GeoConvention
Calgary • Canada • March 7-11 2016
OPTIMIZING RESOURCES

Far-offset AVOAz inversion and the symmetry axis ambiguity

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Summary

Through their anisotropic signature, fractures can be remotely detected using the P-wave seismic amplitude variation with offset and azimuth (AVOAz) technique. One of the issues in estimating the fractures is correctly determining their orientation. The near-offset AVOAz inversion is non-unique with two solutions 90 degrees apart. This paper explores the use of information from larger incidence angles and constraints to solve this problem. By themselves larger angles are not the solution. The key is in incorporating geologic information such as the regional stress field and rock physics constraints.

Introduction

In this study a method to solve the seven-parameter linearized AVOAz inversion is demonstrated. The approach is valid for both transverse anisotropic media with a horizontal symmetry axis (HTI), and vertical fractures in an isotropic (VFI) background medium. The seven parameters to be estimated include: three background parameters such as density, P-wave and S-wave impedance reflectivity; and four anisotropic parameters including an orientation parameter. The HTI Rüger equation (1998) is a subset of this problem. One of the key elements in solving the seven-parameter inverse problem is determining the azimuth of the symmetry axis in the case of HTI media, or of the fracture normal in the case of VFI media. For brevity both azimuths are referred to in this paper as the symmetry axis azimuth ϕ_{sym}.

The inverse problem is nonlinear. Similar to the solution of the near-offset HTI Rüger equation the solution is bimodal. This nonuniqueness manifests itself as a 90 degree ambiguity in the estimate of ϕ_{sym}. This ambiguity potentially biases the remaining six parameters. Through the introduction of constraints based on rock physics relationship or geologic control the most likely solution may be chosen.

I begin by reviewing the seven-parameter AVOAz equation and parameterizations specific to HTI and VFI media. The linearized AVOAz expression is then written in terms of azimuthal Fourier coefficients (FCs) (Downton et al., 2011) in order decompose the problem into simpler parts for analysis. The solution of the near-offset linearization is next reviewed with the objective of introducing the symmetry axis ambiguity. It is shown that a priori knowledge of the regional stress field may be used to preferentially choose one solution over the other. Having reviewed the near-offset case, the more complex far-offset problem is discussed and shown to exhibit the same ambiguity. In this case constraints based on the rock physics of fractured media are employed to help resolve the ambiguity. Both synthetic and real seismic data examples are shown to illustrate the method.

Linearized seven-parameter AVOAz

The seven-parameter HTI and VFI P-wave reflectivity, which varies as a function of incidence angle θ and azimuth ϕ, may be written as the Fourier series (Downton and Roure, 2015)

$$R(\phi,\theta) = r_0(\theta) + \mathrm{r}_2(\theta)\cos\left(2\left(\phi-\phi_{sym}\right)\right) + r_4(\theta)\cos\left(4\left(\phi-\phi_{sym}\right)\right). \qquad (1)$$

where in the linearized form, only the magnitudes of the sinusoids of periodicity n = 0, 2 and 4 are nonzero. The magnitudes in equation (1) are

$$r_0(\theta) = A_0 + B_0 \sin^2\theta + C_0 \sin^2\theta \tan^2\theta, \tag{2}$$

$$r_2(\theta) = B_2 \sin^2\theta + C_2 \sin^2\theta \tan^2\theta, \tag{3}$$

$$r_4(\theta) = C_4 \sin^2\theta \tan^2\theta. \tag{4}$$

where the definitions of the parameters A_0, B_0, C_0, B_2, C_2 and C_4 depend on the form of the anisotropy and are described in Downton and Roure (2015). This paper focuses on the B_2, C_2 and C_4 parameters since they control the Amplitude variation with Azimuth (AVAz). In HTI media $B_2 = 0.5B_{ani}$, $C_2 = 0.25\Delta\varepsilon^{(v)}$ and $C_4 = 1/16\Delta\eta.^{(v)}$ The parameter B_{ani} is the anisotropic gradient, $\varepsilon^{(v)}$ is the Thomsen parameter describing the P-wave anisotropy and $\eta^{(v)}$ represents the anellipticity (Rüger, 2002). All the parameters are evaluated at the interface generating the reflectivity with the symbol Δ denoting the difference operator between the lower and upper medium. The phase of the sinusoids is controlled by the symmetry axis azimuth ϕ_{sym}. In the case of VFI media, the parameters B_2, C_2 and C_4 are parameterized in terms of fracture weakness parameters. Rotationally asymmetric fractures give rise to orthorhombic anisotropy. The medium is described by the vertical, horizontal and normal fracture weakness parameters δ_V, δ_H and δ_N respectively. The transformation linking these parameters is

$$\begin{bmatrix} B_2 \\ C_2 \\ C_4 \end{bmatrix} = \begin{bmatrix} \frac{1}{2}g & 0 & -\frac{1}{2}(1-2g)g \\ 0 & 0 & -\frac{1}{2}g(1-g) \\ 0 & \frac{1}{8}g & -\frac{1}{8}g^2 \end{bmatrix} \begin{bmatrix} \Delta\delta_V \\ \Delta\delta_H \\ \Delta\delta_N \end{bmatrix}, \tag{5}$$

where g is the squared S-wave to P-wave velocity ratio of the background media. The case of rotationally symmetric fractures gives rise to HTI anisotropy. In this case, both the vertical and horizontal fracture weaknesses are equal and are replaced by the single parameter, the tangential fracture weakness δ_T.

Linearized AVOAz Inversion

In order to solve the linearized AVOAz inverse problem it is easier to write the Fourier series in terms of cosine (u_n) and sine (v_n) functions, where n refers to the periodicity of the sinusoid. Rewriting equation (1) in block matrix notation and in terms of the sine and cosine FCs results in

$$\begin{bmatrix} \mathbf{u}_0(\theta) \\ \mathbf{u}_2(\theta) \\ \mathbf{v}_2(\theta) \\ \mathbf{u}_4(\theta) \\ \mathbf{v}_4(\theta) \end{bmatrix} = \begin{bmatrix} \mathbf{1} & \mathbf{x} & \mathbf{z} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{x}\cos(2\phi_{sym}) & \mathbf{z}\cos(2\phi_{sym}) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{x}\sin(2\phi_{sym}) & \mathbf{z}\sin(2\phi_{sym}) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{z}\cos(4\phi_{sym}) \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{z}\sin(4\phi_{sym}) \end{bmatrix} \begin{bmatrix} A_0 \\ B_0 \\ C_0 \\ B_2 \\ C_2 \\ C_4 \end{bmatrix}, \tag{6}$$

where $\mathbf{x}=\sin^2(\theta)$ and $\mathbf{z}= \sin^2(\theta)\tan^2(\theta)$. All the bold faced vectors are functions of incidence angle θ. Although written as a set of linear equations, equation (6) is actually nonlinear due to the ϕ_{sym} dependence in the linear operator. A brute force method to solve this system of equations is to iterate over all possible values of ϕ_{sym} solving the least squares problem for each possible ϕ_{sym}. The solution corresponding to the ϕ_{sym} with the minimum misfit is the global solution.

However, the solution is nonunique since it is bimodal. This is more obvious if only the equations describing the AVAz are considered, namely

$$\begin{bmatrix} \mathbf{u}_2(\theta) \\ \mathbf{v}_2(\theta) \\ \mathbf{u}_4(\theta) \\ \mathbf{v}_4(\theta) \end{bmatrix} = \begin{bmatrix} \mathbf{x}\cos(2\phi_{sym}) & \mathbf{z}\cos(2\phi_{sym}) & \mathbf{0} \\ \mathbf{x}\sin(2\phi_{sym}) & \mathbf{z}\sin(2\phi_{sym}) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{z}\cos(4\phi_{sym}) \\ \mathbf{0} & \mathbf{0} & \mathbf{z}\sin(4\phi_{sym}) \end{bmatrix} \begin{bmatrix} B_2 \\ C_2 \\ C_4 \end{bmatrix}. \tag{7}$$

In the near-offset approximation, the **z**= $\sin^2(\theta)\tan^2(\theta)$ terms are ignored resulting in

$$\begin{bmatrix} \mathbf{u}_2(\theta) \\ \mathbf{v}_2(\theta) \end{bmatrix} = \begin{bmatrix} \mathbf{x}\cos(2\phi_{sym}) \\ \mathbf{x}\sin(2\phi_{sym}) \end{bmatrix} [B_2]. \tag{8}$$

It can be seen by substitution that both $(\hat{\phi}_{\mathrm{sym}}, +0.5\hat{\mathrm{B}}_{\mathrm{ani}})$ and $(\hat{\phi}_{\mathrm{sym}} + \pi/2, -0.5\hat{\mathrm{B}}_{\mathrm{ani}})$ fit the data equally well. Typically, only one of the solutions is output. Figure 1a and 1c show the estimated anisotropic gradient B_{ani} and symmetry axis ϕ_{sym} corresponding to the positive B_{ani} solution for a 3D seismic inline. The azimuth solution oscillates 90 degrees between different layers and hence appears nonphysical. Another approach is to make use of local geologic information to constrain the solution. Zoback (2007) observes that the horizontal stress field changes slowly in a regional sense. For stress-induced anisotropy the slow direction corresponds to the direction of minimum horizontal stress (i.e. ϕ_{sym}). If this orientation is known from local well control or from the world stress map (Heidbach et al, 2008) then this information may be used to constrain the solution. In this case, the solution is chosen which is most consistent with the minimum horizontal stress direction. The symmetry axis azimuth for this solution is shown in Figure 1b. By definition, it fits with the known geologic information much better. A further consequence is that B_{ani} has both positive and negative values which again are more geologically believable.

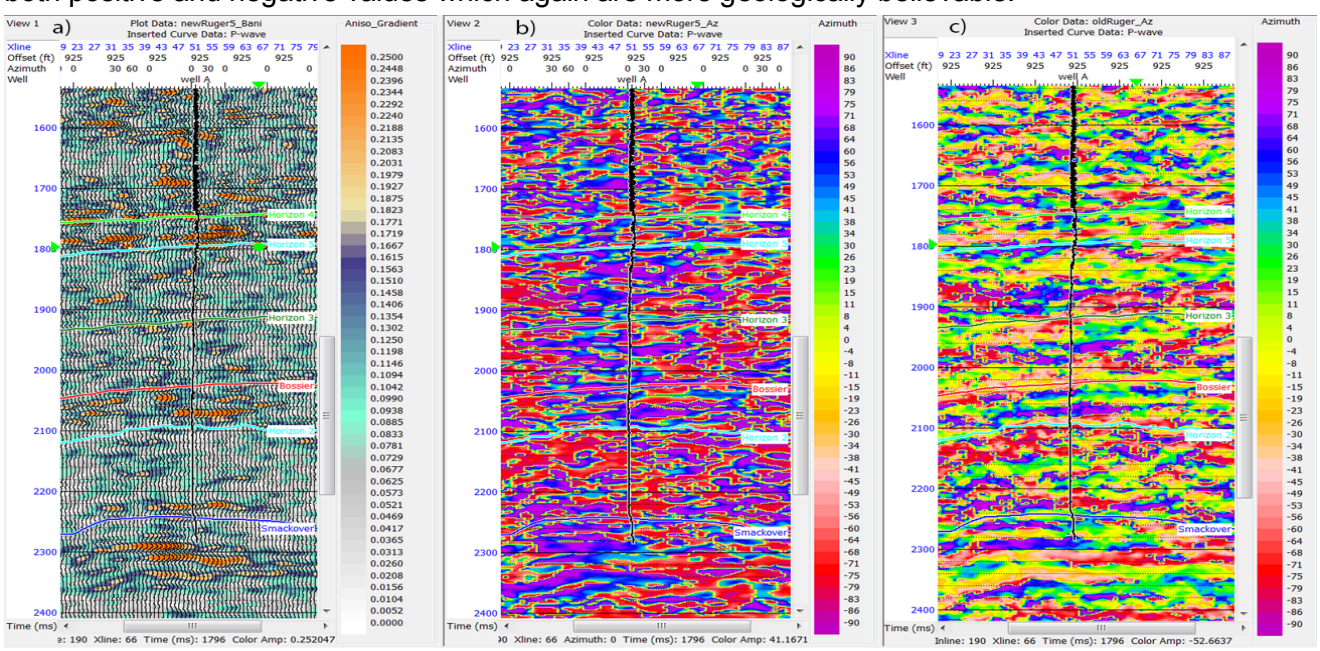

Figure 1. The (a) anisotropic gradient B_{ani}, (b) stress constrained symmetry axis and (c) symmetry axis calculated using the positive B_{ani} solution.

Similar to the near-offset case, the solution to the far-offset equation (7) has two solutions, $(\hat{\phi}_{\mathrm{sym}}, +\hat{\mathrm{B}}_2, +\hat{\mathrm{C}}_2, \hat{\mathrm{C}}_4)$ and $(\hat{\phi}_{\mathrm{sym}} + \pi/2, -\hat{\mathrm{B}}_2, -\hat{\mathrm{C}}_2, \hat{\mathrm{C}}_4)$, as can be seen by substitution. Constraints again may be used to reduce the solution space. Downton et al., (2011) assumed the anisotropy is due to vertical rotationally symmetric fractures. Under this assumption, equation (7) becomes

$$\begin{bmatrix} \mathbf{u}_2(\theta) \\ \mathbf{v}_2(\theta) \\ \mathbf{u}_4(\theta) \\ \mathbf{v}_4(\theta) \end{bmatrix} = \frac{g}{2} \begin{bmatrix} \mathbf{x}\cos(2\phi_{sym}) & ((2g-1)\mathbf{x}+(g-1)\mathbf{z})\cos(2\phi_{sym}) \\ \mathbf{x}\sin(2\phi_{sym}) & ((2g-1)\mathbf{x}+(g-1)\mathbf{z})\sin(2\phi_{sym}) \\ \frac{1}{4}\mathbf{z}\cos(4\phi_{sym}) & -\frac{g}{4}\mathbf{z}\cos(4\phi_{sym}) \\ \frac{1}{4}\mathbf{z}\sin(4\phi_{sym}) & -\frac{g}{4}\mathbf{z}\sin(4\phi_{sym}) \end{bmatrix} \begin{bmatrix} \Delta\delta_T \\ \Delta\delta_N \end{bmatrix}, \qquad (9)$$

This reduction in the number of free parameters leads to a more stable solution and a global minimum, provided the solution does not exist in the null space. In the more general case of rotationally asymmetric fractures, a similar constraint may be used. In this case rather than forcing $\delta_V = \delta_H$, the solution is chosen in which δ_V and δ_H are closest together. Figure 2 shows the application of this constraint to the inversion of equation (6) on synthetic data. The parameters are transformed using equation (5). Alternatively, other empirical rock physics relationships may be used. For example, one popular approximation is to make the P-wave anisotropy $\varepsilon^{(v)}$ approximately equal to the S-wave anisotropy $\gamma^{(v)}$ (Wang, 2002). Constraints may be used in combination. For example, a rock physics constraint may be used in combination with the stress constraint and some spatial continuity constraint.

Discussion

The three-parameter amplitude variation with offset (AVO) inversion is a subset of the seven-parameter AVOAz inversion and thus suffers the same stability issues. In order to get stable three-parameter AVO inversion estimates seismic data with exceptional signal-to-noise and incidence angles in excess of 45 degrees must be acquired and incorporated. The AVOAz inversion problem has the additional requirement of azimuth sampling finer than 22.5 degrees with at least 8 azimuths. In practise, it is probably more stable to work with reduced parameterizations such as equation (9) and shown in Downton et al. (2011).

In practice, the solution of this problem is best implemented as part of a simultaneous azimuthal inversion such as Downton and Roure (2010). In the AVOAz inversion each time sample is treated as an interface and wavelet issues must be dealt with. For example, estimating ϕ_{sym} at the zero crossing is unstable. In addition, it is easier to incorporate greater theoretical complexity, such as allowing the symmetry axis to vary as a function of layer.

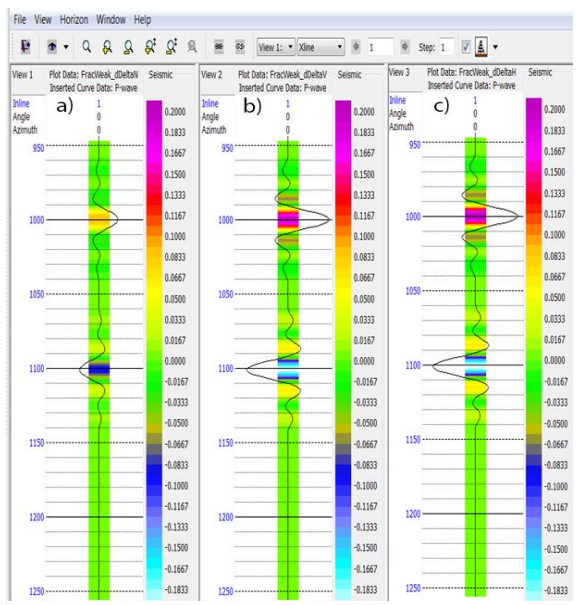

Figure 2: The (a) normal, (b) vertical and (c) horizontal fracture weakness contrast parameters estimated from the seven-parameter AVOAz inversion based on synthetic data. All show the correct phase, this would not be the case if the wrong symmetry axis was chosen. The fracture weaknesses are all positive at the top of the fractured zone (1000 ms) and negative at the base (1100 ms).

Conclusions

Both the near-offset and seven-parameter AVOAz inversions are non-unique and exhibit a 90 degree azimuth ambiguity. This ambiguity biases the remaining six parameter estimates, the size and complexity of which depends on the parameterization. This ambiguity may be reduced by imposing geologic constraints including: the regional stress field, continuity constraints and empirical relations linking the anisotropic parameters. This study showed how to introduce these constraints and also illustrated the solution with both real and synthetic examples.

Acknowledgements

I thank and acknowledge my colleague and collaborator Benjamin Roure for the many discussions that helped lead to this paper. I also thank my new colleague Leonardo Quevedo for some stimulating discussions that helped me look at this problem in a new way.

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