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3D Wave-equation-based wavefront reconstruction from sparse data using a finite-difference reverse-time migration technique

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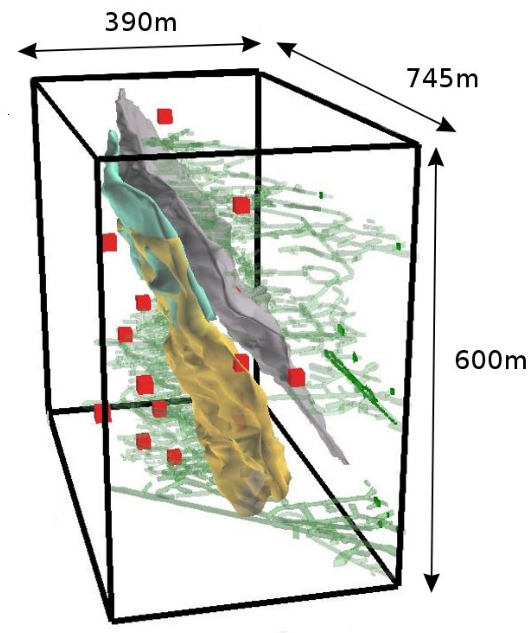
Summary

Extrapolation of seismic wavefields can be very useful for determining the location of the seismic source, as well as how the wavefront propagates given different local conditions. Reverse-time migration can be used effectively to achieve this extrapolation, and does not suffer from the shortcomings of depth migration such as incorrectly migrating steeply dipping layers. Using reverse-time migration, this study aims to reconstruct 3D wavefronts from sparse synthetic data, and to apply this method to real data from Nickel Rim South mine in Sudbury, Ontario. This allows for visualization of the propagation of wavefronts, which is essential for understanding localized effects on seismic waves. Sparse data can also be seen in microseismic arrays involving surface and borehole receivers, which can offer poor azimuthal coverage. Synthetic seismic data will be generated to examine the limit of azimuthal coverage that still enables accurate wavefront reconstruction.

Introduction

A method for extrapolating seismic wavefields backwards in time, known as reverse-time migration (RTM) was introduced by McMechan (1983) and Baysal (1983). Using the two-way wave equation, energy can be propagated through a two-dimensional second order finite-difference grid, thus propagating the wavefront iteratively backwards in time. Prior to RTM, finite-difference algorithms with paraxial approximations had been used in a similar process known as depth-migration, where due to difficulties with these approximations, were not able to correctly migrate steeply dipping layers. The method of reverse-time extrapolation is able to migrate these same dipping layers correctly, as the two-way wave equation, not an approximate one-way wave equation, is used. Using this method, wavefronts can be propagated backwards in time to $t = 0$, where the location of the source may be determined.

Using RTM to reconstruct wavefronts incrementally backward in time may offer insight into localized peak particle velocity (PPV) and peak particle acceleration (PPA) used in hazard assessment. This is a step forward from conventional monitoring, where event triangulation is used and local effects are not usually considered. With microseismic monitoring arrays, used in mining as well as hydraulic fracturing operations, full coverage can be an issue. This leads to sparse data being recorded, and hence more difficulty using migration techniques for imaging purposes. For in-mine microseismic monitoring systems, there is true 3D coverage, but generally it is a sparse array of geophones (see figure 1a). In the case of monitoring hydraulic fracturing wells, vertical seismic profiling is generally used. This involves an array of 3-component (3C) geophones within a borehole, as well as an array of 3C surface geophones (see figure 1b). This specific type of setup may have limited azimuthal coverage in three dimensions. This study will determine a lower-bound to data needed for accurate wave-equation-based reconstruction of wavefronts. In addition to determining source location, the wavefront may be analyzed for local anomalous behaviour throughout the reverse-time migration. For example, within mines with complex heterogeneous settings, seismic waves may experience local amplification and/or attenuation (see figure 2).



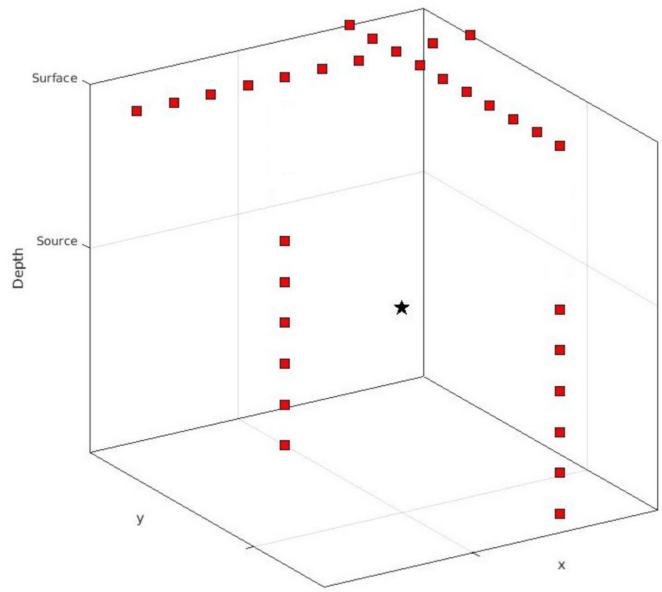


Figure 1a: Model of Nickel Rim South Mine. Shown are 3C geophones as red cubes, the nickel ore body in grey, the copper ore body in yellow, and the mine tunnels in green. This is clearly a sparse array given the volume being monitored.

Figure 1b: 3D data acquisition model of a combination of borehole and surface 3C seismic arrays, surrounding a seismic source. This type of setup is seen in microseismic and hydraulic fracture monitoring.

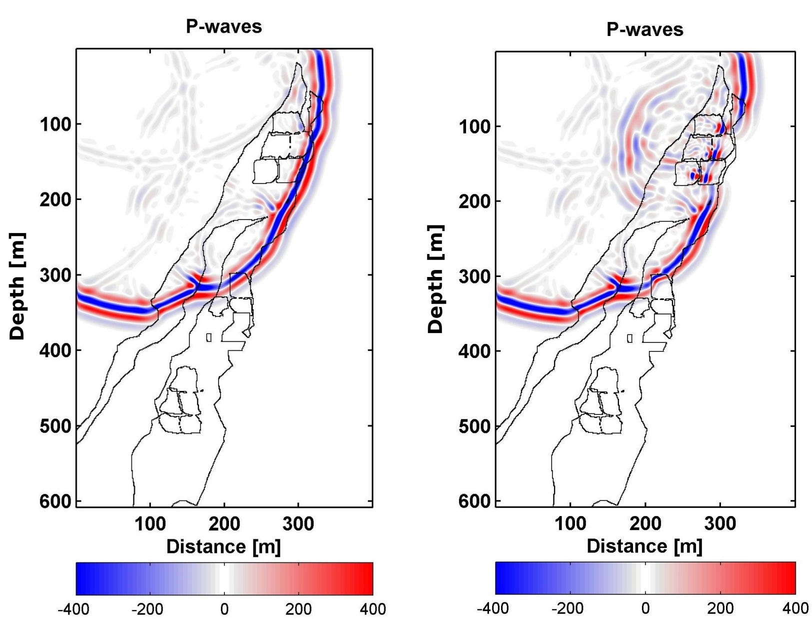


Figure 2: The left image shows p-wave propagation through a mine before production. The right image shows p-wave propagation through a mine after stopes have been added, orebodies have been mined, and voids have been backfilled. (Saleh et al., 2015)

Theory

Reverse-time migration involves an approach to finite-difference migration called boundary value migration (BVM) which was also proposed by McMechan (1983). In this method, energy is propagated through a two dimensional finite-difference grid using the two dimensional wave equation:

$$U_{yy} + U_{zz} = \frac{1}{V^2(y,z)} U_{tt}$$

where U is acoustic pressure, V is velocity, and subscripts denote derivatives with respect to horizontal distance (y), depth (z) and time (t). At a given step in time, the method uses previous wavefields to calculate the current overall wavefield. This involves generating a response wavefield from initial conditions given a known velocity model, which can then be iterated to extrapolate towards an origin time. The following equation gives this response wavefield:

$$U(y_k, z_j, t_i) = 2(1 - 2A^2)U(y_k, z_j, t_{i-1}) - U(y_k, z_j, t_{i-2}) \\ + \; A^2[U(y_{k+1}, z_j, t_{i-1}) + U(y_{k-1}, z_j, t_{i-1}) + U(y_k, z_{j+1}, t_{-1}) + U(y_k, z_{j-1}, t_{i-1})]$$

where A = V(yk, zj)*Δt/h, Δt is the timestep between wavefields, h is the distance between grid lines, and k and j denote vertical and horizontal grid lines indices, respectively. Although formulated in two-dimensions, the above equation can be extended to a third dimension, as seen in Liu (2013). This study will work in three dimensions so as to fully realize localized wavefront behaviour as well as response to sparse data, although the process outlined below is in two dimensions for ease of visualization.

Suggested Methods

The following steps are taken to reconstruct wavefields, in both mining and hydraulic fracturing mining environments. SOFI3D (Bohlen, 2002) is used to generate synthetic seismic data to illustrate how real data can be manipulated. In Figure 3, a source can be seen surrounded by a randomly distributed microseismic array, in which each a 3C seismic wavefront has arrived at each receiver. This is the baseline starting point, using real data from geophones. Each source-receiver distance is determined, and then a reference sphere is created at an intermediate distance, as seen in Figure 4. The waveforms are then phase-shifted in time to the reference sphere as seen in Figure 5. Using these waveforms, the spherical wavefront is calculated by interpolation. Finally, Figures 6 and 7 depict this wavefront being migrated backwards or forwards in time respectively, using reverse-time migration techniques outlined above.

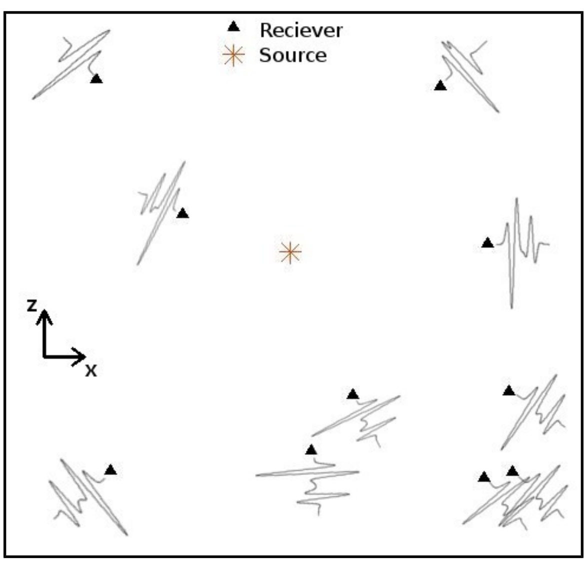


Figure 3: 3C seismic wavefronts at receivers surround seismic source.

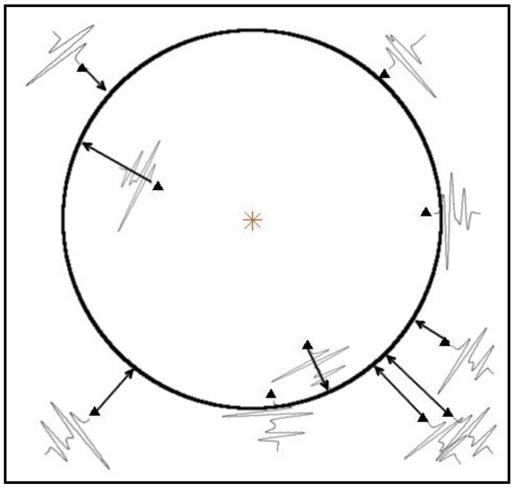


Figure 4: Creation of arbitrary reference radius.

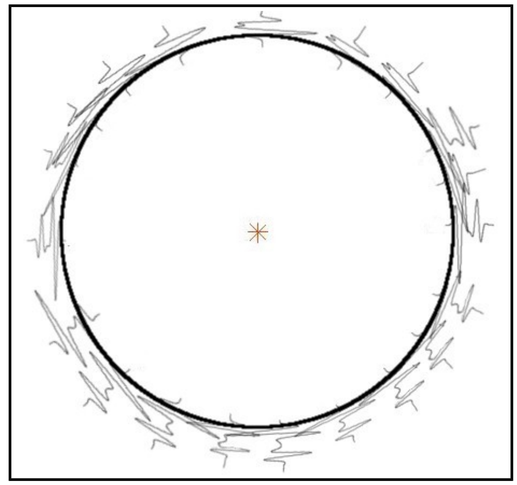


Figure 5: Shift of wavefronts towards reference radius, followed by interpolation.

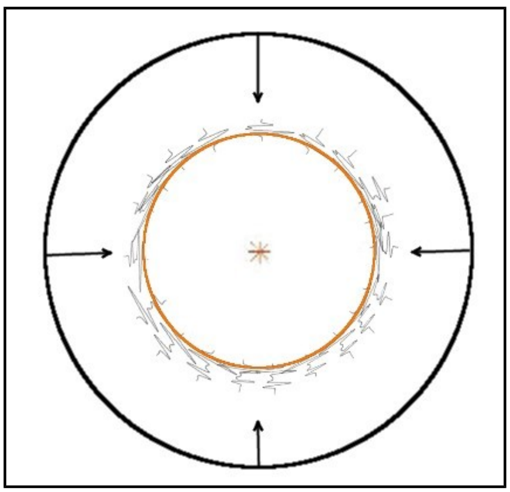


Figure 6: Reverse-time migration.

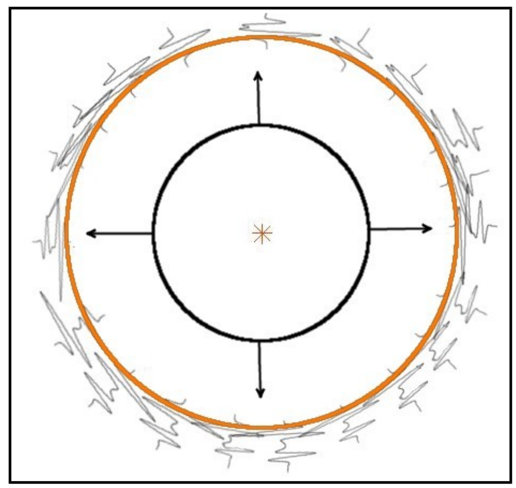


Figure 7: Forward-time migration.

Conclusions

Using finite-difference techniques for reverse-time migration, wavefronts can be reconstructed from sparse data. This is useful for potential hazard assessment of areas where local changes in medium may affect the peak particle velocity or acceleration. Areas like mines and hydraulic fracturing sites need to ensure the highest level of safety, and being able to understand how exactly a wavefront propagates through a specific locality lends itself well to that goal.

Acknowledgements

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