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A Broader Approach to Attenuation Corrections in Seismic Records

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Summary

The correction for attenuation effects in refraction and reflection seismic records is generally a broader problem than the traditional inverse *Q*-filtering. In *Q*-filtering, the effect of attenuation on seismic amplitudes is commonly described by the quality (*Q*) factor. However, in a rigorous sense, the *Q* is only an apparent, i.e. phenomenological property of a wave propagating in a uniform medium. Extending this property to finely layered Earth and treating it as a material property can be difficult and ambiguous.

Another effect of attenuation is the phase-velocity dispersion, which causes noticeable distortions of the propagating or reflected waveforms. It is often believed that the rate of dispersion can be inferred from the *Q* by using the Kramers-Krönig relations. Nevertheless, for practical purposes within the exploration-seismic frequency band, velocity dispersion and *Q* can still be mutually independent. Therefore, to perform attenuation corrections, we need to specify *both* the frequency-dependent *Q* and velocity dispersion *dV*/*df*.

I discuss a general approach to correcting for attenuation effects in seismic records. The approach consists of two steps: 1) modeling the propagating, non-stationary source waveform and 2) deconvolve it from the data in a time-variant manner. In general, the modeling should include not only the effects of a *Q*, but also direct physical mechanisms: wavefront focusing and defocusing, scattering, solid viscosity, and pore-fluid related effects. The method is independent of "tuning" or "reference" frequencies that are present in standard velocity-dispersion laws. The deconvolution can also be performed in several ways. Selecting an appropriate combination of physical mechanisms, frequency dependences of *Q* and dispersion, and inversion algorithms leads to a flexible algorithm for correcting for various types of attenuation effects.

Introduction

Seismic waveforms change during propagation through the Earth, which results in frequency-dependent reduction of amplitudes and phase distortions in seismic records. *Q*-compensation (inverse *Q*-filtering) is a standard procedure often applied to correct reflection data for attenuation effects (Hargreaves and Calvert, 1991; Wang, 2008; van der Baan, 2012). Formulations of the *Q*-filter are usually based on several theoretical wave dispersion relations, such as the Futterman, Azimi-Strickman and constant-*Q* (Kjartansson) models, models of linear solids, Andrade, or Cole-Cole models (e.g., Futterman, 1962; Aki and Richards, 2002; Wang, 2008; Lakes, 2009; Mavko, 2013). Attenuation/dispersion models used in forward wave modeling can be complex (for example, the Generalized Standard Linear Solid (Liu et al., 1976)) but share one key property, namely that their inelasticity is completely described by a single frequency-dependent quality factor, $Q(\omega)$. However, it is in fact unclear how closely such models correspond to wave propagation in realistic layered rock with varying physical properties.

In this paper, I show that a single function $Q(\omega)$ is generally insufficient for describing wave attenuation, and a relatively independent velocity-dispersion law, $V(\omega)$ should be specified by considering viable physical mechanisms of wave propagation. This realization of multiple physical mechanisms leads to a simple, accurate and versatile approach to modeling and correcting for various types of attenuation effects in reflection seismic records.

Theory

Similarly to the traditional forward and inverse *Q* filtering, let us use the correspondence principle (Lakes, 2009). In its broadest form, this principle states that that there exists a simple transformation between waves in elastic and inelastic media. If we start from a harmonic wave in an elastic medium $w_{\mathrm{el}}(x,t) = A_0 \exp(-i\omega t + ik_0 x)$ and turn the inelasticity on, then the waveform becomes modified by changing the amplitude ($A_0 \to A$) and the wavenumber ($k_0 \to k = k_0 + \Delta k$) and adding a spatial logarithmic decrement (attenuation coefficient) α:

$$w(x,t) = A\exp(-i\omega t + ikx - \alpha x). \quad (1)$$

The transformation from $w_{\mathrm{el}}(x,t)$ to $w(x,t)$ can be represented by a linear attenuation operator $\hat{\mathrm{A}}$ (forward *Q* filter; e.g. Hargreaves and Calvert, 1991): $w(x,t) = \hat{\mathrm{A}}\left[w_{\mathrm{el}}(x,t)\right]$. By appropriately selecting the coordinate origin $x = 0$, and geometrical (zero-frequency) attenuation (Morozov, 2008), the amplitude A can be taken equal A_0. Equation (1) then shows that w_{el} is an eigenvector of $\hat{\mathrm{A}}$:

$$\hat{\mathrm{A}}\left[w_{\mathrm{el}}(x,t)\right] = \zeta w_{\mathrm{el}}(x,t), \quad (2)$$

where the eigenvalue equals $\zeta = \exp(i\Delta kx - \alpha x)$. Thus, the modification in the wave shape includes an amplitude decay per unit distance (attenuation coefficient, α) and a phase shift (dispersion, Δk). The phase velocity and *Q* represent combinations of these basic parameters: $V \equiv \omega/(k_0 + \Delta k)$ and $Q \equiv (k_0 + \Delta k)/2\alpha$. The oscillation frequency ω is controlled by the source, and parameters Δk and α, and consequently V and Q also depend on this frequency.

The attenuation-correction procedure consists in finding an inverse filter $\hat{\mathrm{A}}^{-1}$ to transform the inelastic case back into an elastic one. This filter can be constructed by only knowing (measuring, modeling, or postulating) the frequency-dependent waveform attributes Δk and α.

In contrast to the conventional approach, I do not assume that the inelasticity of the medium is described by a *Q* factor of the medium but consider multiple and specific attenuation mechanisms. Consequently, filter $\hat{\mathrm{A}}$ is not always a "*Q* filter" and can be more generally described as an "attenuation filter", or A-filter. The mechanisms considered here include geometric spreading (focusing and defocusing of wavefronts), solid viscosity (Landau and Lifshitz, 1986), scattering, as well as any number of unspecified mechanisms of *Q*-type amplitude decay and dispersion. These mechanisms lead to a factorization of the attenuation operator:

$$\hat{\mathrm{A}} = \hat{\mathrm{A}}_{\mathrm{Geometric}}\,\hat{\mathrm{A}}_{\mathrm{Scattering}}\,\hat{\mathrm{A}}_{Q}\,\hat{\mathrm{A}}_{\mathrm{Viscosity}}\,\hat{\mathrm{A}}_{\mathrm{Dispersion}}. \quad (3)$$

Each of these factors can be evaluated by numerical modeling of the waveform propagating through a layered medium. For each attenuation mechanism, this modeling results in a value of the apparent $Q^{-1}(\omega)$ (attenuation) and $V^{-1}(\omega)$ (phase slowness) at every point within the wave path. It is convenient to combine these two properties in a complex-valued slowness:

$$s^*(\omega) \equiv s(\omega) + is'(\omega) \equiv V^{-1}(\omega)\left[1 + \frac{i}{2}Q^{-1}(\omega)\right]. \quad (4)$$

Among the different factors in eq. (3), the dispersion deserves a special disussion. In the *Q*-filtering literature, it is often stated that "the dispersion is a result of requirement that the wave propagation in an absorbing medium must be causal" (Hargreaves and Calvert, 1991). It is also often inferred from the causality (Kramers-Krönig; K-K) relations that $dV/d\omega$ must be positive and proportional to $1/Q$ within the seismic band (e.g., Wang, 2008). Nevertheless, causality still does not mean that dispersion is caused by attenuation or that it has a definite sign. The K-K relations consist of slowly-converging integrals (Aki and Richards, 2002), and definitive relations between $Q^{-1}(\omega)$ and $V^{-1}(\omega)$ can only be found for certain forms of models defined across infinite frequency bands (Futterman, 1962; Aki and Richards, 2002; Wang, 2008). Within the narrow exploration-seismic frequency band, the K-K relations do not dictate any particular relation between of Q^{-1} and $dV/d\omega$.

The K-K relations represent a fairly weak constraint only expressing the causality of the wave-propagation process. In the time domain, causality simply means that at a travel distance x, the amplitude of the wave equals zero at all times preceding some "wave onset:" $t < t_{\text{onset}} = xV_0^{-1}$, where V_0 is some velocity. In Futterman's (1962) model, V_0 equals $V(\omega_0)$ at an extremely low cutoff frequency ω_0. In several other models (such as linear solids), V_0 is the velocity at infinite frequency (denoted V_∞), and in the constant-Q and power-law models (Kjartansson, 1979; Műller, 1983), $V_0 = \infty$. It seems intuitively clear that such a basic statement cannot constrain the relation between $Q^{-1}(\omega)$ and $V^{-1}(\omega)$ at any given frequency.

In frequency domain, the identity $w(t < t_{\text{onset}}) \equiv 0$ becomes a reciprocal integral relation between the real and imaginary parts of phase slowness in equation (4) (Aki and Richards, 2002):

$$s(\omega) = V_0^{-1} + \mathrm{H}\left[s'(\omega)\right] \text{ , and } \quad s'(\omega) = \frac{\alpha_0}{\omega} - \mathrm{H}\left[s(\omega) - V_0^{-1}\right] \text{ ,} \tag{5}$$

where $\mathrm{H}[...]$ is the Hilbert transform, and $\alpha 0$ is another arbitrary constant of dimensionality [1/distance].

To illustrate the causality constraints in the frequency domain, Figure 1 shows a hypothetical attenuation spectrum $s'(\omega)$ with two peaks and the corresponding phase-delay spectrum $s(\omega)$ evaluated by the first relation (5). Assuming our measurements take place within a relatively narrow frequency band, three characteristic regimes can be recognized (labels in Figure 1). If the observation band is located near the attenuation peak (regime A in Figure 1) or trough (regime B), a near-constant *Q* would be observed, and the velocity dispersion would be positive or negative, respectively. If the observations happen to be made between the peak and trough of Q^{-1}, the dispersion is near zero (regime C in Figure 1). Usual models used in inverse *Q* filtering (e.g., Futterman, 1962) imply a sole peak in $Q^{-1}(\omega)$ (regime A in Figure 1); however, this does not have to be the case in practice.

A gradient $dQ/d\omega$ is difficult to measure within the seismic band, and the asymptotes of $Q^{-1}(\omega)$ at $\omega \to 0$ and $\omega \to \infty$ are unknown. Consequently, regimes A to C can be difficult to identify. In a given measurement, we can expect arbitrary velocity dispersion $dV/d\omega$ alongside a (relatively) arbitrary frequency dependence of *Q*. Within the exploration frequency band, these dependences can be approximated by linear functions of $\log\omega$:

$$s(\omega) = \frac{1}{V_r}\left(1 + s_1 \log\frac{\omega}{\omega_r}\right), \text{ and } \quad s'(\omega) = \frac{1}{2V_r Q_r}\left(1 + s_1' \log\frac{\omega}{\omega_r}\right), \tag{6}$$

where ω_r is some reference frequency, and s_1 and s_1' are dimensionless parameters characterizing the velocity dispersion and frequency-dependence of Q for the wave. The parameterization by the logarithm of frequency is chosen to resemble Futterman's (1962) dispersion relation and similar laws. For measurements on top of a dissipation peak (regime A in Figure 1), s_1 takes on the value of $s_1 \approx -1/(\pi Q_r)$ (Futterman, 1962). With this parameterization, the frequency dependence of $q \equiv Q^{-1}$ is:

$$q(\omega) \approx \frac{1}{Q_r}\left[1 + (s_1' - s_1)\log\frac{\omega}{\omega_r}\right]. \tag{7}$$

Although s_1' is difficult to measure, eq. (7) shows that both s_1' and s_1 are equally important for the effect of attenuation.

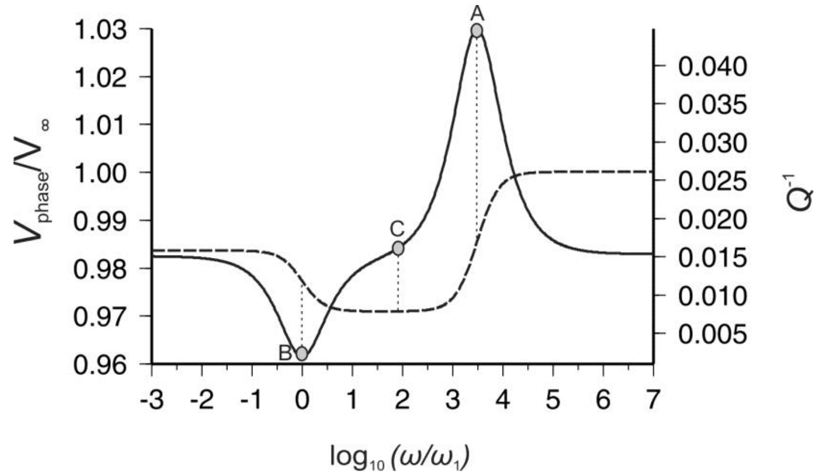

Figure 1. Relations between frequency-dependent inverse *Q*-factor $Q^{-1}(\omega) = -2s'/s$ (solid line) and phase velocity dispersion $V(\omega) = 1/s$ for a simple form of $s'(\omega)$ containing a peak and a trough: $s'(\omega) = 1 + 0.04 f\left(\frac{\omega}{\omega_1}\right) - 0.06 f\left(\frac{\omega}{3000\omega_1}\right)$, where function $f(x) = \frac{x}{1+x^2}$ asymptotically decreases as $O(x)$ for $x \to 0$ and $O(1/x)$ for $x \to \infty$. Dots labeled A, B, and C indicate three possible regimes of narrow-band observation.

Many types of time-variant deconvolution can be used to evaluate the attenuation-corrected waveform $w_{\mathrm{el}}(x,t) = \hat{A}^{-1}\left[w(x,t)\right]$ (eq. (2)). We consider two frequency-domain methods: Wiener filtering (van der Baan, 2012) and its modification known as the “water-level” deconvolution in earthquake studies. In the accompanying paper (Haiba and Morozov, 2016), we also consider time-domain deconvolution bypassing the determination of the inverse filter $\hat{A}^{-1}$ and producing higher-resolution records.

Examples

Attenuation-compensation examples using synthetics and real data are shown in the accompanying paper (Haiba and Morozov, 2016).

Conclusions

Modeling of attenuation effects and correction for them in seismic data is a much broader problem than the traditional forward and inverse *Q* filtering. Within the limited exploration-seismic frequency band, velocity dispersion relations can be relatively independent of the *Q*, and therefore independent amplitude and dispersion corrections need to be considered. In addition to the phenomenological (apparent) *Q*, several non-*Q* type effects can and need to be included: focusing and defocusing of wavefronts (variations of geometric spreading), forward- and back-scattering, solid viscosity, and potentially, pore-flow related effects.

For all types of attenuation and velocity dispersion mechanisms, corrections for their effects on seismic reflection records can be performed by modeling the propagating wavelet during its two-way propagation and deconvolving it from the records.

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