

Effect of Pressure on Electrical Conductivity and Formation Factor in Sandstone

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Summary

Apart from changes due to variations in viscosity of a conducting fluid, electrical conductivity is mostly independent of pressure. However, in a porous medium deforming under changing conditions of stress and pore pressure during injection or production, the opening and closing of crack like pores can lead to changes in conductivity. We build on the theoretical work of Stesky(1986) by incorporating a crack modulus as defined by Gao and Gibson (2012) into the original equation. We show that this model is capable of describing the observed trends from pore pressure cycling experiments. Further experimental work is required to validate this model but the parameters can in theory be estimated from both electrical and sonic data. The integration of these two datasets is of interest due to its role in mass quantification during carbon capture and storage (CCS). Drawing on two different datasets offers to combat non-uniqueness of modeled crack parameters.

Introduction

The dependence of electrical conductivity and seismic wave velocities on pore and confining pressure can provide useful information on both fluid properties and pore geometry. Laboratory measurements under controlled conditions allow us to understand how pore and confining pressure individually affect the mentioned physical properties and relate them to the *in-situ* rock properties. The non-linear behavior of seismic velocity at low effective pressures (up to ~40MPa) is attributed to the closing of micro cracks. Once micro cracks have closed the behavior is generally linear. The term micro crack here refers to uncemented grain boundaries, fractures with size on the order of magnitude of grain sizes and joints.

Electrical resistivity and velocity measurements measure very different properties but are both dependent on porosity, pore structure and pore fluid. While the closing of cracks increases the stiffness of the rock and the measured velocity, it also causes an increase in resistivity of the rock due to a combination of geometrical changes including conduction path tortuosity and crack aperture. This therefore also leads to a change in formation factor, often used as a measure of tortuosity in rocks. Some change in resistivity is expected to be caused by an increase in viscosity of the conducting fluid at high fluid pressures but this is negligible at the P-T conditions used here.

Here, we present preliminary results for an acquisition system being developed to make simultaneous sonic and electrical measurements, on a saturated sample of Berea sandstone. A fracture based model initially presented by Stesky(1986) is adapted to describe the observed trends by incorporating pressure dependent crack compliances presented by Gao and Gibson (2012). We are motivated to make simultaneous measurements of wave speeds and electrical conductivity under in situ conditions due to the needs for both sets of data in monitoring of sequestered greenhouse gases.

Theory and Method

Experiments were conducted on two samples. The first is Berea sandstone and the second is a porous ceramic rod. The ceramic has greater porosity (50%) compared to the Berea Sandstone (19%)

but the Berea sample has greater air permeability of 237.65mD versus 96.94mD for the ceramic rod (Yam, 2011).

The samples were subject to confining pressures of 10, 20 and 30 MPa and had the pore pressure increased to within 1 MPa of the confining pressure in each case. *P* and *S* wave travel times were simultaneously recorded using piezoelectric transducers and electrical resistivity measurements were taken immediately after the sonic measurements, under the same T-P conditions. Refer to (Yam, 2011) for details on the ultrasonic measurements. Here we report electrical conductivity values obtained at 10000Hz. Conductivity (sigma or σ) is simply the reciprocal of resistivity (Rho or ρ). Figure 1a shows the measurement circuit employed using the two electrode method. Electrode polarization is a common problem in two electrode measurement systems. To avoid this problem we chose a frequency where there was virtually no observable phase lag between the applied and sample voltage. The rocks were saturated with 1.56 S/m brine.



Figure 1- (a) Measurement circuit for electrical resistivity. (b) Schematic of a rock with a single fracture. Geometric terms are defined in the text. (Stesky, 1986)

Figure 2 shows Vp vs. confining pressure and Vp vs. resistivity for the ceramic sample (right) and Berea sandstone (left). The strong correlation between the velocity and resistivity data for the sandstone suggests that there is elastic deformation due to pressure causing changes in both properties. The ceramic sample shows no such correlation and the small change in velocity suggests that the sample is very stiff undergoing little deformation. This makes sense given that the sandstone has elongate pores while the ceramic has equant pores. (Yam, 2011; Prasad and Manghnani, 1997)

To attempt modeling the dependence of conductivity on differential pressure (confining pressurepore pressure) we build on the work of Stesky (1986) who looked at the effect of fractures on rock conductivity and developed a simple parallel resistor model for the conductivity of fractured rock. Equation 8 in Stesky (1986) modified for the case of many fractures is given as

where,

$$\sigma_{fr} = (\sigma_s - \sigma_r) f \bar{a} \bar{w} \frac{1 - \overline{Ac}}{1 + \overline{Ac}} + \sigma_r \tag{1}$$

$$f = \frac{n}{A}$$

In the original formulation σ_{fr} is the conductivity of the fractured rock, σ_s is the conductivity of the fluid in the fracture and σ_r is the conductivity of the wall rock. f is the fracture density defined as the number of fractures per unit area (A). \bar{a} is the fracture aperture and \bar{w} is the width. These parameters can be seen in Figure 1b which is taken from Stesky (1986) where his aperture e is the same as \bar{a} here. Bars over quantities represent the average over all cracks. Ac is the contact area of asperities on the crack wall. This is similar to equivalent channel models in that it uses a representative or averaged crack to model a geometry that is certainly more complex in reality and scales it based on the number of cracks. The number of cracks can be expressed in terms of crack porosity φ_c .

$$n = \frac{\varphi_c A}{\overline{a \, w}} \tag{2}$$



Figure 2- (Top) P wave velocity vs. Pressure (Pd) for Berea Sandstone (left) and porous ceramic rod (right). (Bottom) Cross-plot of P wave velocity vs. resistivity for Berea sandstone (left) and porous ceramic rod (right).

Crack aperture is a pressure dependent quantity. In order to calculate it at different pressures we use the pressure dependent crack modulus as derived by Gao and Gibson (2012) where cracks are represented as rough surfaces in contact as it is in Stesky (1986). There are some differences with regard to how the authors choose to describe the crack properties, namely in the distribution of asperity heights and contact area of asperities. For consistency we use only the representations of those quantities as they are presented in Gao and Gibson (2012). The distribution of Asperity heights are therefore described using a power law distribution and the contact area of asperities are described by the analytic solution of Gao and Gibson as opposed to the empirical relation used by Stesky. The crack aperture is related to its modulus as given by equation 3 below where Mn is the crack modulus, and *a* is the aperture. Equation 3 assumes that the total volume change of the crack due to differential pressure is in the aperture. This is reasonable for a flat crack and has been defined this way by previous authors (Gangi and Carlson 1996).

$$Mn = -a\frac{dP}{da} \tag{3}$$

Rearranging and integrating equation 3 gives,

$$a = \frac{a_0}{e^{C(p)}} \tag{4}$$

where,

$$C(p) = \int_{P_0}^P Bn \, dP$$

And Bn is just the inverse of Mn. The integral is evaluated numerically here. Substituting equations (4) and (2) into (1) gives,

$$\sigma_{fr} = (\sigma_s)\varphi_c e^{-C(p)} \frac{1 - \overline{Ac}}{1 + \overline{Ac}} + \sigma_r$$
(5)

and using Archie's law for the equant porosity gives

$$\sigma_r = a\sigma_s \varphi^m \tag{6}$$

where a and m are Archie's cementation exponent and tortuosity factor. Calculating the crack modulus at different pressures requires three parameters; the exponent controlling the power-law distribution of asperity heights, the closure modulus of the crack which is simply the modulus of the crack when the two faces are mated and the crack porosity. In the above form, all 3 parameters can in theory be determined by either velocity (as in Gao and Gibson (2012)) or electrical conductivity as shown here.

Examples

Figure 3 (top) shows measurements made during up and down pore pressure cycles with both sets of data are fitted by the model with rms error of 0.0018 S/m for the up cycle data which does not fit as well as the down cycle data. Bottom shows the formation resistivity factor changing as is expected.



Figure 3- (Top) Conductivity vs. pressure modeled with the modified equation from Stesky (1986). (Bottom) Formation factor vs. pressure.

Conclusions

The electrical conductivity measured on sandstone cores under in-situ conditions is a necessary prerequisite to conduct a CO_2 mass quantification from geophysical field data. For an estimation of CO_2 saturations from time-lapse geoelectric or electromagnetic data, one needs an inverse petrophysical relation that is calibrated with data from laboratory experiments on representative core samples of the storage reservoir.

The success of the proposed model offers the potential for integration of electrical and sonic data to provide a highly constrained, robust determination of reservoir rock parameters. Combining two datasets offers an added constraint to combat the non-uniqueness of the parameters used to determine the crack modulus. Future work includes gathering further experimental data and investigating other models for crack compliance.

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References

Gangi, A. F., & Carlson, R. L. (1996). An asperity-deformation model for effective pressure. Tectonophysics, 256(1-4), 241–251.

Gao, K., & Gibson, R. L. (2012). Pressure-dependent seismic velocities based on effective compliance theory and an asperity deformation model. *Geophysics*, 77(6), D229.

Prasad, M. (1997). Effects of pore and differential pressure on compressional wave velocity and quality factor in Berea and Michigan sandstones. *Geophysics*, *62*(4), 1163.

Stesky, R. M. (1986). Electrical conductivity of brine-saturated fractured rock. Geophysics, 51(8), 1585.

Yam, H.,(2011), CO2 rock physics: A laboratory study (Master's Thesis), Dept. of Physics, University of Alberta: 285 pp.