

## **Least-squares extended imaging with surface-related multiples**

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### **Summary**

Common image gathers are used in building velocity models, inverting anisotropy parameters, and analyzing reservoir attributes. Often primary reflections are used to form image gathers and multiples are typically attenuated in processing to remove strong coherent artifacts generated by multiples that interfere with the imaged reflectors. However, researchers have shown that, if correctly used, multiples can actually provide extra illumination of the subsurface in seismic imaging, especially for delineating the near-surface features. In this work, we borrow ideas from literatures on imaging with surface-related multiples, and apply these ideas to extended imaging. This way we save the massive computation cost in separating multiples from the data before using them during the formation of image gathers. Also, we mitigate the strong coherent artifacts generated by multiples which can send the migration velocity analysis type algorithms in wrong direction. Synthetic examples on a three-layer model show the efficacy of the proposed formulation.

### **Introduction**

An extended image is a multi-dimensional correlation of source and receiver wavefields, as a function of *all* subsurface offsets (see Sava and Vasconcelos, 2011, and references therein for a recent overview). There are many different applications in which extended images are used extensively like construction of angle-domain common-image gathers (ADCIGs) and migration velocity analysis (MVA) (Biondi and Symes, 2004; Shen and Symes, 2008; Symes, 2008; Sava and Vasconcelos, 2011; Kumar et al., 2013, 2014). Other than that, extended images are used to study the rock properties and fluid indicators by estimating the amplitudes of reflected waves as a function of incident angle at the interface. One of the current limitations is that the extended imaging conditions cannot handle multiple reflections. However, Lu et al. (2014) showed that primaries may not provide enough illumination to image near-surface targets. To overcome this situation Lu et al. (2014) proposed to use multiples only data where they apply the same imaging condition to multiple-only data by putting in the primaries as source and the multiples as receivers. However, efforts during seismic data processing to extract multiple wavefields are always challenging, computationally expensive and can risk damage to the underlying signal. Therefore, to mitigate these impediments and to get benefit from the multiples, we follow earlier work on joint imaging of primaries and surface-related multiples (Tu et al., 2013; Tu and Herrmann, 2015) and adapt it to extended imaging which potentially leads to better images. The idea is to combine EPSI (Estimation of Primaries via Sparse Inversion) (van Groenestijn and Verschuur, 2009) with migration as a way to benefit from surface-related multiples. Following the same ideas, we show that we can exploit the EPSI relationship while forming the image gathers. As a result, we need to move to a least-squares extended imaging rather than a simple correlation-based imaging condition. The proposed way of imaging the surface-related multiples along with primaries do not increase the dominant computational cost, which is the solution of the wave-equation. Finally, we show the efficacy of the proposed formulation on a three-layer synthetic velocity model.

### **Extended imaging with free-surface multiples**

Given the time-harmonic source and receiver wavefields as matrices  $\mathbf{U}$  and  $\mathbf{V}$ , extended images at a *single frequency*, for *all* subsurface offsets and for *all* subsurface points can be written as the outer product of these two matrices, i.e., we have

$$\mathbf{E} = \mathbf{V}\mathbf{U}^*, \quad (1)$$

where  $\mathbf{U}, \mathbf{V}$  are calculated using the two-way wave-equation and each column of  $\mathbf{U}, \mathbf{V}$  corresponds to a source experiment. The explicit expression of  $\mathbf{E}$  can be written as

$$\mathbf{E} = \mathbf{H}^{-*} \mathbf{P}_r^T \mathbf{D}_0 \mathbf{Q}^* \mathbf{P}_s \mathbf{H}^{-*}, \quad (2)$$

where  $\mathbf{H}$  is a discretization of the Helmholtz operator ( $\omega^2 \mathbf{m} + \nabla^2$ ),  $\mathbf{m}$  is the squared slowness, the matrix  $\mathbf{Q}$  represents the source function,  $*$  represents the conjugate-transpose,  $\mathbf{D}_0$  is the primary reflection data matrix and the matrices  $\mathbf{P}_s, \mathbf{P}_r$  sample the wavefield at the source and receiver positions (and hence, their transpose injects the sources and receivers into the grid). Note here that  $\mathbf{H}^{-1}$  involves PDE solves and constitutes the main computational cost in equation (2). In order to incorporate multiples along with primary, we follow the SRME (surface-related multiple estimation) formalism proposed by Verschuur et al. (1992) where the total upgoing pressure wavefield  $\mathbf{D}$  (note that we overload  $\mathbf{D}$  to denote the total upgoing data now), the Green's function and the source are related as follows

$$\mathbf{D} = \mathbf{G}(\mathbf{Q} - \mathbf{D}), \quad (3)$$

where  $\mathbf{G}$  represents the Green's function,  $\mathbf{P}$  the total up-going wavefield,  $-\mathbf{D}$  represents the down-going receiver wavefield at the surface (note that the minus sign in the front comes from the surface reflectivity, assumed to be -1) that acts as a generalized *areal-source* wavefield for the surface-related multiples and  $\mathbf{Q}$  is the down-going point-source wavefield. In this expression, we assume that the source is the same for all shots, i.e.,  $\mathbf{Q} = \mathbf{q}\mathbf{l}$ . Note that each quantity in the above expression represent monochromatic variables. Therefore, we can replace the primary reflections data matrix in equation(2) with the total up-going data matrix  $\mathbf{P}$  and redefine extended image  $\mathbf{E}$  as

$$\mathbf{E} = \mathbf{H}^{-*} \mathbf{P}_r^T \mathbf{D}(\mathbf{Q} - \mathbf{D})^* \mathbf{P}_s \mathbf{H}^{-*}, \quad (4)$$

In 2D, the extended image is a 5-dimensional function of *all* subsurface offsets and temporal shifts. So even in this case, it is prohibitively expensive to compute and store the extended image for all the subsurface points. To overcome this problem, we select  $l$  columns of  $\mathbf{E}$  implicitly by multiplying this matrix with the tall matrix  $\mathbf{W} = [\mathbf{w}_1, \dots, \mathbf{w}_l]$  yielding,

$$\mathbf{E}' = \mathbf{E}\mathbf{W}, \quad (5)$$

where  $\mathbf{w}_i = [0, \dots, 0, 1, 0, \dots, 0]$  represents a single scattering point with the location of 1 corresponding to the  $i^{th}$  grid location of a point scatterer. Each column of  $\mathbf{E}'$  represents a common-image point gather (CIPs) at the locations represented by  $\mathbf{w}_i$ . We follow van Leeuwen and Herrmann (2012) to efficiently compute  $\mathbf{E}'$  by combining equations (4) and (5) as

$$\mathbf{E}' = \mathbf{E}\mathbf{W} = \mathbf{H}^{-*} \mathbf{P}_r^T \mathbf{D}(\mathbf{Q} - \mathbf{D})^* \mathbf{P}_s \mathbf{H}^{-*} \mathbf{W}. \quad (6)$$

As we can see the computational cost of calculating  $\mathbf{E}'$  is  $2l$  PDE solves plus the cost of correlating the areal-source and the data matrices. Thus, the cost of computing the CIPs does not depend on the number of sources or the number of subsurface offsets, as it does in the conventional methods for computing image gathers (Sava and Vasconcelos, 2011). This is particularly beneficial when we are interested in computing only a few CIPs. Here, equation (6) defines an extended migration operator that maps the total up-going data matrix to an extended image. The corresponding extended demigration operator,  $\mathcal{F}$ , is defined as

$$\mathcal{F}(\mathbf{E}') = \mathbf{P}_r \mathbf{H}^{-1} \mathbf{E}' ((\mathbf{Q} - \mathbf{D})^* \mathbf{P}_s \mathbf{H}^{-*} \mathbf{W})^*, \quad (7)$$

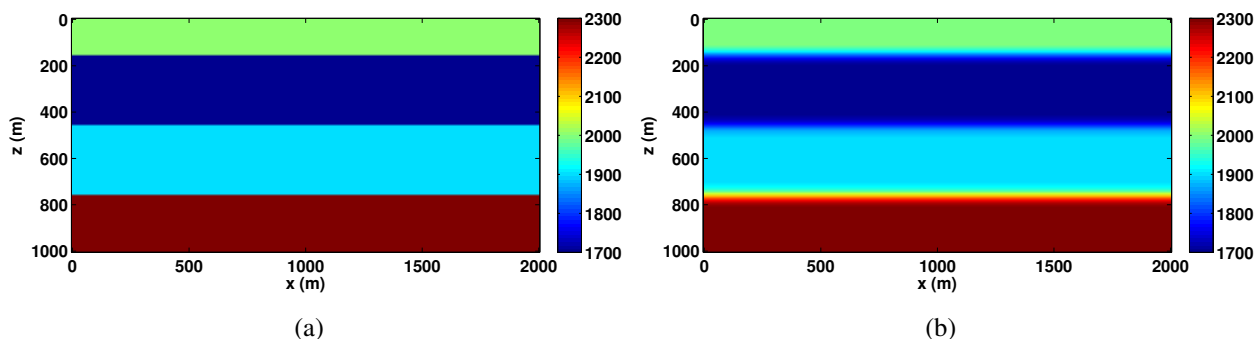
where  $\mathcal{F}$  is a linear operator w.r.t the sources. To compute reliable amplitudes for extended image, we estimate the least-squares extended image by solving

$$\text{minimize}_{\mathbf{E}'} 1/2 * \|\mathbf{D} - \mathcal{F}(\mathbf{E}')\|_F^2, \quad (8)$$

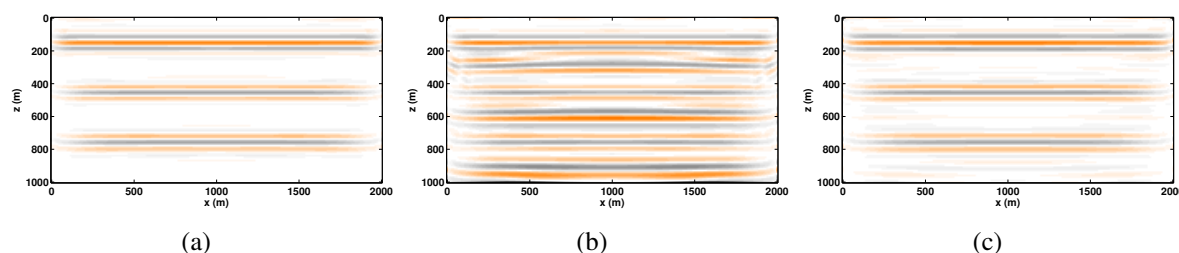
where  $\|\cdot\|_F$  is the Frobenius norm of the matrix (sum of the squared entries). In the above expression we assume that  $\mathbf{Q}$  is known, however, in the proposed framework  $\mathbf{Q}$  can be estimated as well (Aravkin et al., 2013).

## Examples

To test the proposed algorithm for computing common-image point gathers, we use a synthetic three-layer velocity model (with a grid sampling of 10 m) as shown in Figure 1(a). This model is used to generate seismic data. We placed the sources and receivers on the surface with a spatial interval of 10m. We use a finite-difference frequency-domain code to generate the synthetic data sets. The source signature is a Ricker wavelet with a peak frequency 10 Hz. Figure 1(b) shows the velocity model used to form the least-squares reverse-time migration (RTM) and common image gathers (CIGs). We perform 15 iterations of LSQR to invert RTM and equation 8. We construct common-image gathers (CIGs) at  $x = 1000\text{m}$  for all  $z$ . Figures 2(a), 3(a) show the least-squares RTM and CIGs when we only use the primary reflection data  $\mathbf{D}_0$  and  $\mathbf{Q}$  as the source function. As expected, RTM image and image gather is fully focused when using the correct velocity model. Next, we use the total up-going (primaries and multiples) data  $\mathbf{D}$  and still use  $\mathbf{Q}$  as the source function. Since multiple reflection data are not properly dealt with during the imaging condition so acausal artifacts created by multiples destroy the focusing of image point gathers as shown in Figures 2(b), 3(b). Finally, we form the RTM image and image gathers where we use the total up-going data  $\mathbf{D}$  along with the areal source function  $\mathbf{Q} - \mathbf{D}$ . We can see (Figures 2(c), 3(c)) that by including the areal source in extended imaging, we can mitigate the acausal artifacts.



**Figure 1** (a) True velocity model. (b) Background velocity model used in extended imaging.

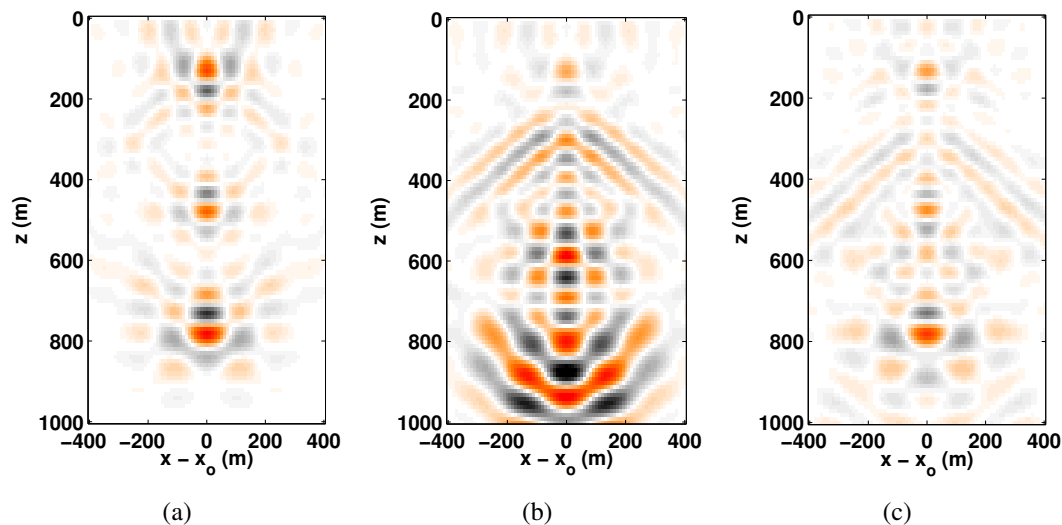


**Figure 2** Least-squares reverse-time migration. (a) Primary data only. (b) Total up-going data is used but areal source is not used. We can clearly see the ghost reflector when we do not use the areal source. (c) Total up-going data is used along with the areal source. Incorporation of areal source remove the ghost reflector.

## Conclusions

In this abstract, we have demonstrated how can we incorporate the surface-related multiples along with the primary reflection data to get benefit from the extra illumination of the subsurface. This will lead to a conceptual transition from multiple removal to using multiples in the future while working with extended images. Since formation of extended images for *all* subsurface offsets and *all* subsurface points is computationally very expensive, we compute image gathers for a few subsurface points without explicitly computing the source and receiver wave fields for all the sources. The main benefit of this approach is that

the computational complexity mainly depends on the number of image points and not on the number of sources or desired number of subsurface offset samples. Future work is to test the proposed formulation on the realistic field data set and extend this work to migration velocity analysis (MVA) to better construct the velocity models using both primary and multiple reflections.



**Figure 3** Least-squares common-image gather extracted along  $x = 1000\text{m}$  and for all  $z$ . (a) Primary data only. (b) Total up-going data is used but areal source is not used. (c) Total up-going data is used along with the areal source. We can clearly see the effect of ghost reflectors (in middle) which is removed (in right) via incorporating the areal source.

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