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Multi-component seismic data registration by optimization

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Summary

Mapping PS-wave data to PP-wave traveltime is a critical step before joint PP-wave and PS- wave data interpretation and inversion. Registration is usually performed with provided Vp/Vs ratios. However, accurate information of velocity ratios is absent in most cases. One can solve the problem of seismic data registration by minimizing the difference between PP-wave and warped PS- wave images with the constraint of a smooth warping function. In order to avoid undesirable foldings or rapid changes in warped PS-wave images, we generally require the warping function to be monotonic and smooth. These requirements are difficult to satisfy in common registration methods. In this paper, we propose to invert Vp/Vs ratios directly from the PP and PS data rather than estimating it from the warping function. The seismic data registration is posed as constrained non-linear optimization problem. Furthermore, we propose to represent V_p/V_s ratios by spline functions, in this way, the number of unknowns are significantly reduced and the convergence to a smooth solution is guaranteed.

Introduction

Multi-component seismic data registration is an important step before quantitative seismic data interpretation and joint AVO analysis. Multi-component seismic data registration is typically performed by cross-correlation. For instance, Gaiser (1996) introduced a correlation-based method to determine the long wave-length component of the V_p/V_s ratio. Fomel et al. (2003) showed how one can warp PS-wave events to match PP-wave events by minimizing the differences between PP-wave data and warped PS-wave data. Seismic data registration by least squares techniques is a highly non-linear problem. All the gradient-based algorithms may easily get trapped in a local minimum before converging to a global minimum. In order to avoid this problem, Hale (2013) proposed a dynamic warping algorithm to align PS-wave traces to their corresponding PP-wave traces. The sequence of time shifts estimated by this method are a global solution to the non-linear optimization problem.

All the aforementioned methods attempt to invert time shifts by minimizing the difference between PP-wave and PS-wave data. Then the V_p/V_s ratio is estimated from the inverted sequence of time shifts. In this paper, we propose a new method which inverts the V_p/V_s ratio directly from the PP and PS data. Rapid changes in the inverted V_p/V_s ratio are avoided by adopting a cost function that is constrained by a smoothing operator. We also decrease the number of unknowns of the problem by re-parametrizing the V_p/V_s ration in terms of splines.

Theory

We start with the cost function for multi-component seismic data registration

$$J = \iint (e_{pp}(t,x) - e_{ps}(w(t,x),x))^2 \, dt \, dx + \mu \iint \left(\frac{\partial^2 w(t,x)}{\partial t^2} + \frac{\partial^2 w(t,x)}{\partial x^2} \right)^2 dt \, dx \,, \quad (1)$$

where t and x are the PP-wave traveltime and the spatial variable, respectively. The fields $e_{pp}(t,x)$ and $e_{ps}(t,x)$ are used to indicate the envelope functions of PP-wave and converted-wave data, respectively. Similarly, $w(t,x)$ is the warping function which builds the mapping relationship between

PP-wave samples and PS-wave samples. The scalar μ is the trade-off parameter that controls the relative weight between the regularization term $\iint \left(\frac{\partial^2 w(t,x)}{\partial t^2} + \frac{\partial^2 w(t,x)}{\partial x^2}\right)^2 dt\,dx$ and the data difference term $\iint (e_{pp}(t,x) - e_{ps}(w(t,x),x))^2\,dt\,dx$. By penalizing the second order derivative of the warping function, we enforce smoothness on the warping function in t and x. This is clear because the warping function $w(t,x)$ is related to V_p/V_s ratio $\gamma(t,x)$ through the following expression (Fomel et al., 2003)

$$\gamma(t,x) = 2\frac{\partial w(t,x)}{\partial t} - 1\,. \tag{2}$$

The latter can be expressed as follows

$$w(t,x) = \int p(t',x)dt'\,, \tag{3}$$

where $p(t,x) = (\gamma(t,x)+1)/2$. We now proceed to vectorize equation (3)

$$\mathbf{w} = (\mathbf{I}_n \otimes \mathbf{A})\mathbf{P} = \mathbf{G}\mathbf{p}\,, \tag{4}$$

where **w** is the warping function for all traces after being reshape into a vector. Similarly, $p(t,x)$ is expressed in vector form as **p**. The matrix $\mathbf{I}_n$ is an identity matrix with dimension $n \times n$, n is the number of traces. The matrix **A** is the integration operator for one trace. In this way, $\mathbf{I}_n \otimes \mathbf{A}$ represents integration for all traces. The symbol $\otimes$ indicates the Kronecker product that is used to extend integration over one trace to all traces.

We also represent the function $p(t,x)$ in terms of smooth function (Hill et al., 2001; Maintz and Viergever, 1998; Klein et al., 2010). In this work, we propose to use B-spline functions to represent the 2D function $p(t,x)$ (Sederberg and Parry, 1986),

$$p(t,x) = \sum_{j_1=1}^{m_1}\sum_{j_2=1}^{m_2} c_{j_1,j_2} b^{j_1}(t) b^{j_2}(x)\,, \tag{5}$$

where c_{j_1,j_1} indicates the coefficient of the basis spline function, m_1, m_2 indicate the number of splines in the time and in the spatial direction, respectively. Similarly, $b^{j_1}(t)$ and $b^{j_2}(x)$ are the basis B-spline functions for the temporal and spatial variables, respectively. We also write equation (5) in matrix form

$$\mathbf{p} = \mathbf{B}\mathbf{c}\,, \tag{6}$$

where **c** is the vector of spline coefficients and **B** is the spline basis also written in matrix form. Therefore, the final warping field can be expressed as follows

$$\mathbf{w} = \mathbf{G}\mathbf{B}\mathbf{c}\,. \tag{7}$$

Now we can write the cost function in vector form

$$J = \|\mathbf{e}_{pp} - \mathbf{e}_{ps}(\mathbf{w})\|_2^2 + \mu\|\nabla^2\mathbf{w}\|_2^2\,. \tag{8}$$

The Gauss-Newton method is used to minimize the objective function with respect to the vector of spline coefficients

$$\begin{aligned}\mathbf{c}^{k+1} &= \mathbf{c}^k + \alpha\Delta\mathbf{c}\\ \Delta\mathbf{c} &= -\mathbf{H}^{-1}\mathbf{g}\end{aligned}\,, \tag{9}$$

where the vector $\mathbf{g}$ is the gradient of J with respect to the coefficients $\mathbf{c}$. Similarly, $\mathbf{H}$ is the Hessian matrix of the problem. The index k indicates iteration number and α is the step-size. The step size is determined by Armjio's line search method. The update $\Delta\mathbf{c}$ is computed via the conjugate gradients method. It can be easily shown that the Hessian matrix $\mathbf{H}$ is sparse. Therefore, the solution $\Delta\mathbf{c}$ can be efficiently computed via the conjugate gradient method with fast matrix-times-vector products in sparse format.

Example

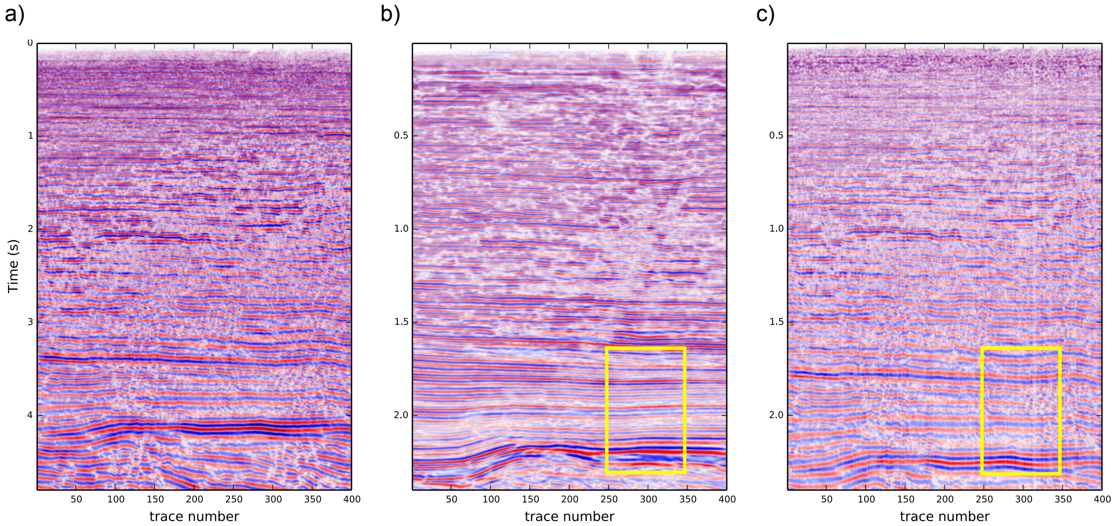

Figure 1: (a) PS-wave data before registration. (b) PP-wave data. (c) Warped PS-wave data, the reflection events of the PS-wave are mapped to PP-wave traveltime.

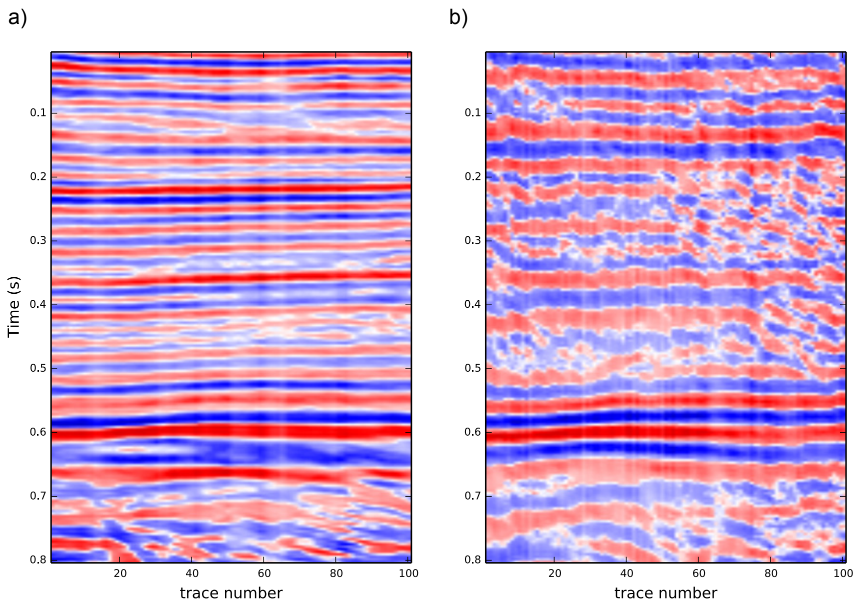

Figure 2: Zoom of the seismic data indicated by the yellow box in Figure 1. (a) PP-wave data. (b) PS-wave data.

The proposed method is tested on a real data set from East China. The data are shown in Figure 1. Figure 1a is the PS-wave data before registration. The PP-wave is shown in Figure 1b. Figure 1c represents warped PS wave data. These data sets consist of 400 traces. The PP-wave section contains traces of 600 samples. The PS-wave section includes traces of 1300 samples each. In this example, we used 33 and 23 B-splines nodes to represent V_p/V_s ratios. The initial model for the V_p/V_s ration is obtained from a simple linear initial velocity trend model. The 1D initial model is shown in Figure 3a. The V_p/V_s ratio decreases with increasing depth. After registration, the reflections in PS-wave section are mapped to PP-wave time with good correlation between the PP-wave and the warped PS-wave main horizons. The final smooth V_p/V_s field is shown in Figure 3b. To further

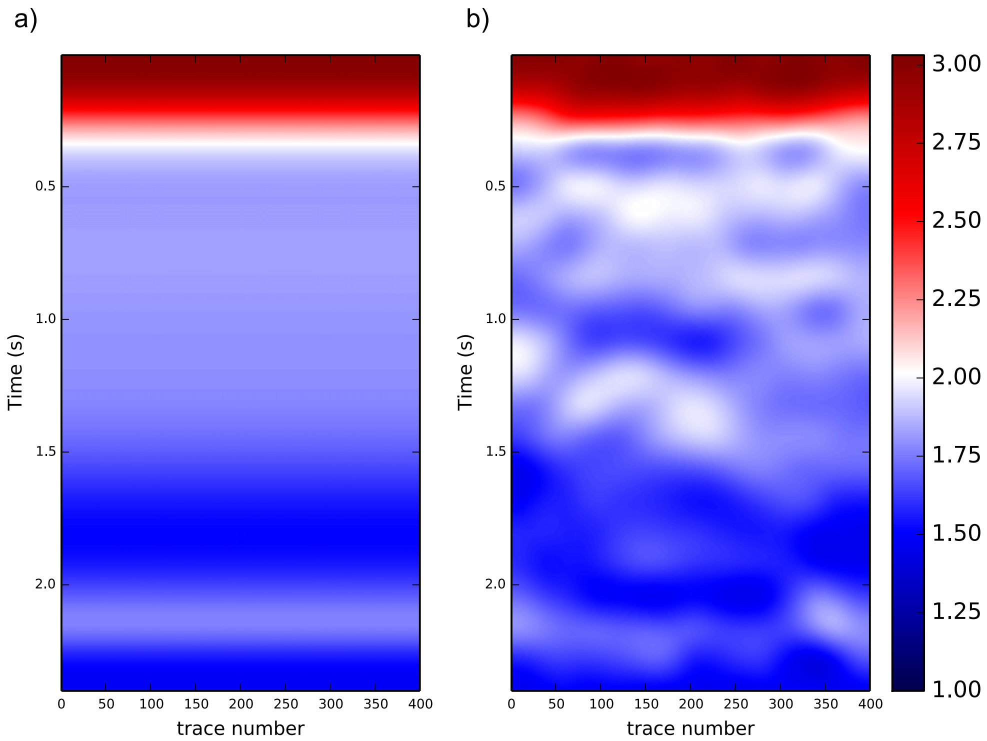

Figure 3: (a) Initial velocity ratio. (b) Final velocity ratio.

compare the result of the proposed registration, we provide a magnified portion of the PP-wave and PS-wave images in Figures 2a-b.

Conclusion

We have presented a new algorithm for multi-component seismic data registration. The algorithms inverts for a smooth V_p/V_s ratio. In our algorithm the velocity ratio is represented by splines. The registration entails minimizing a non-quadratic cost function via the Gauss-Newton method. For the multichannel registration case, the Gauss-Newton algorithm uses sparse matrices multiplication in conjunction with a conjugate gradients solver. The latter makes our algorithm extremely efficient in terms of computational time and applicable for the registration of a large number of traces.

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