GeoConvention 2016
Calgary • Canada • March 7-11
OPTIMIZING RESOURCES

Least squares migration in an extended image domain

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Summary

We study the problem of computing angle domain gathers via the least squares migration technique for shot-profile migration. For this purpose, one can compute angle gathers by estimating a reflectivity function that depends on angle of propagation. A polynomial interpolation method is adopted for constructing an operator that maps shot dependent reflectivity to a reflectivity model that depends on propagation angle. The angle of propagation is estimated by forward modelling the source-side wavefield on the smooth background migration velocity model. Specifically, the gradient of the source-side wavefield is used to estimate the angle of propagation. Finally, a pair of migration and de-migration operators that pass the dot product test (to guarantee adjointness) are used in conjunction with the method of conjugate gradients to solve the least squares migration problem.

Introduction

Different least squares migration (LSM) methods have been proposed to improve the resolution of seismic images. Initial efforts in the area of LSM adopted Kirchhoff migration and de-migration operators (Nemeth et al., 1999) to estimate an image that honours pre-processed seismic data. LSM methods that utilize one-way wave equation operators have been proposed by Kuehl and Sacchi (2003) and Kaplan et al. (2010). LSM can be written as a quadratic optimization problem with a regularization constraints that permits to estimate high-quality common image gathers. Both quadratic and non-quadratic constraints can be incorporated into the formulation of the problem to enhance the vertical resolution (Wang and Sacchi, 2007) and the lateral continuity of seismic images (Wang and Sacchi, 2009). LSM experiments on field data have shown an important uplift on image quality. The latter has been reported by Wang et al. (2005), Wang and Sacchi (2009) and, more recently, by Salomons et al. (2014).

LSM with one-way propagators have been also proposed to image elastic data (Stanton and Sacchi, 2015). Recent progress in the area also includes adopting recursive solutions (Kazemi and Sacchi, 2015), and the implementation of LSM via forward and adjoint reverse time migration operators (Yao and Jakubowicz, 2012a,b; Li et al., 2015; Wong et al., 2014). In this work, we are interested in a strategy to compute common image gathers parametrized in terms of angle of propagation.

Shot profile least squares migration

In shot profile LSM an image of the subsurface is obtained by solving a linear inverse problem of the form

$$\mathbf{d}_j = \mathbf{L}_j \mathbf{m}, j = 1, 2, \cdots, N_s \tag{1}$$

where $\mathbf{d}_j$ denotes the prestack data for source j, $\mathbf{L}_j$ is the demigration (modelling) operator for source j and $\mathbf{m}$ is the image of the subsurface. Classical migration can be considered as the application of the adjoint operator $\mathbf{L}_j^*$ to the data followed by stacking over sources

$$\tilde{\mathbf{m}} = \sum_j \mathbf{L}_j^* \mathbf{d}_j \tag{2}$$

LSM entails defining a cost function that is minimized with respect to the unknown image $\mathbf{m}$

$$J = \sum_j ||\mathbf{L}_j\,\mathbf{m} - \mathbf{d}_j||_2^2 + \mu||\mathbf{W}\,\mathbf{m}||_2^2 \tag{3}$$

where μ is a tradeoff parameter and $\mathbf{W}$ is a weighting matrix. Equation 3 is minimized via the method of Conjugate Gradients (CG), which finds the solution that minimizes J by a series of steps that require the application of the forward and adjoint operator to vectors that belong to model and data space, respectively. In our method, one-way wave operators are employed to downward and upward continue wavefields. Lateral variations of velocities are handled via screen corrections (Xie and Wu, 1998). Equation 3 can also be written in terms of a source dependent image by simple replacing $\mathbf{m}$ by the image associated to each individual shot $\mathbf{m}_j$. Then the new cost function is given by

$$J = \sum_j ||\mathbf{L}_j\,\mathbf{m}_j - \mathbf{d}_j||_2^2 + \mu\sum_j ||\mathbf{W}\,\mathbf{m}_j||_2^2\,. \tag{4}$$

The image $m(x,z)$ was replaced by an extended image $m(x,z,s)$, where s is the the source position. The extended image adds unknowns to the problem and makes the problem underdetermined. The latter greatly simplifies the least-squares data fitting process (Kaplan et al., 2010). However, the model $m(x,z,s)$ does not include propagation (angle) information. In addition, solving equation 4 is equivalent to solving N_s independent least-squares problems with the addition of a regularization term.

In this paper, we design a new extended image which is a function of the propagation angle θ. We denote the new image $r(x,z,\theta)$. One can express the re-parameterization of the reflectivity in terms of propagation angles as follows $\mathbf{r} = \mathbf{A}\,\mathbf{m}$ where $\mathbf{A}$ is the matrix of interpolation coefficients used to map energy in (x,y,s) to $(x,y,\theta(x,z))$. To implement the aforementioned mapping one needs to estimate propagation angles. The gradient of the wavefield determines the energy flow direction at each point in space. Consequently, it can be used to specify the angle of propagation (Jia and Wu, 2009). In a 2D media, it is

$$\begin{aligned} \nabla u &= (\frac{\partial u}{\partial x}, \frac{\partial u}{\partial z}) = (u_x, u_z) \\ \theta &= tan^{-1}(|u_x|/|u_z|)\,. \end{aligned} \tag{5}$$

The new parametrization leads to the following cost function for the LSM problem

$$J = \|\mathbf{L}\,\mathbf{A}\mathbf{r} - \mathbf{d}\|_2^2 + \mu\|\partial_\theta \mathbf{r}\|_2^2\,. \tag{6}$$

In the last equation we have incorporated a physical regularization constraint in the form of smoothing along the angular coordinate (∂_θ) for each common image gather (Kuehl and Sacchi, 2003). After solving the LSM problem, the estimated image can be used for the reconstruction of missing data.

Example

To test our method, we generated a group of synthetic data that consists of 10 common shot gathers. The velocity model is shown in figure 1.

The location of the shots ranges from $s = 500m$ to $s = 1400m$ at $z = 0m$ with an interval of $100m$. The data gathers were modelled by the forward operator. The common image gathers in the extended propagation angle domain were obtained after running the method of conjugate gradients for 15 iterations. The regularization parameter is $\mu = 0.0001$. We portray results for different propagation angles in figure 2. To demonstrate the application of the method to data reconstruction, we remove the CSG at position $s = 900m$ and repeat the test. Based on the inverted image, we can estimate the missing data via forward modeling. The input data with the missing CSG and the the reconstructed CSG are shown in figure 3.

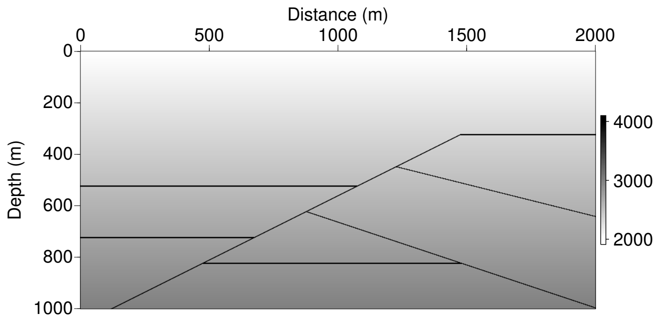

Figure 1 *Velocity model for migration. 10 shot gathers were generated over the model.*

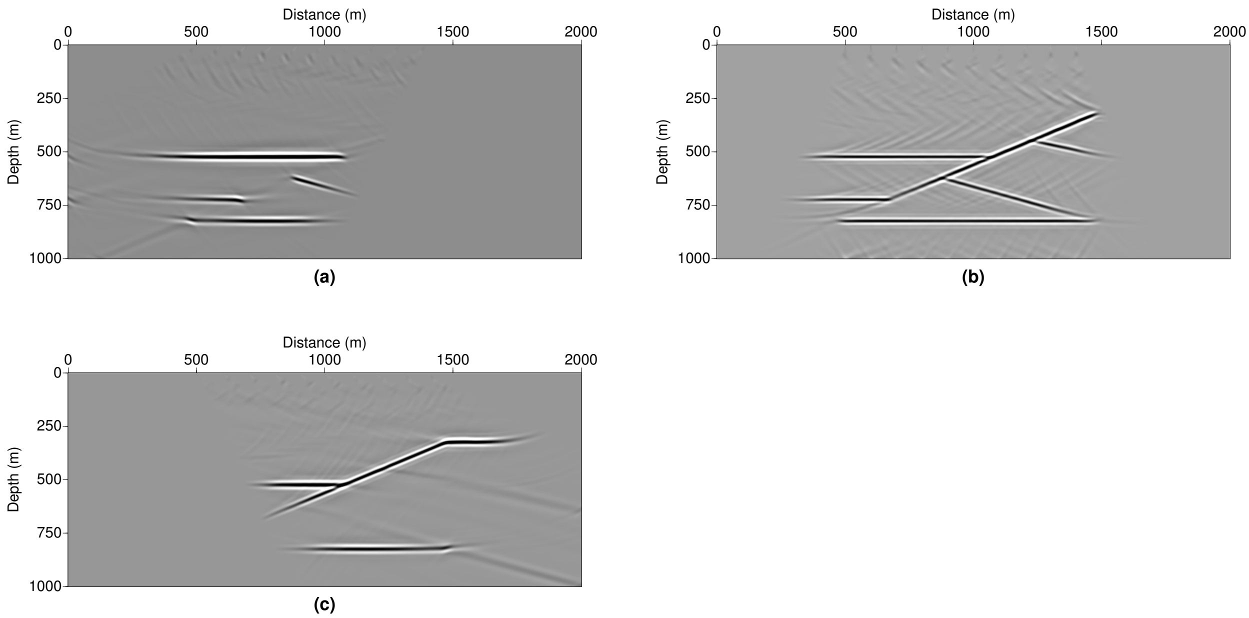

Figure 2 *Extension image gained by CG method of 15 iterations, at* $\theta = -40^\circ$ *(a),* $\theta = 0^\circ$ *(b) and* $\theta = 40^\circ$ *(c).*

Conclusion

A method was introduced that permits to estimate common image gathers in terms of angle of propagation using least squares migration. We use the wavefield gradient to calculate the propagation angle. The method can be used to reconstruct missing data. Encouraging results have been obtained with synthetic data. Future work entails regularized and robust estimation of propagation angles and applications to more complex synthetic data sets and real data.

Acknowledgements

We appreciate the support from the sponsors of Signal Analysis and Imaging Group (SAIG) at the University of Alberta.

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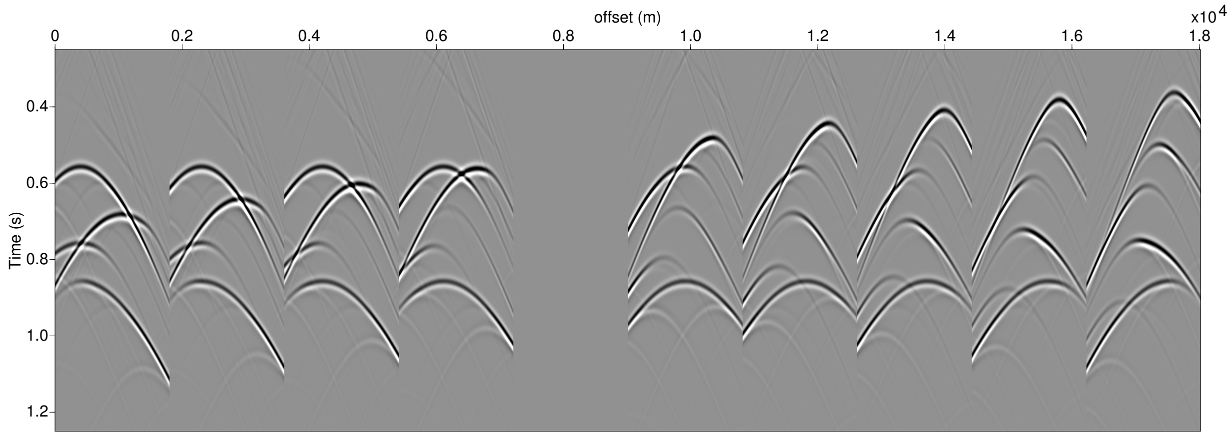
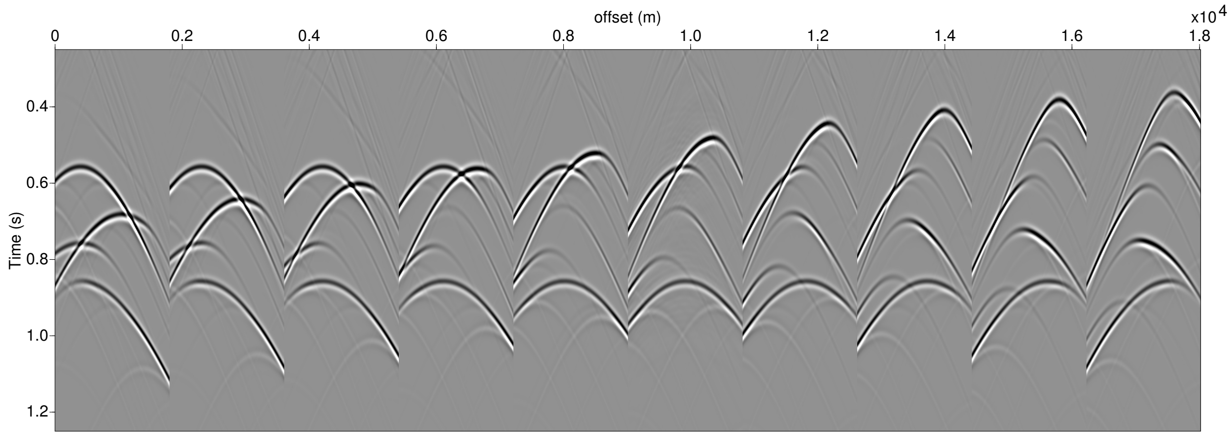


(a)



(b)

Figure 3 *(a) Input data with missing common shot gathers at $s = 900m$. (b) Output data with reconstructed common shot gathers at $s = 900m$.*

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