

# Application of Kirchhoff approximation in iterative multiparameter elastic waveform inversion

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## Summary

Elastic Full Waveform Inversion (FWI) is an iterative method that simultaneously uses seismic travel time and amplitude to recover subsurface elastic properties. This work is intended to reduce the computational cost associated with Finite Difference Time Domain (FDTD) forward modeling and Reverse Time Migration (RTM)-based techniques *used in FWI method*. Therefore, we propose a method which makes use of prestack time migrations (PSTM) of multicomponent data and associated inversion in an iterative scheme. In this study, the gradient function of reflection impedance from formation boundary is obtained using transformation of volume integral inversion strategy of Tarantola (1986) into the surface integral Kirchhoff approximation.

## Introduction

The procedure of FWI by FDTD operators has been the subject of much research and in particular the computational time burden (Vigh and Starr, 2008) is an ongoing challenge. Attempts were made to improve the computational efficiency by implementing cheaper numerical schemes for forward modeling and migrations. The asymptotic Born approximations of wave equation are shown to be an efficient strategy in iterative inversion algorithms (Beydoun and Mendez, 1989, Jin et al., 1992, Thierry et al., 1999, Lambare et al., 2003 and Operto et al., 2003). In this paper, an inversion method based on Kirchhoff approximation has been developed. The Kirchhoff approximation considers the differential of model parameters across bedding interfaces compared to the Born approximation that considers variation of model parameters with a reference medium. Consequently, the shape of perturbation in Born approximation is step-like while the shape perturbation in Kirchhoff approximation (e.g., reflectivity function) is spike-like with respect to reflectors normal vector (See e.g., Beylkin and Burridge, 1985, Jaramillo and Bleistein, 1999, Kroode, 2013 and Shaw and Sen, 2004). For the inversion, the steepest descent inversion scheme and the gradient functions are obtained by setting the inversion of the data residual and updating the operators at each iteration. Here, first perturbation of Born approximation is compared to Kirchhoff approximation. Then, the technique is applied on gradient function of Tarantola (1986) for P- and S-waves impedance.

## Born approximation vs Kirchhoff approximation

Consider an isotropic medium, with true elastic properties  $m_{\text{true}}(\lambda_{\text{true}}, \mu_{\text{true}}, \rho_{\text{true}})$ , that can be described by perturbation in elastic properties  $\delta m(\delta\lambda, \delta\mu, \delta\rho)$  using

$$\begin{aligned}\lambda_{\text{true}}(\mathbf{x}) &= \lambda_0(\mathbf{x}) + \delta\lambda(\mathbf{x}), \\ \mu_{\text{true}}(\mathbf{x}) &= \mu_0(\mathbf{x}) + \delta\mu(\mathbf{x}), \\ \rho_{\text{true}}(\mathbf{x}) &= \rho_0(\mathbf{x}) + \delta\rho(\mathbf{x}).\end{aligned}\tag{1}$$

where  $\rho(\mathbf{x})$  stands for density,  $\lambda(\mathbf{x})$  and  $\mu(\mathbf{x})$  are elastic constants of the stiffness tensor,  $c_{ijkl}$ . The subscript 'true' refers to the true model to be found and the subscript '0' refers to the initial model to be updated and used as input to subsequent iterations. The vector  $\mathbf{x} = (x, y, z)$  is the subsurface coordinate and the terms  $\delta\rho$ ,  $\delta\lambda$  and  $\delta\mu$  are the perturbations of the elastic properties which contain the higher

frequencies of the true model. In our problem, the terms  $\delta\rho$ ,  $\delta\lambda$  and  $\delta\mu$  are assumed to generate the perturbation of the total scattered elastic wavefield,  $\delta d(\mathbf{x}_s, \mathbf{x}_r, t)$ , from the medium, where  $\mathbf{x}_s$  and  $\mathbf{x}_r$  refer to the source and receiver locations. The single scattered wavefield components of the elastic waves can be expressed by (Beylkin and Burridge, 1990)

$$\delta d_{jk}^{IR} = -\partial_t^2 \int \mathbf{S}_{lm}^{IR} A_{jl}^I A_{km}^R \delta(t - \phi^I - \phi^R) d\mathbf{x}, \quad (2)$$

where,  $\mathbf{S}_{lm}^{IR}$  is the scattering matrix, superscripts  $I$  and  $R$  refer to incident and reflected waves, the subscripts  $jk$  indicate the displacement in the  $k$ -direction due to a point force in the  $j$ -direction.

Generally, the derived angle-dependent scattering potentials  $\mathbf{S}$  has the form

$$S^{IR}(x) = h_k^I S_{ki}^{IR} h_i^R, \quad (3)$$

where,  $S_{ki}^{IR}$  is defined as

$$S_{ki}^{IR} = \frac{\delta\rho(x)}{\rho_0(x)} \delta_{ik} + \frac{\delta c_{ijkl}(x)}{\rho_0(x)} p_j^R p_l^I, \quad (4)$$

where  $p$  is slowness unit vector parallel to the path of the rays. Equation (4) is a general description of the scattering potential for various types of waves. Coupling the definition of the stiffness,  $c_{ijkl}$ , for the type of model, e.g. elastic, isotropic and anisotropic, with the slowness,  $p$ , as well as the polarization vectors,  $h$ , of the incident and scattered waves has been the basis for modeling and inversion from the amplitude radiation pattern of the wavefields (Shaw and Sen, 2004).

Formulation of Born and Kirchhoff approximation are asymptotically similar given a smooth error (Beylkin and Burridge, 1990, Jaramillo and Bleistein, 1999, Shaw and Sen, 2004 and Kroode, 2012). As shown in Figure 1 by dividing the volume integral into two upper lower volumes  $D^+$  and  $D^-$  upon reflection surface  $\Sigma$ , the scattering operator in Kirchhoff approximation is expressed as

$$S(\mathbf{x}) = \left[ \frac{\Delta\rho^+(\mathbf{x}) - \Delta\rho^-(\mathbf{x})}{\rho_0(\mathbf{x})} \delta_{ik} + \frac{\Delta c_{ijkl}^+(\mathbf{x}) - \Delta c_{ijkl}^-(\mathbf{x})}{\rho_0(\mathbf{x})} p_j^R p_l^I \right] h_k^I h_i^R, \quad (5)$$

which shows that while the radiation pattern of scattering potential of Born approximation in (4) is step-like whereas the radiation pattern of Kirchhoff approximation is spike-like resulting from singularities over the surface  $\Sigma$ .

### Continuity of perturbed model in elastic Born approximation vs Kirchhoff approximation

A typical perturbation  $\delta m$  is considered in Figure 1. Note that here,  $\delta m = m_{\text{true}} - m_{\text{background}}$  where,  $m_{\text{background}}$  is a background model. As seen in Figure 1c, the first order Born approximation consider the perturbation independently from its surroundings which infers that although the background model of Born approximation is continuous, however, the continuity of the perturbed model with its surrounding is neglected at bedding interfaces by assumption of smooth surface. Continuity of perturbation across the bedding interface is considered in the Kirchhoff approximation as illustrated in Figure 1d, where the differential of both particles  $\delta m^+$  and  $\delta m^-$  with respect to layer boundary, i.e.,  $\delta m^+ - \delta m^-$ , is considered. Hence, for this perturbation the surface integral Kirchhoff approximation considers the bedding interfaces within the perturbed model. In iterative inversion algorithm, after the first iteration, the background model parameters are updated which produce the reflection boundaries. In this situation, Kirchhoff approximation is more useful for forward modeling. Consistent with forward modeling, the inversion is based on reflectivity function of Zoeppritz equations (Aki and Richards, 1980). Therefore, the boundaries of updated model produces reflection to be used in estimation of gradient function (Khaniani, 2015).

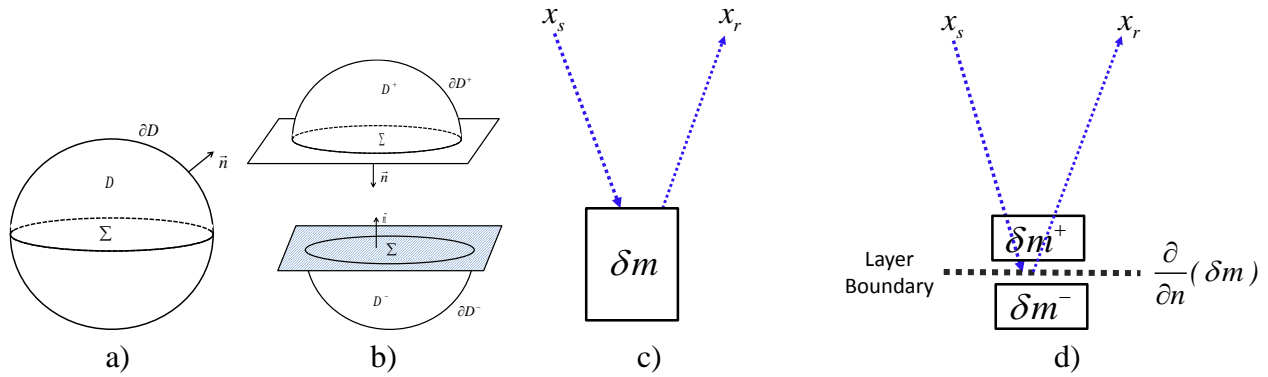


Figure 1: The relationship between the a) Born approximation (volume integral) and b) the Kirchhoff approximation (surface integral with the Born perturbation in c) and Kirchhoff reflection boundaries in d).

Comparison of equations (4) and (5) shows that we can transform the volume integral elastic inversion strategy of Tarantola (1986) into the surface integral Kirchhoff approximation. The approach is consistent with earth model which the change in elastic properties at formation boundaries create reflection data. Compared with AVO inversion that is based on radiation pattern of scatterpoints, the technique simultaneously uses all components of recorded data and estimates the gradient for P- and S-wave impedance by dilatation of the P-waves and rotation (curl) of the S-waves respectively.

## SYNTHETIC EXAMPLE

A synthetic numerical model was created from a two layer model as shown in Figure 2. The elastic FDTD forward modeling (Manning, 2007) is used to produce a single shot record, with a maximum of 1000m offset in split spread configuration with a receiver spacing of  $Dx=2$  m. For illustration purposes an explosive source is positioned in depth of 700 below surface. This helps to have primary PP and PS data and avoid the interference of other types of waves such as SS, SP and their multiple reflection with them. A zero phase wavelet with a dominant frequency of 35 Hz was arbitrarily chosen. The PSTM scatter point imaging of Khaniani (2015) is used for migration of PP and PS data from radial ( $U_x$ ) and vertical ( $U_z$ ) components in Figure (3). Note that a time to depth conversion is used to illustrate the migrated data in the vector form. The background color represents the dilatation and rotation of multicomponent data. The continuity and intensity of dilatation and rotation are consistent with P- and S-wave types and their strength can be correlated with reflectivity. In Figure (3a), primary PP data has strong dilatation before critical angle but after critical angle it also experiences rotation as seen in Figure (3b). This correlation can be due to change of supercritical phase or other numerical features that is subject to further studies. In migrated PS domain of Figure (3d) stronger rotation is observed for S-waves. In addition, the multiple PP is observed to have stronger dilatation in Figure (3c) and weaker rotation values in Figure (3d) which is consistent with P-wave behavior.

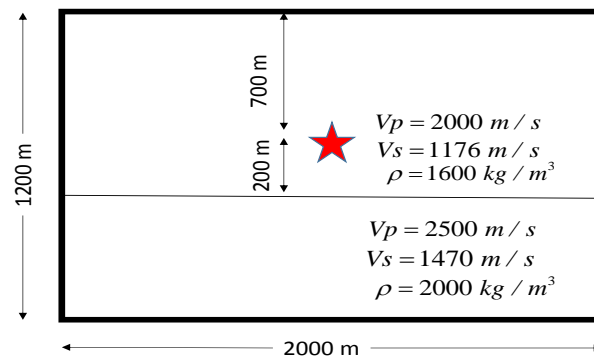


Figure 2: The elastic two layer model with an explosion source buried at depth

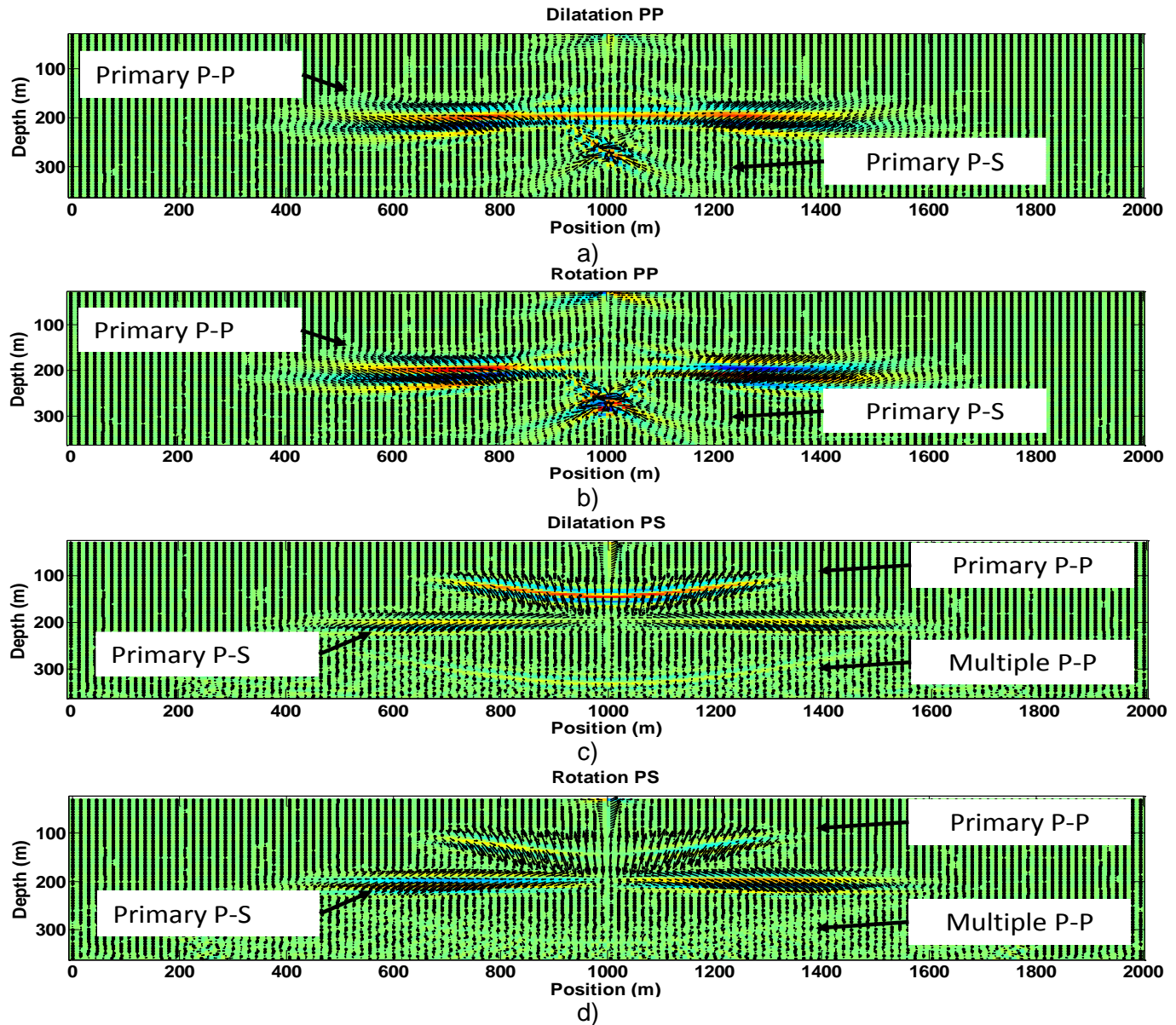


Figure 3: Multicomponent PSTM migration and inversion of PP and PS data. The sections are converted to depth for illustration purposes. The vectors illustrate the migration of  $U_x$  and  $U_z$  components and the background color are computed dilatation and rotation values to be used as gradient function in FWI method. a) Dilatation of PP data b) Rotation of PP data c) Dilatation of PS data d) Rotation of PS data.

## Conclusions

We proposed a multiparameter inversion of the elastic properties based on surface integral Kirchhoff approximation. The procedure is analytically compared with volume integral Born approximation. The approach is more realistic and stable procedure for inversion because the solution is based on reflection boundary. All components of recorded wavefield are simultaneously used for migration and inversion. The output of inversion can be used as the gradient functions for iterative FWI method.

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