Migration velocity analysis of incomplete data using LSPSM CIGs

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Summary

Migration velocity analysis by measuring coherencies on common image gather (CIG)s is extended to the least squares prestack migration (LSPSM) CIGs. It is shown that when the data are incompletely or irregularly sampled, the LSPSM shot domain CIGs give higher resolution in the unnormalized crosscorrelation panel for choosing the best velocity than conventional migration CIGs. The velocity information extracted by these methods is accurate enough to give a good convergence in the least squares conjugate gradients and also a good reconstruction of missing data.

Introduction

The ability of easily handling incompletely sampled seismic data is probably the main advantage of the prestack Kirchhoff to the other methods of migration. However, incomplete data produce migration artifacts and give a blurred image of the earth subsurface reflectivity. This shortcoming of Kirchhoff can be improved by using a generalized inverse as an approximation to the exact inverse (Tarantola, 1984). This approach is called least squares migration (Nemeth et. al., 1999, Duquet et. al., 2000).

Kirchhoff seismic modelling can be expressed in the general form of:

$$\boldsymbol{d} = \boldsymbol{G}\boldsymbol{m}, \tag{1}$$

where $\boldsymbol{d}$ is the observed seismic data, $\boldsymbol{m}$ is the earth reflectivity model and $\boldsymbol{G}$ is an operator that contains diffraction information. The inverse process recovers the earth model or reflectivity from the seismic data. Since matrix $\boldsymbol{G}$ may not be square or non-invertible or it may be extremely large, calculating the inverse of $\boldsymbol{G}$ if it is not impossible, will be difficult. Thus, approximations to the exact inverse can be used. The first approximation uses the transpose (adjoint) of $\boldsymbol{G}$:

$$\widehat{\boldsymbol{m}} = \boldsymbol{G}^{\mathbf{T}}\boldsymbol{d}, \tag{2}$$

where $\widehat{\boldsymbol{m}}$ is the migrated image and $\boldsymbol{G}^{\mathrm{T}}$ is the transpose of $\boldsymbol{G}$ or the migration operator. In the LSPSM, the difference between the observed data, $\boldsymbol{d}$, and the modeled data, $\boldsymbol{G}\widetilde{\boldsymbol{m}}$, expressed by $|\boldsymbol{G}\widetilde{\boldsymbol{m}} - \boldsymbol{d}|$, where $\widetilde{\boldsymbol{m}}$ is an approximation to $\boldsymbol{m}$, is minimized. In the general, an objective function in the following form is minimized (Nemeth et. al., 1999):

$$J(\widetilde{\boldsymbol{m}}) = \|\boldsymbol{G}\widetilde{\boldsymbol{m}} - \boldsymbol{d}\|^2 + \mu^2\mathcal{R}(\widetilde{\boldsymbol{m}}). \tag{3}$$

where $\mathcal{R}(\widetilde{\boldsymbol{m}})$ is the regularization function and μ is the regularization weight. Minimum norm is the simplest form of the regularization function, ($\mathcal{R}(\boldsymbol{m}) = \|\boldsymbol{m}\|_2^2$), which leads to the damped least squares solution, $\boldsymbol{m}_{\boldsymbol{DLS}}$, to the problem:

$$\boldsymbol{m}_{\boldsymbol{DLS}} = (\boldsymbol{G}^{\mathrm{T}}\boldsymbol{G} + \mu^2\boldsymbol{I})^{-1}\boldsymbol{G}^{\mathrm{T}}\boldsymbol{d}. \tag{4}$$

Nemeth et. al. (1999) showed that LSPSM can be used to reduce the migration artifacts in the seismic data. The resulted high resolution image can be used for the data reconstruction, as well.

The performance of least squares seismic inversion requires solving an usually large system of linear equations. The conjugate gradient (CG) method (Hestenes and Steifel, 1952) is an efficient solver and is widely used for solving the LSPSM problems. However, LSPSM requires accurate subsurface velocity information. The high sensitivity of LSPSM to the accuracy of velocity and its low sensitivity to the incompleteness and irregularities of data are the key points for using that for the migration velocity analysis of irregularly sampled data.

Migration and LSPSM velocity analysis

Any advanced imaging technique requires relatively accurate velocity information. In LSPSM Kirchhoff modelling and migration operators, $\boldsymbol{G}$ and $\boldsymbol{G}^T$, must be predefined. These operators need a good estimation of velocity function to perform migration and modelling as accurately as possible. Proper subsurface velocity information leads to a well-focused migration image. An inaccurate velocity model distorts the migration image. However, these distortions have some useful information about the velocity and can be used to update the velocity model used for imaging.

Migration velocity analysis can be implemented by performing constant velocity migration several times using a range of plausible velocities and creating many migration images. Then comparing best image focusing for each time, a migration velocity function at each CMP position is achieved. In a better way this can be done by doing velocity analysis on each migration CIG. This study extends the method to the LSPSM CIGs. Different types of CIG can be used for velocity analysis (Biondi, 2007). Our focus is on the shot domain CIGs in which the horizontal axis is the distance between image point and source. Shot domain CIGs can be can be easily obtained by prestack Kirchhoff migration. Same improvement achieved with using common offset CIGs (Yousefzadeh et. al., 2011).

To see the effect of the velocity accuracy on LSPSM, consider a case when the velocity used for LSPSM is %10 more than the true velocity. With the acquisition geometry in Figure 1a which shows 16 sources and 96 receivers per sources and with the velocity model in Figure 1b data are produced. The source interval is 187.5 m and receiver interval is 15.625 m and sampling rate is 2 ms. After adding %1 random noise, the data are decimated by removing %75 of the traces, randomly.

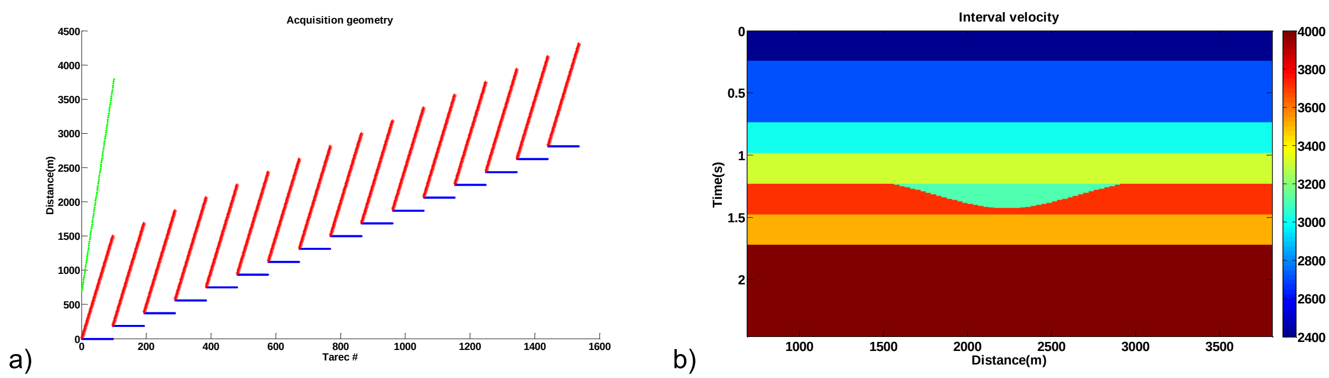

Figure 1. a) Acquisition geometry used to generate synthetic data. blue: sources, red: receivers, green: CMPs . b) Velocity model used to create synthetic data.

With these irregularly sampled data, LSPSM produced a high resolution image (Figure 2a). LSPSM of complete data, with using %10 higher than the true velocity is shown in Figure 2b. LSPSM not only did not improve the image resolution, but also introduced more noise to the resulted image if compared with the corresponding migration.

Figure 3 compares the convergence rate of the CG method when the true velocity is used in LSPSM when implemented velocity is %10 higher or lower than the true velocity. The residuals converges to %11 in three iterations when using true velocity. The residuals will not be less than %75 with %10 higher velocity and %85 with %10 lower velocity is used in the LSPSM.

Velocity analysis on LSPSM Shot domain CIGs

We showed that the resolution, data reconstruction and the convergence rate of LSPSM strongly depend on the velocity accuracy. However LSPSM is less sensitive to the incompleteness of data. Therefore, LSPSM CIGs can be used as an effective tool for the velocity analysis of incomplete data. Due to strong incoherency of shot domain CIGs with using incorrect velocity, the semblance which is an efficient method for measuring coherencies in the CMP gather and common offset CIGs, is not an appropriate tool for velocity estimation of the shot domain CIGs. We found the “unnormalized crosscorrelation sum” (Yilmaz, 2008) (XC), as a useful quantity to measure the coherency in the shot domain migration and LSPSM CIGs.

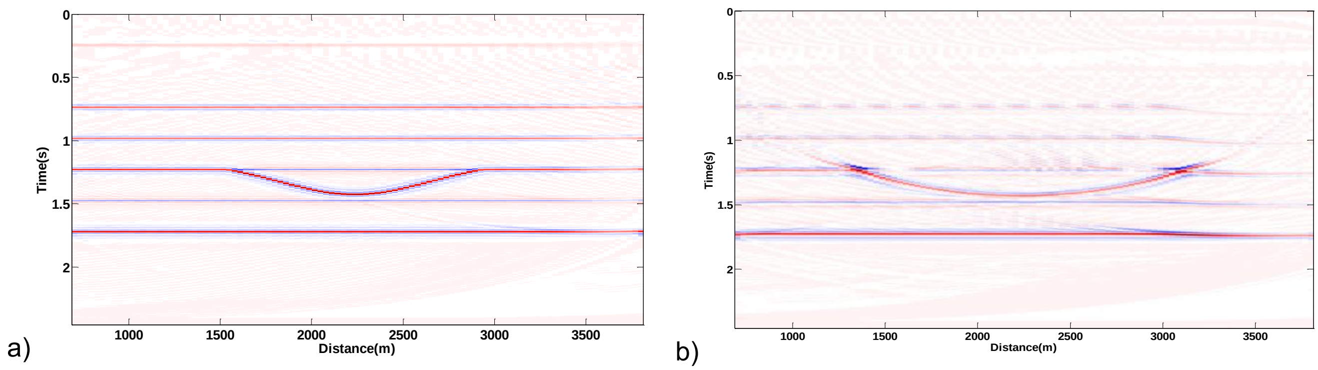

Figure 2: a) LSPSM image of %75 decimated data with the true velocity. b) LSPSM of complete data with the velocity %10 higher than the true velocity.

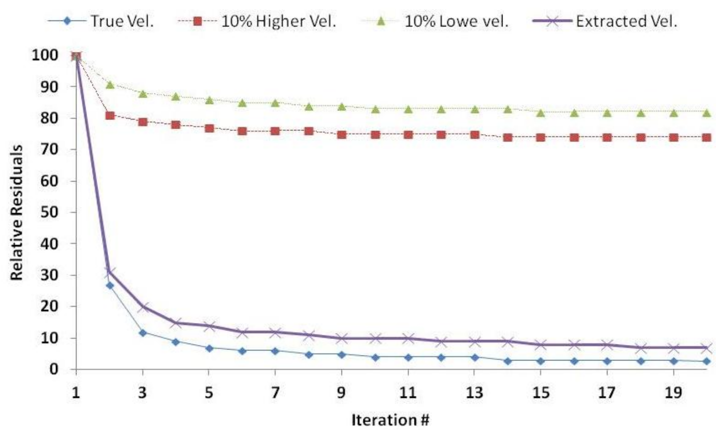

Figure 3: Convergence of LSPSM in 20 iteration with the true (blue), %10 more (red), and %10 less (green) and the extracted (purple) velocity.

Using constant velocities within a range that starts at 2000m/s and with the increment of 25m/s increases to 4500m/s, we performed constant velocity migration and LSPSM on the data and we achieved a XC spectrum for each CIG at each CMP. Figure 4 compares the XC spectrums for the migration and LSPSM shot domain CIGs, at the position $x = 900\,m$, at the edge of the model, and $x = 2250m$, at the middle of the model, when only %10 of data is used. The improvement in the resolution of XC spectrum for velocity analysis by using LSPSM shot domain CIGs instead of migration shot domain CIGs is noticeable.

With the extracted velocity using this method, LSPSM performed on data. Result is a high resolution image and the data reconstruction succeeds. The convergence rate of the LSPSM using true velocity and extracted velocity is compared in Figure 3. In one iteration LSPSM with the extracted velocity converge to %31 of the original difference where this amount with true velocity is %27. Finally in 20 iterations, the extracted velocity cause the convergence rate goes down to %7, only %3 more than when the true velocity is to do LSPSM.

Conclusions

Semblance method on the offset domain or unnormalized crosscorrelation on the shot domain migration CIGs can be used for velocity analysis in the area with dipping layers. We showed that doing velocity analysis on the migration CIGs, is a robust tool for migration velocity analysis can be extended to the LSPSM CIGs. This gives better result when the data are irregularly or incompletely sampled. The velocity extracted by this method is accurate enough to give a high resolution image in LSPSM and works well for the data reconstruction. The ability of data reconstruction and convergence rate of CG iteration are useful tools to measure the accuracy of the estimated velocity.

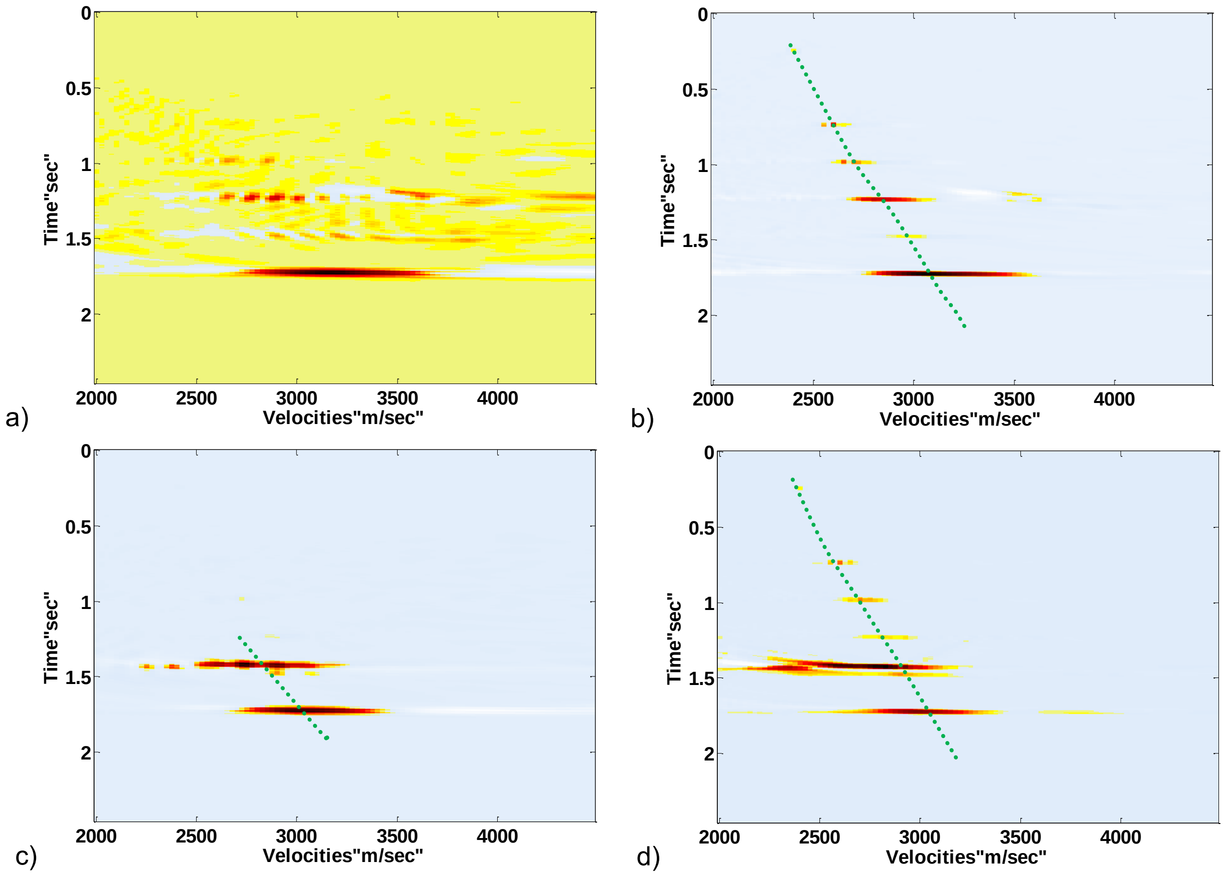

Figure 4: XC spectrum for the migration and LSPSM shot domain CIG with %10 of data. a) migration, b) LSPSM at $x = 900\ m$, c) migration, and d) LSPSM at $x = 2250\ m$.

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