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Fast time domain hyperbolic Radon transforms

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Summary

Seismic data can be modeled using a forward Radon transform. The coefficients needed to synthesize the data in the Radon domain are estimated by solving an inverse problem. In this paper, we present a strategy to drastically speed up the time domain hyperbolic Radon transform. The method is based on a restriction of the domain used to perform the inversion. Furthermore, this approach also helps to focus the data representation and improves the signal-to-noise ratio enhancement. We apply our algorithm to estimate the coefficients of the apex-shifted hyperbolic Radon transform. Our fast implementation performs 20 times faster than the classical approach.

Introduction

Radon transforms have vast applications in seismic data processing (Trad et al., 2003). Originally, they were proposed to increase the SNR of seismic records (Thorson and Claerbout, 1985). Later, Radon transforms gained popularity as a technique for multiple attenuation (Hampson, 1986; Yilmaz, 1989) and data reconstruction (Kabir and Verschuur, 1995). In recent years, multiple applications have been proposed. For instance, seismic data deblending (Ibrahim and Sacchi, 2014, 2015), anti-aliasing for reverse time migration (Wang and Nimsaila, 2014), de-ghosting (Wang et al., 2014) and microseismic signal detection and denoising (Sabbione et al., 2015). Two type of Radon transforms have been proposed. The classical implementation of the Radon transform in the frequency domain is often used to compute parabolic and linear Radon transforms. On the other hand, a time domain implementation is generally reserved for hyperbolic and apex-shifted hyperbolic Radon transforms. In this paper, we will discuss a fast implementation of time domain hyperbolic Radon transforms and pay particular attention to the apex-shifted hyperbolic Radon transform.

Time domain Radon transforms are computed via iterative solvers. In each iteration of a typical solver to compute the Radon transform, one requires two linear operators in implicit form: the forward and the adjoint Radon operators. This paper studies an *on the flight* strategy to compute both operators faster. The proposed method uses the adjoint operator in conjunction with amplitude thresholding to specify the set of coefficients in the Radon space that are active in the representation of the data.

Method

Common-shot gathers can be represented by a superposition of apex-shifted hyperbolas in the data domain (t,x) according to $t^2 = \tau^2 + (x-a)^2/v^2$. Here, τ is the two-way traveltime, and v and a are processing parameters used to focus reflections in the Radon domain. Following this assumption, we can model the data with the time domain apex-shifted hyperbolic Radon transform:

$$d(t,x) = \sum_v \sum_a m(\tau = \sqrt{t^2 - \frac{(x-a)^2}{v^2}}, v, a), \tag{1}$$

$$m_{adj}(\tau,v,a) = \sum_x d(t = \sqrt{\tau^2 + \frac{(x-a)^2}{v^2}}, x). \tag{2}$$

Equation (1) represents the Radon forward operator and equation (2) the adjoint operator of the transform. We can express them in matrix-times-vector form as follows

$$\mathbf{d} = \mathbf{L}\mathbf{m} \tag{3}$$

$$\mathbf{m}_{adj} = \mathbf{L}^T\mathbf{d} \tag{4}$$

In our discussion, $m(\tau, v, a)$ represents the Radon space coefficients required to model the data via expression (1). Thus, the problem reduces to the estimation of $m(\tau, v, a)$ from $d(t, x)$ (Thorson and Claerbout, 1985), and is solved by minimizing the following cost function

$$J(\mathbf{m}) = \|\mathbf{L}\mathbf{m} - \mathbf{d}\|_2^2 + \mu\mathscr{R}(\mathbf{m})\,. \tag{5}$$

We have introduced a trade-off parameter μ and a regularization term $\mathscr{R}(\mathbf{m})$. The regularization term is needed to estimate a unique and stable solution. The classical solution of the Radon coefficients via damped-least squares adopts the l_2 regularization term: ($\mathscr{R}(\mathbf{m}) = \|\mathbf{m}|_2^2$). The high resolution Radon transform, on the other hand, adopts either an l_1 norm or a Cauchy (Sacchi and Ulrych, 1995) constraint to impose sparsity on the distribution of Radon domain coefficients. Independently of the adopted regularization term, one must minimize equation (5) using an iterative solver where the operators $\mathbf{L}$ and $\mathbf{L}^T$ must be provided in matrix-free form. In what follows, we define a strategy to speed up the application of $\mathbf{L}$ and $\mathbf{L}^T$ and, therefore, decrease the computational cost associated to the estimation of $\mathbf{m}$ via iterative solvers.

Both the forward and adjoint operators can be computed by series of nested loops on the variables x, τ, v and a. If we maintain constant the size of the data $\mathbf{d} \in R^{N_t \times N_x}$, the cost of the application of $\mathbf{L}$ and $\mathbf{L}^T$ is proportional to the size of Radon coefficients vector $\mathbf{m} \in R^{N_\tau \times N_v \times N_a}$. In the proposed algorithm, we first determine which elements of the Radon domain are relevant in fitting the data. This is done by applying thresholding to the Radon coefficients obtained via the adjoint operator: $\mathbf{m}_{adj} = \mathbf{L}^T\mathbf{d}$. Clearly, according to equation (2), the coefficients $m_{adj}(\tau, v, a)$ will be relatively large when integrating over reflections and small when integrating over random zero-mean noise. In the first pass of our algorithm we use amplitude thresholding of $m_{adj}(\tau, v, a)$ to define a restricted Radon space $\mathbb{A}$ of active coefficients. The latter is represented as $\mathbb{A} = \left\{(\tau, v, a) \text{ such that } \frac{1}{N_x} \mid m_{adj}(\tau, v, a) \mid > T\right\}$, where T is a threshold parameter that we need to set and satisfies $0 < T < 1$. Note that the data has to be normalized to unity in order to adopt a bounded parameter T. The restricted operators $\mathbf{L}_{\mathbb{A}}$ and $\mathbf{L}_{\mathbb{A}}^T$ are then used by our iterative solver to compute the Radon coefficients needed to model the data:

$$\mathbf{m}_{inv} = \underset{\mathbf{m}}{\operatorname{argmin}} \left[\, \|\mathbf{L}_{\mathbb{A}}\mathbf{m} - \mathbf{d}\|_2^2 + \mu\|\mathbf{m}\|_2^2 \,\right], \tag{6}$$

$$\hat{\mathbf{d}} = \mathbf{L}_{\mathbb{A}}\mathbf{m}_{inv} \tag{7}$$

In our simulations, we have minimized the cost function given by (5) via the method of conjugate gradients (Hestenes and Stiefel, 1952). It is worth noting that this strategy is similar to the algorithm proposed by Liu and Sacchi (2002). Nonetheless, unlike in our approach, Liu and Sacchi (2002) define areas of interest in both domains that change from iteration to iteration.

Example

To illustrate our method, we generated a synthetic data example from a simple 2D model with three reflectors. The data consists of 31 traces separated by $\Delta x = 50$ m with sampling interval $\Delta t = 4$ ms. We used a Ricker wavelet with central frequency $f_0 = 20$ Hz and added band-limited zero-mean random noise to the data with signal-to-noise ratio by amplitude equal to 2. For the first reflector, we set $\tau = 0.3$ m/s, $v = 1500$ m/s and $a = 390$ m; for the second $\tau = 0.5$ m/s, $v = 2400$ m/s and $a = 460$ m;

Parameter		Method	
n_v	n_a	HRT	FHRT
21	41	22.66	1.28
21	61	33.59	1.66
21	81	45.08	2.25
31	41	33.18	1.62
31	61	49.38	3.14
31	81	67.31	3.27
41	41	43.63	2.17
41	61	66.10	3.47
41	81	88.44	4.48

Table 1: Computing time required to model the data with the HRT and with the FHRT. Times are given in seconds.

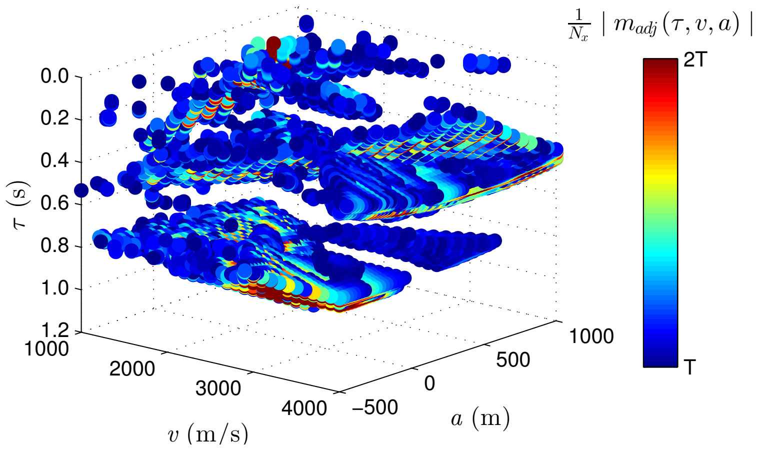

Figure 1: Radon coefficients $m_{adj}(\tau, v, a)$ that form the restricted domain $\mathbb{A}$. The colorbar scale of the plot was limited to $(T, 2T)$.

and $\tau = 0.8$ m/s, $v = 3000$ m/s and $a = -375$ m for the third one. Regarding the inversion, the time parameter of the Radon domain was scanned from $\tau = 0$ to $\tau = 1.2$ s, the velocity from $v = 1000$ to $v = 4000$ m/s and the apex from $a = -500$ to $a = 1000$ m.

We calculated the computational cost of the conventional hyperbolic Radon transform (HRT) and the new fast hyperbolic Radon transform (FHRT). The results are shown in Table (1) in terms of the computing time needed to model the data for different sets of scanning parameters. In all cases, $\Delta\tau$ was set equal to Δt such that all the time samples were scanned. Thus, we only varied the number of velocities n_v and the number of apexes n_a as shown in the table. The threshold parameter T was set equal to 0.08, and we used the same trade-off parameter μ and truncated the inversion after 10 iterations of the conjugate gradients inversion in all the examples. In general, we observe that the proposed method is about 20 times faster than the traditional one.

To gain an insight into the proposed method, Figure (1) shows the Radon coefficients that form the subset $\mathbb{A}$ we used to invert the data. The colorbar scale was limited to $(T, 2T)$ to facilitate the visualization of the plot. This figure, and the following ones, correspond to the test with $n_v = 31$ and $n_x = 61$. Note that only a low percentage of the Radon coefficients are used for the inversion. Figure (2) shows the three panels of the 3D Radon domain that correspond to the velocities used to generate the data. Top panels show the Radon coefficients after equation (2) and the bottom ones the inversion using the restricted Radon domain $\mathbb{A}$. Note that the plots that correspond to the inversion are highly focused on the reflection energy. Finally, the noisy input data and the output of the modeled data are shown in Figure (3). The first plot corresponds to the original noisy data, the second plot to the data inverted using all the Radon coefficients (HRT) and the third one to the inversion using the restricted domain (FHRT). Note that new method not only speeds up the data processing, it also noticeably helps to focus the inversion around the reflection energies and improves the denoising.

Conclusions

The algorithm presented in this work considerably speeds up the application of the time domain Radon transform to reflection data. In addition, it helps to focus the inversion. This result is achieved by restricting the Radon coefficients used in the inversion. We identify the active set of coefficients that stack reflections using the adjoint operator followed by amplitude thresholding. This method improves the utilization of time-domain hyperbolic Radon transforms to perform multiple removal, denoising, interpolation and other applications.

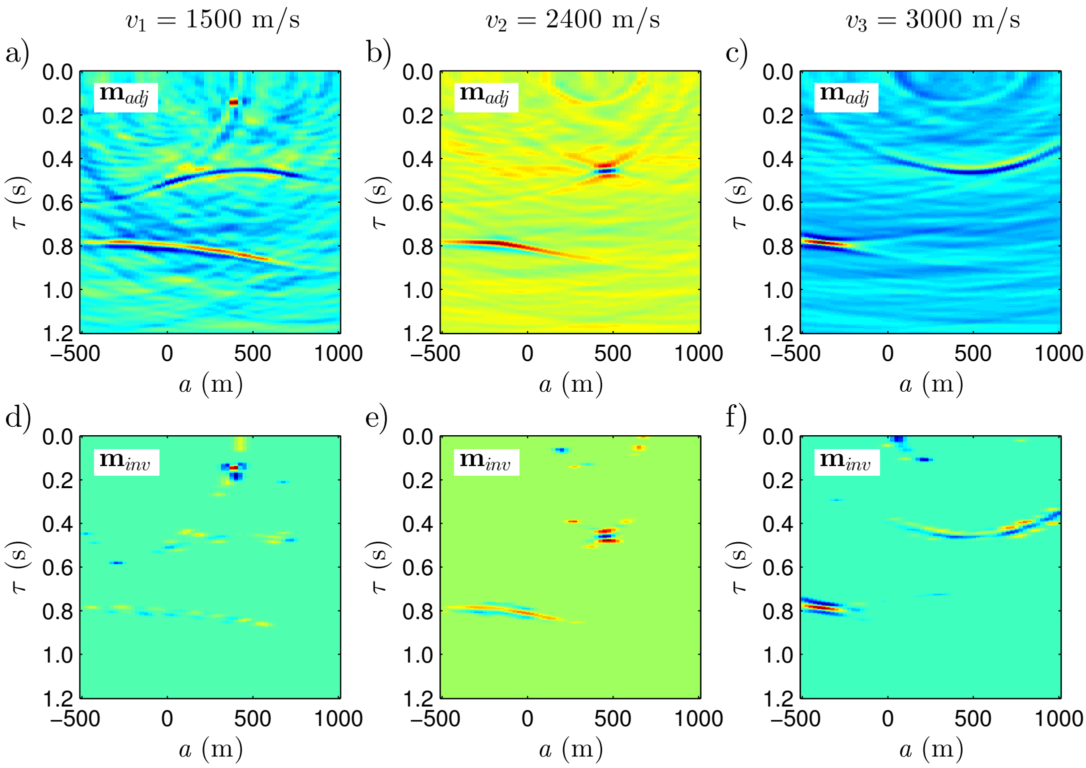

Figure 2: Panels of the 3D Radon domain for constant velocities $v_1 = 1500$ m/s, $v_2 = 2400$ m/s, and $v_3 = 3000$ m/s. a), b) and c) Radon coefficients obtained using the adjoint operator (2). d), e) and f) Inverted model in the restricted domain.

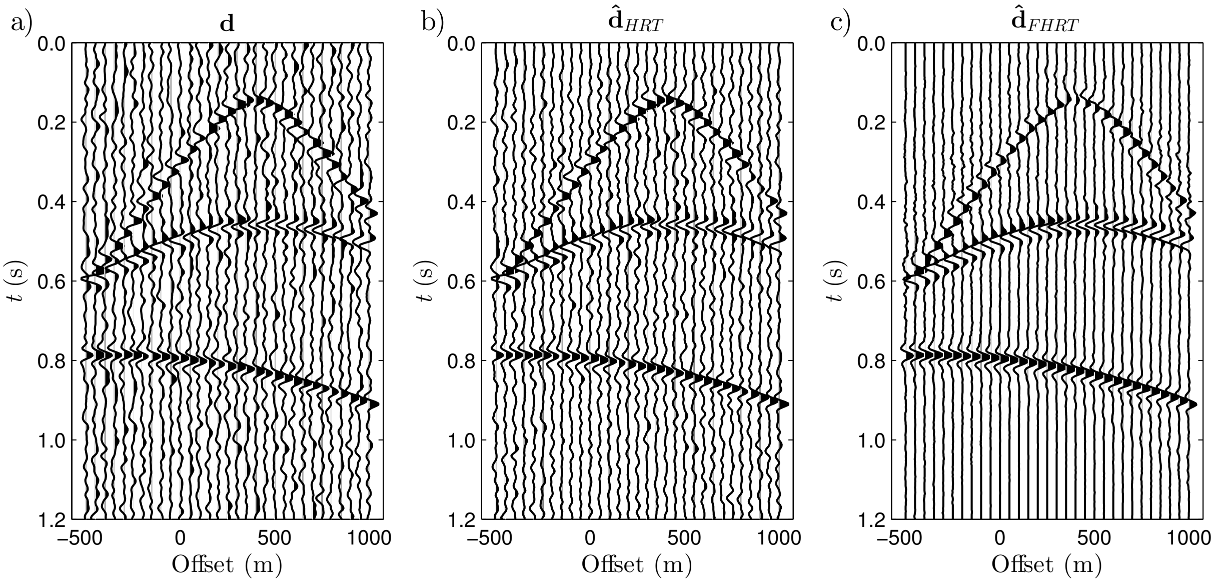

Figure 3: a) Original data with three reflections. b) Data predicted after traditional HRT using all the Radon coefficients. c) Data predicted after FHRT using restricted Radon domain.

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