



# Suppress Parameter Cross-talk for Elastic Full-waveform Inversion: Parameterization and Acquisition Geometry

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## Summary

Full-waveform inversion (FWI) promises to provide high-resolution estimates of subsurface properties but remains exposed to a range of challenges. Involving multiple physical parameters in FWI increases the non-linearity of the inversion process. The parameter cross-talk arising from the coupling effects between different physical parameters makes the inverse problem more undermined. Proper parameterization and acquisition geometry should be determined to manage the parameter cross-talk. In this research, we analyze the parameter cross-talk associated with different parameterizations for elastic FWI. We also discuss the influence of acquisition geometry in managing the parameter cross-talk. We present numerical examples to show that impedance parameterization is most efficient for elastic FWI in mitigating parameter cross-talk.

## Introduction

Full-waveform inversion (FWI) allows to reconstruct high-resolution velocity models of the subsurface through the extraction of the full information content of the seismic data (Lailly, 1983; Tarantola, 1984; Virieux and Operto, 2009). FWI iteratively minimizes a L-2 norm misfit function, which measures the difference between the modelled data and observed data (Pratt et al., 1998; Pan et al., 2014b, 2015a, 2015b).

Shorter-term FWI is employed to provide high-resolution velocity model for seismic imaging. Longer-term FWI is expected to inverse more elastic parameters for reservoir characterization in exploration geophysics. Compared to mono-parameter FWI, multi-parameter FWI is a more challenging task. Simultaneous inversion of multiple parameters increases the non-linearity of the inversion process. The coupling effects (or parameter cross-talk) between different physical parameters make the inverse problem more undetermined. The overlap of the partial derivative wavefields (or scattering patterns) associated with different physical parameters give rise to the parameter cross-talk difficulty.

The multi-parameter Hessian in FWI is a square and symmetric matrix with a block structure. It carries more information than a mono-parameter Hessian. Within the multi-parameter Gauss-Newton Hessian approximation, the off-diagonal blocks measure the correlation of Fréchet derivative wavefields with respect to different physical parameters, and they act to mitigate the coupling effects between these parameters. In this research, the scattering patterns of the elastic constants in general anisotropic media are given for parameter cross-talk analysis. It is verified that the multi-parameter Gauss-Newton Hessian and its parameter-type approximation can reduce the parameter cross-talk in multi-parameter FWI (Pan et al., 2015b, 2016).

We also compare different parameterizations for effective inversion in elastic FWI. The numerical experiments show that the impedance parameterization is more effective for inversion and avoiding parameter cross-talk, compared to velocity and Lamé constants parameterizations.

## Review of non-linear least-squares inverse problem

As a non-linear least-squares optimization problem, FWI seeks to estimate the subsurface parameters by iteratively minimizing the difference between the synthetic data and observed data. The misfit function is formulated in a least-squares form:

$$\Phi(\mathbf{m}) = \frac{1}{2} \sum_{\mathbf{x}_s} \sum_{\mathbf{x}_r} \sum_{\omega} \|\Delta \mathbf{d}(\mathbf{x}_r, \mathbf{x}_s, \omega)\|^2, \quad (1)$$

where  $\mathbf{x}_r$  and  $\mathbf{x}_s$  indicate the locations of receivers and sources,  $\Delta \mathbf{d}$  is the data residual vector and  $\omega$  is the angular frequency. To minimize the quadratic approximation of the misfit function, the updated model at  $k+1$  th iteration can be written as the sum of the model at the  $k$ th iteration and the search direction:

$$\mathbf{m}_{k+1} = \mathbf{m}_k + \mu_k \Delta \mathbf{m}_k, \quad (2)$$

where  $\mu_k$  is the step length, a scalar constant calculated through a line search method satisfying the weak Wolfe condition. The gradient can be constructed by cross-correlating the forward modelled wavefield and back-propagated wavefield. The search direction is the solution of the Newton linear system:

$$(\tilde{\mathbf{H}}_k + \varepsilon \mathbf{A}) \Delta \mathbf{m}_k = -\mathbf{g}_k, \quad (3)$$

where  $\mathbf{g}_k$  is the gradient,  $\tilde{\mathbf{H}}_k$  is the Gauss-Newton Hessian approximation, and  $\varepsilon \mathbf{A}$  is the damping term. The Gauss-Newton Hessian is the correlation of two partial derivative wavefields:

$$\tilde{\mathbf{H}}(\mathbf{x}, \mathbf{x}') = \frac{\partial \mathbf{u}^\dagger}{\partial \mathbf{m}(\mathbf{x})} \frac{\partial \mathbf{u}}{\partial \mathbf{m}(\mathbf{x}')}. \quad (4)$$

## The multi-parameter Hessian

The multi-parameter Hessian has a block structure. Considering the 2D subsurface model with  $N_x \times N_z$  nodes and  $N_p$  physical parameters are assigned to describe the properties of each node. The multi-parameter Hessian is a  $N_x N_z N_p \times N_x N_z N_p$  square and symmetric matrix with  $N_p$  diagonal blocks and  $N_p(N_p - 1)$  off-diagonal blocks. Each block is a  $N_x N_z \times N_x N_z$  square matrix. For isotropic and elastic media, three different physical parameters  $\mathbf{m}_1, \mathbf{m}_2, \mathbf{m}_3$  can be used to describe the properties of the media. Thus, the Newton linear system can be written as:

$$\begin{bmatrix} \tilde{\mathbf{H}}_{11} & \tilde{\mathbf{H}}_{12} & \tilde{\mathbf{H}}_{13} \\ \tilde{\mathbf{H}}_{21} & \tilde{\mathbf{H}}_{22} & \tilde{\mathbf{H}}_{23} \\ \tilde{\mathbf{H}}_{31} & \tilde{\mathbf{H}}_{32} & \tilde{\mathbf{H}}_{33} \end{bmatrix} \begin{bmatrix} \Delta \mathbf{m}_1 \\ \Delta \mathbf{m}_2 \\ \Delta \mathbf{m}_3 \end{bmatrix} = - \begin{bmatrix} \mathbf{g}_1 \\ \mathbf{g}_2 \\ \mathbf{g}_3 \end{bmatrix}. \quad (5)$$

The off-diagonal blocks in the multi-parameter Hessian indicate the correlation of partial derivative wavefields with respect to different physical parameters and they predict the coupling effects of these physical parameters. Thus, applying its inverse to the gradient can mitigate the parameter cross-talk difficulty.

## Parameterization and acquisition geometry issue for elastic FWI

Compared to mono-parameter FWI, multi-parameter FWI is a more challenging task. Simultaneous inversion of multiple parameters increases the non-linearity of the inversion process. The coupling effects between different physical parameters make the inverse problem more undetermined. The overlap of the partial derivative wavefields associated with different physical parameters give rise to the parameter cross-talk difficulty. Proper parameterization and acquisition geometry should be determined for managing the parameter cross-talk. A good choice of parameterization will give three different scattering patterns which are as different as possible, to allow easy identification of these parameters.

In this research, we define three different parameter classes for elastic FWI, which are velocity parameterization:  $\alpha, \beta, \rho$ , Lamé constants parameterization:  $\lambda, \mu, \rho$  and impedance parameterization:  $I_p, I_s, \rho$ . Figure 1 shows the P-P scattering patterns for different parameter classes (Tarantola, 1986). Theoretically, the impedance parameterization should give best inversion result (Tarantola, 1986). We also design a reflection survey and a transmission survey for comparison.

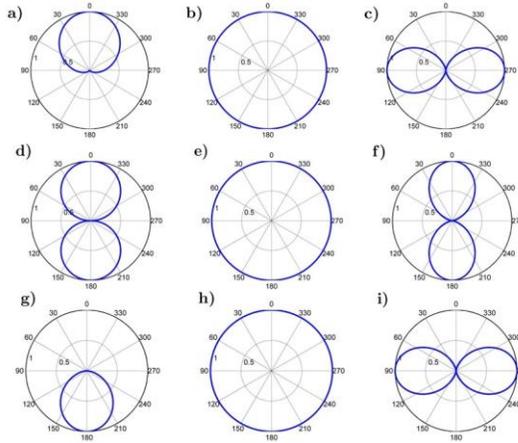


Figure 1. (a), (b) and (c) show the P-P scattering patterns for  $\alpha, \beta, \rho$ . (d), (e) and (f) show the P-P scattering patterns for  $\lambda, \mu, \rho$ . (g), (h) and (i) show the P-P scattering patterns for  $I_p, I_s, \rho$ .

### Examples

In this section, we illustrate with numerical examples to compare the elastic FWI inversion results associated with different parameterizations. We also compare the inversion result by reflection survey with that by transmission survey. Figure 2 shows the true model perturbations for different physical parameters  $\mathbf{m}_1, \mathbf{m}_2, \mathbf{m}_3$ . For velocity parameterization,  $\mathbf{m}_1, \mathbf{m}_2, \mathbf{m}_3$  represent  $\alpha, \beta, \rho$ . For Lamé constants and impedance parameterizations,  $\mathbf{m}_1, \mathbf{m}_2, \mathbf{m}_3$  indicate  $\lambda, \mu, \rho$  and  $I_p, I_s, \rho$ . The initial models are homogeneous.

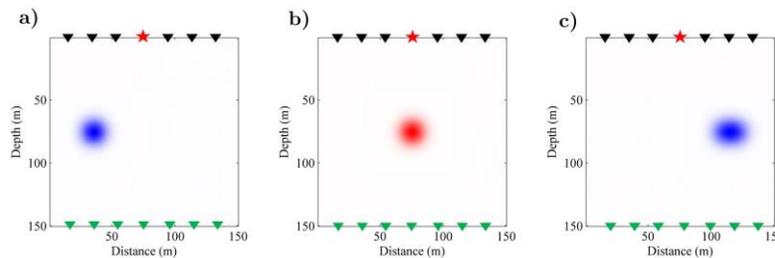


Figure 2. (a), (b) and (c) show the true model perturbations for  $\mathbf{m}_1, \mathbf{m}_2, \mathbf{m}_3$ . The red stars represent the source locations. The black and green triangles indicate the receiver locations for reflection and transmission surveys.

We performed reflection survey and transmission survey for inversion respectively and Gauss-Newton method is used. Figure 3a, 3b and 3c show the Gauss-Newton Hessian of velocity parameterization, Lamé constants parameterization and impedance parameterization for reflection survey. Figure 3d, 3e and 3f show the Gauss-Newton Hessian of different parameterizations for transmission survey. Figure 4 shows the velocity parameterization inversion results with reflection survey and transmission survey. As we can see, the inverted P-wave velocity is still suffered from parameter cross-talk. Figure 5 shows the Lamé constants parameterization inversion results with reflection survey and transmission survey. We can see

that the parameters  $\lambda$  and  $\mu$  are difficult to be recovered. Figure 6 show the inversion results for impedance parameterization. We observe that different physical parameters are be inverted very well without parameter cross-talk artifacts.

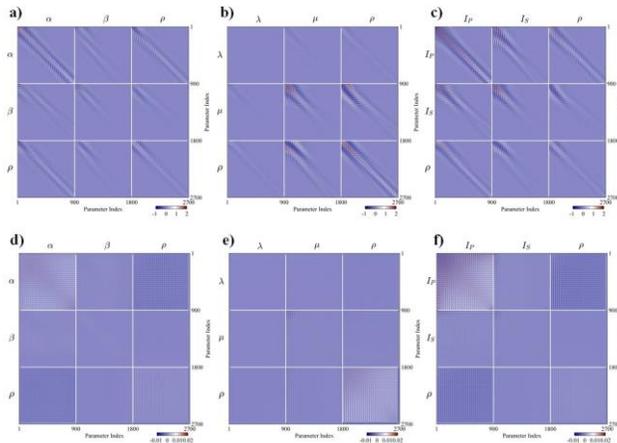


Figure 3. (a), (b) and (c) show the Gauss-Newton Hessian of velocity parameterization, Lamé constants parameterization and impedance parameterization for reflection survey. (d), (e) and (f) show the Gauss-Newton Hessian for transmission survey.

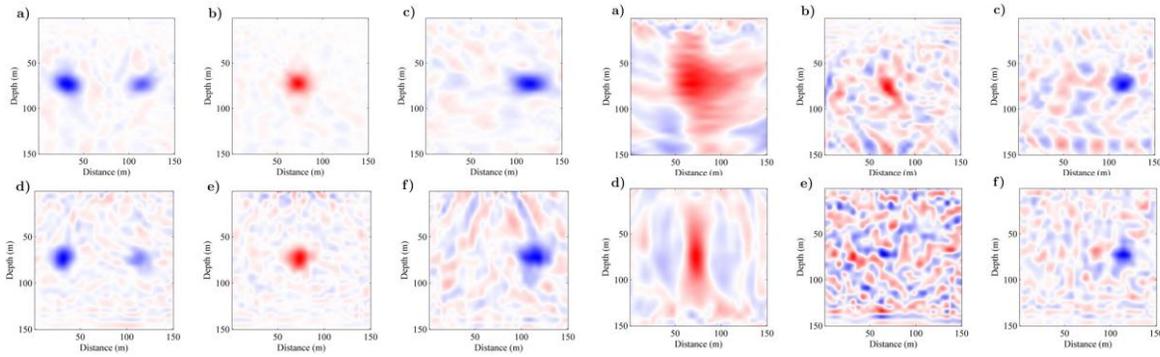


Figure 4. Velocity parameterization inversion results with reflection survey and transmission survey.

Figure 5. Lamé constants parameterization inversion results with reflection survey and transmission survey.

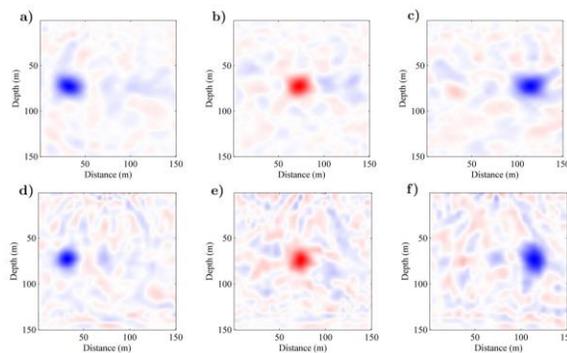


Figure 6. Impedance parameterization inversion results with reflection survey and transmission survey.

## Conclusions

In this research, we discuss the parameter cross-talk difficulty for elastic FWI. We also consider the performances of different parameter classes in managing parameter cross-talk. It is concluded that the impedance parameterization works better for elastic FWI than other parameterizations.

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