mathbf{U}_{r,k}^\alpha$ represents the first k columns of $\mathbf{U}_r^\alpha$ and superscript H is the hermitian transpose operator. Next, the interpolated spatial data at frequency f is obtained by

$$\mathbf{d}_{int}(f) = \mathscr{A}(\mathbf{Q}_{r,k}), \quad (5)$$

where $\mathscr{A}$ is an operator that averages the matrix elements in anti-diagonal direction. At this stage the amplitudes of interpolated data will be less than the amplitudes of the original data because of the addition of $\alpha - 1$ zero values between each available spatial data samples. In order to alleviate the amplitude loss we deploy an iterative routine by reinserting the original available samples back to the interpolated data. The reinsertion algorithm can be summarized as

$$\begin{aligned} &\text{Initialization} \\ &\mathbf{d}_{int}^0(f) = \mathbf{G}\mathbf{d}(f), \\ &\text{For} \quad i = 1,2,3,\ldots,n_{iter} \\ &\quad \mathbf{Q}_{r,k}^i = \mathbf{U}_{r,k}^\alpha \mathbf{U}_{r,k}^{\alpha\ H}\{\mathbf{M}_r|\mathbf{d}_{int}^{i-1}(f)\}, \\ &\quad \mathbf{d}_{int}^i(f) = \mathbf{L}[\mathscr{A}(\mathbf{Q}_{r,k}^i)] + \mathbf{G}\mathbf{d}(f). \\ &\text{End} \end{aligned} \quad (6)$$

where matrix $\mathbf{L}$ is defined as

$$\mathbf{L} = \begin{pmatrix} \mathbf{1}-\Delta & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{1}-\Delta & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1}-\Delta & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{1}-\Delta \end{pmatrix}_{\alpha N \times N}, \tag{7}$$

on which $\mathbf{1} = [1, 1, 1, \cdots, 1]^T_{1\times\alpha}$. The parameter n_{iter} represents the number iterations. In algorithm 6 at each iteration the original available data samples are reinserted into the interpolated data. Applying the above formulas for all of the frequencies in the *f-x* domain results in a beyond-alias rank-reduction interpolation method.

Examples

to examine the performance of proposed reconstruction method we created a synthetic seismic section composed of three linear events in Figure 1a. Figure 1b shows the original data after interlacing three zero traces between each pair of available traces. Figure 1b shows the interpolated data using the de-aliased Cadzow reconstruction method. Figures 1d-f show the *f-k* spectra of the data in Figures 1a-c, respectively. It is clear from the *f-k* spectra plots that the original has been successfully de-aliased by the proposed method.

Figure 2a shows an original real shot gather from a land survey. The interpolated data using de-aliased Cadzow reconstruction method is shown in Figure 2b. The data was interpolated using small spatial windows composed of eight traces with three overlapping traces between the adjacent windows. The amplitude fidelity of the interpolated and original traces is good for most of the events. Deploying a time windowing scheme can further improve the interpolation results.

Conclusion

We introduced a strategy for beyond-alias interpolation of seismic records using rank-reduction methods. The method entails extracting the rand-reduced eigenstates from the low frequency portion of the data and using them to remove the aliased energy from high frequencies. Synthetic and real data examples show the effectiveness of the proposed reconstruction method.

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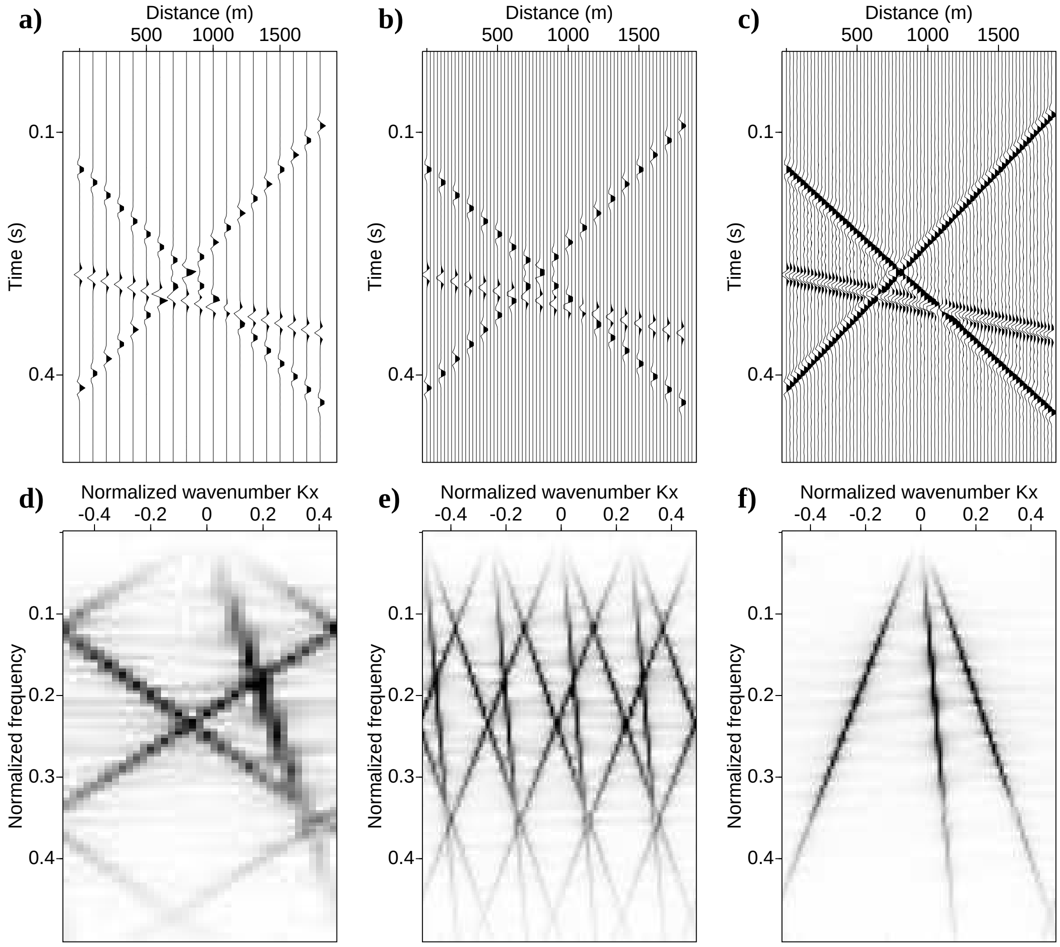

Figure 1 *: a) Original synthetic seismic record. b) The data after interlacing 3 zero traces between available traces. c) Interpolated data using de-aliased Cadzow reconstruction method. d-f) are the f-k spectra of a-c.*

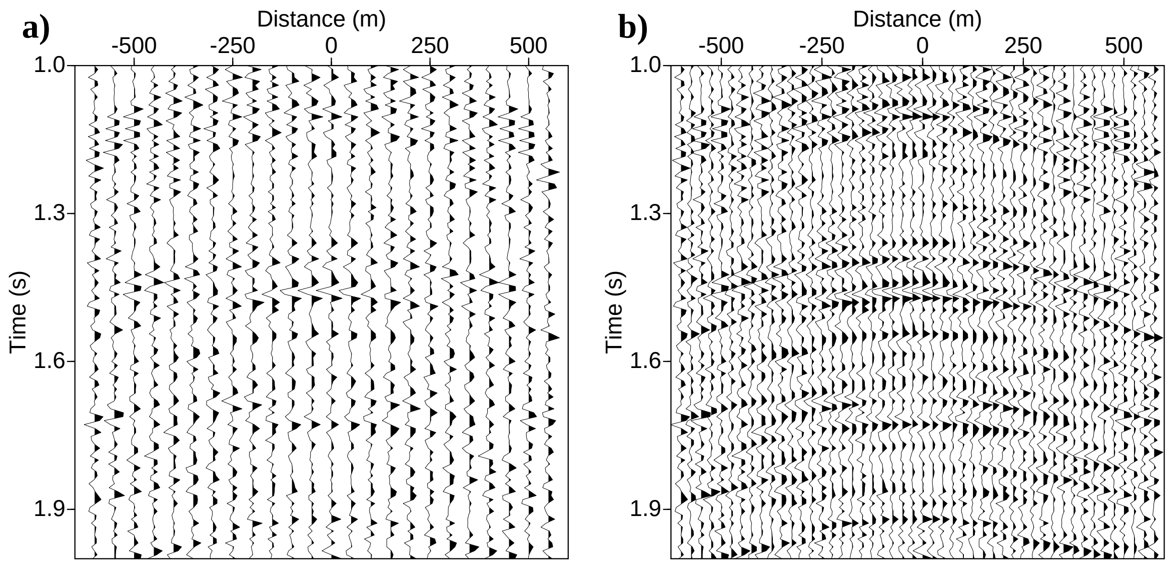

Figure 2 *: a) Original shot gather from a land survey. b) Interpolated data using de-aliased Cadzow reconstruction method..*