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Estimation of Anisotropic Elastic Coefficients from Seismic Data

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Summary

In our attempt to better understand the elastic anisotropy in the subsurface, we have developed an algorithm in which elastic coefficients of an anisotropic media is estimated reflectivity data. To do this, we have implemented our in-house forward modeling algorithm *ARTC* along with Pseudo-linear iterative least-square optimization. Due to the complexity of the relationship between elastic coefficients and the reflectivity, the inverse kernel is calculated based on the sensitivity of the data to the elastic parameters. We applied this method to the various models with different anisotropic symmetries and analyzed.

Introduction

Elastic anisotopy exhibits everywhere in the solid Earth media from the subsurface to the mantle and the core, after Backus (1965), Savag (1999) and Schijns et al (2012). There is no need to emphasise the need to better understand wave properties in anisotropic media for global and exploration seismology purposes. For more than a century scientist have been studiying elastic anisotropy and its effects on seismic waves, refer to Christoffel (1877**)**. Despite the fact that its importance is widely recognized there are very few tools available to deal with it, Mustgrave (1970), Auld (1973), Thomsen (1987), Rüger (1997) and Tsvankin (1997) and Vavrycuk (1999) are among the most prominent approaches.

Understating of the elastic anisotropy of the subsurface is becoming increasingly vital for the proper processing and interpretation of seismic data. For example, the analysis for move-out corrections in even basic seismic processing complicated by anisotropy even for horizontally laying sediments. On top of this, the physics of wave propagation is further complicated by differences in the plane-wave (phase) and ray (group) velocities in each polar direction. For pre-stack inversion and any kind of attributes based on amplitude analysis of the seismic data Zoeppritz (1919), or subsequent approximations of Shuey (1985) or of Aki and Richards (1980), strictly only describe the reflectivity from a interface between two isotropic solids. Anisotropy critically influences reflectivity responses that cannot be capture from above mentioned formulas alone.

For example, most fractured reservoirs will not have only a single set of vertical fractures that can be characterized as horizontal transverse isotropy (HTI). Some may have one or more fracture and joint sets that dip obliquely and hence might more appropriately be described as by tilted transverse isotropy (TTI). Indeed, multiple fracture sets can produce rock masses of even lower symmetry than transverse isotropy (TI). The solution for reflectivity in complex anisotropic media is highly non-linear and complicated; hence many researchers have tried to linearize the solution by introducing assumptions about the geometry of the anisotropy on each side of interface. Readers can follow up for example Thomsen (1988, 1986) who linearized Daley and Hron's (1977) full solution using a 'weak boundary contrast' assumption. Rüger's (1997, 1998) solutions for the reflectivity from the contact between an isotropic and either HTI or VTI layers has been popular for the inversion of amplitude versus azimuth (AVAZ) data. Ursin and Haugen (1996) as well as various other authors extended this approach for PP (P-wave incident and reflected) approximation in weak elastic anisotropy and weak boundary contrast in

TI and orthorhombic media. Consequently, there remains a need to expand investigations of reflectivity for more general cases. In this paper we are using Malehmir and Schmitt (2015) algorithm for reflectivity and transmissivity calculator (ARTc) which analytically solves for plane-wave properties in general anisotropic media up to triclinic with twenty one elastic coefficients.

Through seismic inversion of reflected waves, we are able to quantitatively calculate subsurface properties, such as velocity, density. Seismic inversion in isotropic media has been implemented as a standard procedure on both pre-stack and post-stack seismic data to find seismic velocities and density inside the Earth, Shuey (1985), Zeopprits (1919), Aki and Richard (1981). Knowing that most of minerals and rock formations generated seismic anisotropy, isotropic approach might be an oversimplification. In this section, we will introduce an algorithm to estimate elastic coefficients of an anisotropic medium based on perturbed least square inversion.

Theory

Although the theory is borrowed from mathematicians such as Sherman and Morrison (1949 its application in perturbation theory in waveform modeling by Ökeler et al (2009) motivated us to take advantage of this algorithm in anisotropic inversion. The goal for using perturbation theory is to build a pseudo-linear kernel (**G**) to create reflectivity data (*d*) from elastic parameters (*m*), equation 1.

$$\mathbf{G}_{N\times M}\ m_{M\times 1} = d_{N\times 1}, \qquad (1)$$

Each cell of the $\mathbf{G}_{ij}$ reflects the sensitivity of the data d_i versus small variation of the model parameter m_j, equation 2.

$$\mathbf{G}_{ij} = \frac{\delta d_i}{\delta \boldsymbol{m_j}}, \qquad (2)$$

Then ***m*** is solved by minimizing cost function $J(\boldsymbol{m})$, equation 3.

$$\mathrm{J}(\boldsymbol{m}) = |\mathbf{G}\boldsymbol{m} - d|^2 + \mu|\boldsymbol{m}|^2 \qquad (3)$$

where μ is damping factor which controls the $J(\boldsymbol{m})$ to either better fit the data and minimize the misfit $|\mathbf{G}\boldsymbol{m} - \boldsymbol{d}|^2$ or minimize the second norm of the model $|m|^2$. Proper value for μ is acquired through χ^2 tests, after Ulrych and Sacchi (2005). Inverted elastic coefficient is then calculated from damped least square inversion, equation 4.

$$\widehat{\boldsymbol{m}} = (\mathbf{G}^T\mathbf{G} + \mu I)^{-1}\ \mathbf{G}^T d, \qquad (4)$$

This is an iterative approach in which we should update the kernel matrix, ideally in each step.

Examples

In this paper we setup a model with two horizontal slabs in which the lower medium contains unknown elastic properties and anisotropic symmetry. We used this inversion algorithm to find elastic parameters of a media from reflected qP-wave calculated by ARTc in various azimuthal and incidental directions. In order to understand the effect of anisotropic symmetry on the convergence of the algorithm, we applied this technique on various anisotropic models beneath the known upper half-space isotropic model. Without any doubts that algorithm converges faster when in contact with another isotropic medium, which due to the sensitivity of the reflectivity to non-diagonal elastic coefficients, it takes much longer time to converge in most of the anisotropic symmetries Table 1, ***misfit*** values for each anisotropic class with the amount of time it took to converge, where *Nc* and m_o stands for number of independent elastic coefficients that are optimized and true elastic coefficients of the medium, respectively.

Table 1. Results from Pseudo Linear Least Square Inversion of PP reflected data form different anisotropic medium. Results are shown for the iteration (*N_itr*) in which the cost function is stabilized.

Nc	$\lvert \mathbf{Gm}\text{-d}\rvert^2$	$\lvert \mathbf{m}_o\text{-}\mathbf{m}\rvert^2$	N_iter
2 (ISO)	5.9 E+1	2.6 E-3	200
5 (HTI)	1.28E+2	9.213 E+1	2196
9 (ORT)	7.479 E+2	1.491 E+2	2467

Conclusions

In this contribuion we developed our pseudo-linear least square inversion algorithm to find elastic coefficeints of an anisotropic medium from reflectivity data. The algorithm uses ARTc to calculate reflectivited wave-modes from anisotropic model with plane-wave incident wave. This designed inversion algorithm uses qP-wave reflectivity data in pseudo least squre procedure to find elastic coefficiets of a media by minimizing summation of second norm of misift and model. Future works will be dedicated in incorporating more data into the inversion scheme by considering multicomponent data.

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