# FDEM Modelling of Thermal Wellbore Instabilities within Shale Formations

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## Summary

It has been demonstrated by Lisjak et al. (2014) that the hybrid finite-discrete element method (FDEM) modelling approach provides a realistic representation of rock mass deformation and fracturing processes. However, until recently, it was not able to account for thermal stresses in rock masses. The analytical solution for the temperature and stresses for a thick-walled hollow cylinder was computed and compared to the FDEM simulation and it was found to produce similar results. Subsequently, the FDEM calibrated material properties for a shale formation, namely Opalinus Clay, from the Mont Terri Project in Switzerland will be used to model thermally-induced fracturing around a borehole. The effect of anisotropic material properties and the critical temperature of drilling fluids causing fracturing around the hole will be analyzed and discussed.

## Introduction

In the oil and gas industry, wellbore stability has long been recognized as an issue which results in significant yearly expenditure (Peng & Zhang, 2007). Instabilities during drilling and production lead to temporal and monetary losses, including equipment losses, washouts, borehole collapse, problematic logging, project delays and shutdowns. These instability issues are more often observed when drilling through shale formations. As the hole is being excavated, drilling fluids have an integral role in stabilizing the hole. The drilling fluids are used to cool the drill bit, remove drill cuttings, prevent formation fluids from entering the well and prevent borehole collapse (United States Department of Labor, n.d.).

Inherently, thermal exchange between the drilling fluids and the formation occur which can lead to wellbore instability. As the rock expands or contracts when it is heated or cooled, thermal stresses develop around the hole. These stresses develop as a function of the temperature difference between the drilling fluids and rock. If these induced stresses exceed the strength of the rock, fractures can nucleate, potentially leading to hole instability.

The hybrid finite-discrete element method (FDEM), which is able to realistically model rock mass deformation and fracturing processes, as demonstrated by Lisjak et al. (2014), has recently been extended to the analysis of fracture development due to thermal stresses.

In this work, the numerical approach to simulate thermal stress in a borehole will be validated against the one-dimensional analytical solution for a thick-walled hollow cylinder using arbitrary material properties. Analysis of the Opalinus Clay (OPA) from the Mont Terri Project in Switzerland will be performed using FDEM calibrated parameters by Lisjak et al. (2014). The effect of the anisotropic properties of the OPA on fracturing due to thermal stress will be discussed in addition to the temperature difference between the drilling fluid and host rock which initiates fracturing.

## Theory and Numerical Method

The analytical solution for one-dimensional thermal stresses in an elastic hollow cylinder is used to verify the FDEM model. The plane strain case for a thermally stressed thick-walled hollow cylinder will be employed for the analysis as it is representative of the geometry of a borehole. Plane strain assumes that the body is uniform and the z-dimension is much larger than the xy-dimension (Timoshenko & Goodier, 1951, p. 11) and thus it implies that $\varepsilon_{zz} = \gamma_{rz} = \gamma_{\theta z} = \tau_{rz} = \tau_{\theta z} = 0$, where $r$ is the radial direction and $\theta$ is the tangential direction (Nadai, 1963, p. 179).

The steady state temperature field of the cylinder is determined using the following relationship:

$$T(r) = \frac{T_a - T_b}{\ln \dfrac{b}{a}} \left( \ln \frac{b}{r} \right) + T_b \ , \tag{1}$$

where $T_a$ is the temperature inside the cylinder, $T_b$ is the outside temperature, $a$ is the radius of the hole, $b$ is the outer radius and $r$ is the distance from the centre of the cylinder (Hetnarski & Eslami, 2009, p. 257).

Since the stress distribution of the cylinder is symmetric about the centre point in the z-direction, the governing equations of equilibrium and the strain-displacement relationship are shown in (2), where $\sigma_{rr}$ is the radial stress, $\sigma_{\theta\theta}$ is the tangential stress, $\varepsilon_{rr}$ is the radial strain, $\varepsilon_{\theta\theta}$ is the tangential strain and $u$ is the radial displacement (Timoshenko & Goodier, 1951, p. 407).

$$\frac{d\sigma_{rr}}{dr} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = 0$$

$$\varepsilon_{rr} = \frac{du}{dr} \qquad \varepsilon_{\theta\theta} = \frac{u}{r} \tag{2}$$

The stress-strain relationships of a hollow cylinder are as follows:

$$\sigma_{rr} = \frac{E}{(1+v)(1-2v)} \left[ (1-v)\varepsilon_{rr} + v\varepsilon_{\theta\theta} - (1+v)\alpha T \right]$$

$$\sigma_{\theta\theta} = \frac{E}{(1+v)(1-2v)} \left[ (1-v)\varepsilon_{\theta\theta} + v\varepsilon_{rr} - (1+v)\alpha T \right] \tag{3}$$

where $v$ is Poisson's ratio and $\alpha$ is the coefficient of linear thermal expansion (Hetnarski & Eslami, 2009, p. 254). The boundary conditions for traction free surfaces ($\sigma_{rr} = 0$ at $r = a$ & $r = b$) are applied.

Subsequently, the closed-form solution of the radial and tangential stresses for a hollow cylinder with fixed ends and at steady state temperatures are (Hetnarski & Eslami, 2009, p. 257)

$$\sigma_{rr} = \frac{E\alpha(T_a - T_b)}{2(1-v)\ln\dfrac{b}{a}} \left[ \ln\frac{b}{r} + \frac{a^2}{b^2 - a^2}\left(1 - \frac{b^2}{r^2}\right)\ln\frac{b}{a} \right]$$

$$\sigma_{\theta\theta} = \frac{E\alpha(T_a - T_b)}{2(1-v)\ln\dfrac{b}{a}} \left[ 1 - \ln\frac{b}{r} - \frac{a^2}{b^2 - a^2}\left(1 + \frac{b^2}{r^2}\right)\ln\frac{b}{a} \right] \tag{4}$$

In the FDEM simulation, the nodes of the mesh are assigned an initial temperature and the radial position of the node from the centre is calculated based on its xy-coordinates. Subsequently, the temperature at the inner radius ($T_a$) is increased or decreased by a constant temperature per time step. Thus, the temperature field at that time step is determined using equation (1), based on the incremental change of the inner temperature.

The plane strain thermal stress is then calculated as

$$\sigma_{Thermal} = \frac{E\alpha\Delta T}{1 - 2\nu} , \tag{5}$$

and the mesh is deformed accordingly. This process is reiterated until the last time step, where the inner temperature is equivalent to the desired temperature of analysis.

## Example

To validate the model, an example comparing the temperature and stresses computed from the closed-form solution will be compared to the FDEM model. The geometry of the model is shown in Fig. 1 and the relevant parameters of the model are shown in Table 1. Since the temperature in the inner radius is less than the temperature on the outer boundary, tensile stresses will result from contraction of the material. To guarantee purely elastic behaviour, the tensile strength and internal cohesion are set to extreme values.
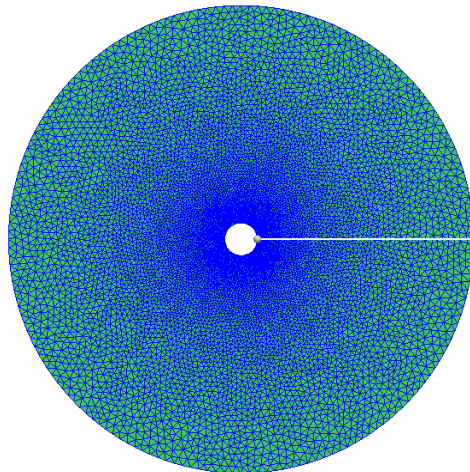


**Fig. 1:** Screenshot of the discretized mesh with element size equal to 1.5 mm and 8 mm at the internal and external boundary, respectively. In total, there are 19,886 elements and 10,030 nodes. The model is left completely unconstrained. The inner and outer radii are equal to 15 mm and 215 mm, respectively. The horizontal white line in the image depicts the plot line of the temperature and stresses in the model.

**Table 1:** Summary of the parameters used in the FDEM simulation and in the analytical solution.

| Arbitrary Material Properties | |
|---|---|
| **Parameter** | **Value** |
| **Young's Modulus** | 50 GPa |
| **Poisson's Ratio** | 0.25 |
| **Friction Coefficient** | 0.5 |
| **Density** | 2700 kg/m$^3$ |
| **Cohesion** | 10 GPa |
| **Tensile Strength** | 50 GPa |
| **Coefficient of Linear Thermal Expansion** | 1 x 10$^{-5}$ /°C |

| Model Parameters | |
|---|---|
| **Parameter** | **Value** |
| **Time Step Size** | 1 x 10$^{-5}$ ms |
| **Model Temperature Change** | -1.5 x 10$^{-2}$ °C/step |
| **Inner Temperature** | 0 °C |
| **Outside Temperature** | 20 °C |
| **Inner Radius** | 15 mm |
| **Outer Radius** | 215 mm |

The temperature, radial and tangential stress are compared to the analytical solution, as shown in Fig. 2. Comparing these three parameters, it is evident that the model agrees well with the analytical solution.
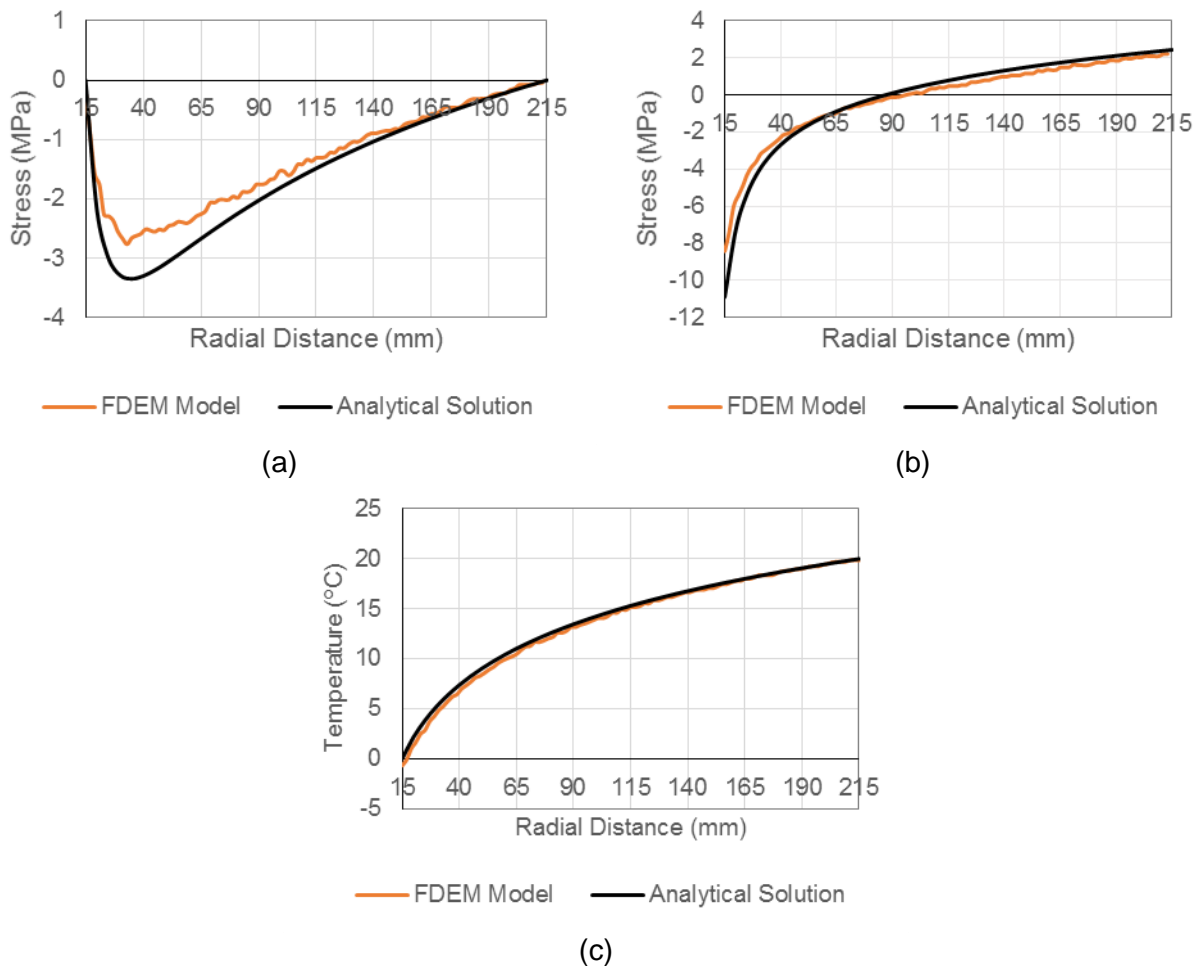


(a)



(b)



(c)

**Fig. 2:** Graphs comparing the (a) radial stress (b) tangential stress and (c) temperature between the FDEM simulation and analytical solution, where positive stress is compressive and negative stress is tensile.

## Conclusions

It has been demonstrated that the FDEM approach is able to accurately reproduce expected temperatures and stresses for a thick-walled hollow cylinder. The next application of the FDEM approach will be to model the OPA from the Mont Terri Project in Switzerland using calibrated parameters by Lisjak et al. (2014). The objective of the analysis will be to roughly determine the temperature difference which will initiate fracturing around the excavation. In addition, the effect of anisotropic material properties on damage around the hole will be analyzed and discussed.

## Acknowledgements

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