Multi-scale 2D acoustic full waveform inversion with high frequency impulsive source

Vladimir N. Zubov*, University of Calgary, Calgary AB

vzubov@ucalgary.ca

and

Michael P. Lamoureux, University of Calgary, Calgary AB

mikel@ucalgary.ca

and

Gary F. Margrave, University of Calgary, Calgary AB

margrave@ucalgary.ca

Summary

The ability to combine together different grid scaling in full waveform inversion (FWI) is examined in the present study, using synthetic data for the acoustic wave equation problem with boundary. Different frequency impulsive sources are put together through domain decomposition with overlap to verify the ability of FWI sub-problems to maintain numerical stability while obtaining convergence through a Newton method root search applied to misfit data correlated with the forward propagation wave.

Introduction

A useful mathematical model in mathematical physics can often be based on fundamental conservation laws of the corresponding physical problem, ensuring the preservation of the most important properties of the problem such as symmetry, monotonicity and conservation of physical laws where it is possible. These properties give the advantage in further theoretic investigation of the mathematical problem and provide a solid base for stability studies and estimations for numerical solutions using iterative methods of linear algebra and numerical analysis, adapted for self-adjoint and positive definite linear problems.

In the present study, we begin with the acoustic wave equation problem (including boundary) written in the self-adjoint form, which provides numerical stability for both the forward and adjoint problem solutions using an implicit factorization scheme. This also ensures the mass conservation property for the medium while using numerically approximate spatial derivatives.

The FWI approach implemented in this study is based on a Newton method root search applied to the inner product of the misfit data and forward wave instead of misfit function norm minimization usually used in similar FWI studies (Virieux and Operto 2009).

Theory and/or Method

The mathematical model developed and studied here is based on fundamental results in wave equation FWI derived by Tarantola (1984) and based on the theory of adjoint operators. A simplifying assumption in this study is that the impulsive source is considered as known; generalizing to an unknown source needs future research and such work is not presented here. To summarize, without source estimation, the mathematical model we use in here consists of three key modules:

- Forward wave propagation with unconditionally stable factorization
- Backward (in time) wave propagation with factorization (adjoint problem)

- Newton method root search for a cross-correlation between the forward and adjoint problem solutions

A forward wave propagation boundary problem (equation 1) is formulated for the acoustic equation using a positive definite self-adjoint operator preserving these properties with Dirichlet boundary conditions. An impulsive point source is located away from the air-soil boundary, deep into the spatial area, to minimize any contribution of the non-absorbing boundary layer. The differential equations are given as:

$$\begin{cases} u_{tt} = div(k\nabla u) + f, \ x \in \Omega \subset R^2, t \in [0,T] \\ u(t=0) = 0, \ u(\vec{x} \in \partial\Omega) = 0 \end{cases} \tag{1}$$

where u represents the acoustic pressure as a function of time variable t and space variable x, $k = k(x)$ represents the physical properties of the transmitting medium which we wish to recover In the FWI, and the impulsive source function is given as a frequency modulated Gaussian localized in space, $f = \delta(\vec{x} - \vec{x}_0) \cdot e^{-\lambda^2(t-t_0)^2} \sin \omega(t-t_0)$.

This boundary problem is approximated with a 4th order finite difference scheme in space and 2nd order factorization scheme in time (Kalitkin 1978, Samarskii 2001):

$$\begin{aligned} (I - \tau^2\sigma\Lambda_1)(I - \tau^2\sigma\Lambda_2)u^{n+1} &= (2 + \tau^2(1 - 2\cdot\sigma)(\Lambda_1 + \Lambda_2))u^n \\ &- (1 - \tau^2\sigma(\Lambda_1 + \Lambda_2))u^{n-1} + \tau^2\sigma \cdot f^{n-1} + \tau^2(1-2\sigma) \cdot f^n + \tau^2\sigma \cdot f^{n+1} \end{aligned} \tag{2}$$

where Λ_1 and Λ_2 are differential analogs of spatial derivatives in corresponding spatial directions. These matrices are self-adjoint and positive definite and the factorization scheme is stable for $\sigma \in [0.25; 0.5]$.

The backwards propagation boundary problem corresponding to (equation 1) is given as

$$\begin{cases} f_{tt} = div(k\nabla f) + g, \ x \in W \subset R^2, t \in [0,T] \\ f(t=T) = 0, \ f(\vec{x} \in \partial W) = 0 \end{cases} \tag{3}$$

where the source function g is an arbitrary function in general, corresponding to the basis vector of the inverse problem. As many other researchers do, we further specify the source function to be misfit data limited with some window:

$$g = \begin{cases} \Delta d = d_{obs} - d_{cal}, \ t < t_0 \\ 0, t \geq t_0 \end{cases} \tag{4}$$

Cross-correlation between the forward and adjoint problem solutions is computed in the traditional way as a numerical integration

$$I_{t_0}(\vec{x}, k) = \int_0^{t_0} (\nabla u, \nabla \varphi)dt \tag{5}$$

The first order Newton method root search (Bjoerck and Dahlquist 2008) is implemented for a projection of the cross-correlation function on the spatial 1D curve Λ corresponding to the bottom line of the cross-correlation being a non-zero function so that $I_{t_0}(\vec{x}, k) = 0$ for each $\vec{x}$ below Λ in 2D and $I_{t_0}(\vec{x}, k) \neq 0$ for $\vec{x} \in \Lambda$. Thus, on each iteration

$$\Delta k = -\hat{J}(I_{t_0})^{-1} \cdot P_\Lambda I_{t_0} \tag{6}$$

where $\hat{J}$ is a diagonal of the Jacobean $\left(\partial I\!\left(\vec{x}_i\right)/\partial k_j\right)_{i,j}$ matrix and P_Λ is a projection operator on the Λ.

Examples

Computational experiments were implemented in order to verify numerically the analytical mathematical model on synthetic data.

The layer of soil in 2D region Ω from (equation 1) is split into 2 independent layers: a high resolution weather layer near the soil surface and low resolution layer deeper in the subsurface. The solution for the weather layer inversion problem is implemented using a higher frequency impulsive source (Fig 2a)) and layer below is inversed with lower frequency impulsive source (Fig 2b)).
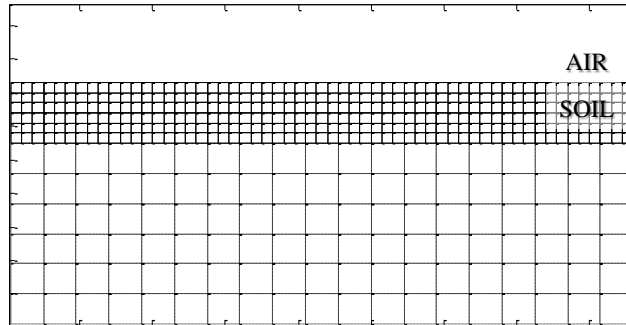

Figure 1: Spatial grid for velocity coefficients $k\!\left(\vec{x}\right)$ in soil layer.

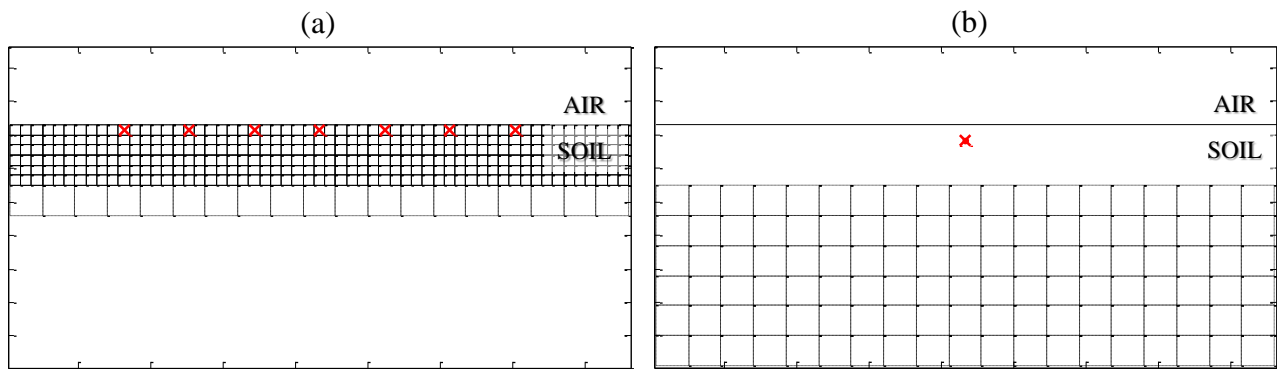(a)   (b)

Figure 2 Spatial domain decomposition with corresponding impulse sources positions (x):
a) – high resolution grid for $k\!\left(\vec{x}\right)$ in weather layer; b) – low resolution grid for $k\!\left(\vec{x}\right)$.

For both the high resolution weather layer and the lower resolution layer with known exact solution of FWI (Fig. 3) and simple initial approximation of the velocity field (Fig. 4) the full waveform inversion is implemented as an iterative procedure the standard way described in many papers and studies (e.g. Margrave et al. 2011) with grid scaling playing the role of velocity field smoother.

Both wave propagation boundary problems (equation 1) and (equation 3) are solved on the same spatial grid 173×97 for each sub-problem in domain decomposition. As a result, temporal grid discretization includes 425 points but for each particular window $[0, t_0]$ corresponding sub-grid is used.

Velocity field for each sub-problem is defined on a robust grid 20×9 with a scaling factor 9×9 used in grid-scaling approach.
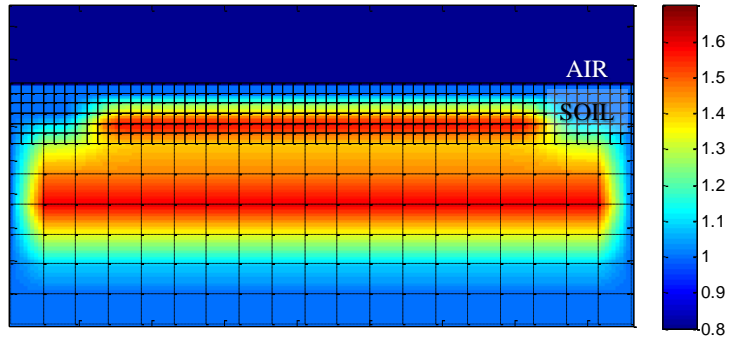
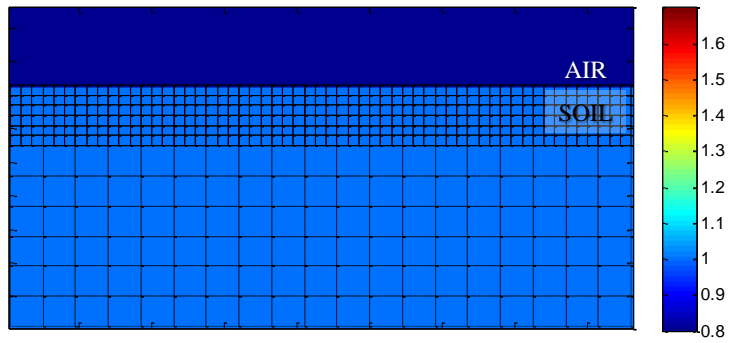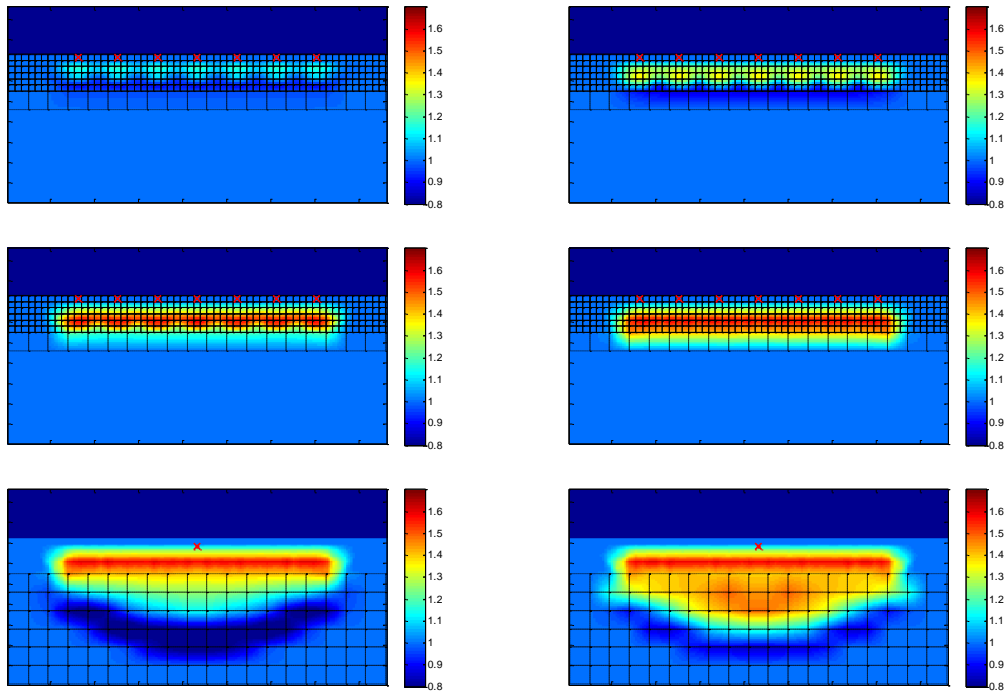Figure 3: Exact velocity field $k_{EXACT}\left(\vec{x}\right)$.

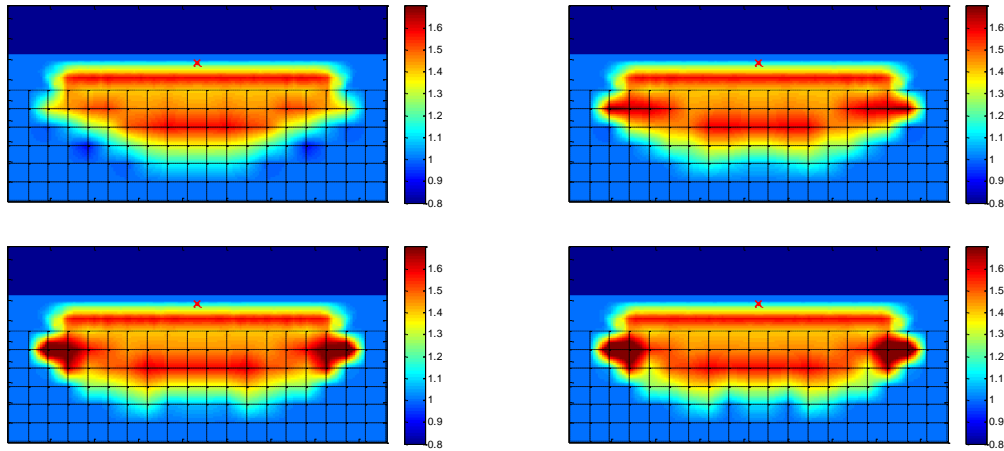Figure 4: Initial velocity field $k_{APPRX}\left(\vec{x}\right)$.

Figure 5. The convergence of an approximate solution $k_{APPRX}\left(\vec{x}\right)$ to $k_{EXACT}\left(\vec{x}\right)$.

The results of a numerical experiment as a FWI iterative convergence are presented in Fig. 5. The total number of FWI iterations is 5000 including 1000 iterations in the high frequency weather layer FWI without low resolution deeper layer FWI. It means that high frequency impulsive source FWI is used as a preconditioner for the deeper low frequency source FWI. Each FWI iteration includes three forward problems (equation 1) numerical solutions, one backward problem (equation 3) numerical solution and one iteration of Newton root search (equation 6). Moreover, each wave propagation problem is solved in corresponding temporal window $[0, t_0]$ which makes different FWI iterations have different computational cost.

Conclusions

From numerical experiments, one of which is presented in previous section, we see that the Newton root search applied to forward wave and residual cross-correlation function shows a good efficiency when applied to 2D acoustic FWI with an impulsive point source. Moreover, the $P_\Lambda I_{t_0}$ iterative root search (equation 6) can be used as a good alternative to well-studied and widely used misfit data norm $\left\|\Delta d\right\|_{L2}$ minimization.

Moreover, in the 1D acoustic FWI case or in other cases implicitly reduced to 1D by increasing the number of impulsive sources, Newton root search applied to cross-correlated forward wave and the residual numerically is expected to provide a good convergence rate comparable with misfit data iterative norm minimization.

Acknowledgements

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