



Multi-trace 1D Laterally Constrained Impedance Inversion

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Summary

We introduce a new approach that uses a lateral constraint to suppress noise and improve the fidelity of formation boundaries in 2D impedance models. A general 1D unconstrained inversion (1D-LUI) framework typically relies on the use of a 1D forward model with each trace being inverted independently following which the 1D impedance estimates are combined together to create a 2D image. This method is fast, however in some cases (e.g. high noise), the resulting 2D impedance model can be noisy making it difficult to distinguish between formation boundaries. To overcome this limitation, while preserving the advantage of low computational cost, a 1D laterally constrained inversion algorithm (1D-LCI) is used to insure that the neighboring 1D models have lateral continuity. Solving the 1D-LCI problem involves the simultaneous inversion of multiple 1D traces producing layered sections with laterally smooth transitions. In addition to enforcing lateral continuity in the inversion model, this algorithm allows for the inclusion of a-priori knowledge from boreholes. We demonstrate the effectiveness of this algorithm on a synthetic 2D model as well as a field seismic dataset.

Introduction

Seismic inversion techniques designed to estimate impedance are common and available for geophysicists through many commercial packages. In most cases, the inversion techniques consider each trace individually, using a 1D forward model. The trace-by-trace inversion results are sensitive to data noise and when combined to form a 2D image, the resulting image can be noisy and mask important geologic features. In this work we refer to this trace-by-trace technique as Laterally Unconstrained Inversion (1D-LUI). Motivated by this issue of noisy impedance models, we present a modification to the 1D-LUI that recognizes spatial correlation between the output impedance models. The laterally constrained inversion (LCI) algorithm was developed for resistivity data to suppress static effects, to reduce the noise resident in the inversion of 1D resistivity data and to decrease the number of equivalent model parameters (Auken et al., 2005). In our work, we build on this study and apply 1D laterally constrained inversion to post-stack seismic impedance inversion. Solving the 1D-LCI problem involves the simultaneous inversion of multiple 1D traces. During the inversion of the seismic traces, we add a constraint on the spatial continuity of the impedance models. This results in reduced noise between adjacent 1D impedance models. We use a second-order derivative approach as a constraint between traces with an a priori reference model derived from borehole logs. In this paper, we present the 1D-LCI algorithm, as well as demonstrate the effectiveness of this approach on 2D synthetic models as well as 2D field examples. In this study, the models produced by the 1D-LCI approach improved in some areas when compared with the laterally unconstrained inversion (1D-LUI) approach. This method is fast, effective, and easy to use.

Theory and/or Method

Most seismic inversion methods utilize some form of the regularized least-squares method to estimate the impedance model (e.g. Gosselin et al. 2003). These methods focus on minimizing an objective function of the form given by equation 1.

$$\theta = \left\| \mathbf{W}_d (\mathbf{s}_j^{obs} - \mathbf{s}_j) \right\|^2 + \lambda^2 \left\| \mathbf{W}_m (\mathbf{l}_j - \mathbf{l}_{ref}) \right\|^2 \quad (1)$$

\mathbf{W}_d is a diagonal matrix of the data error standard deviation (σ_d) and \mathbf{w}_m is a diagonal matrix of the model standard deviation (σ_m), \mathbf{s}^{obs} is the observed seismic trace, \mathbf{s} is the predicted seismic trace, λ is a regularization parameter, \mathbf{l}_j is the natural logarithm of the impedance model parameters and \mathbf{l}_{ref} is the reference impedance model. For this work, the reference model is generated by removing the low frequency trend from the well log data. As a result, the reference model contains only the natural logarithm of the high frequency component of the impedance obtained from the available well log data.

We modify equation 1 to include a lateral constraint that serves to enforce spatial correlation between adjacent traces. This amounts to adding another term to equation 1 and the objective function will be:

$$\theta = \left\| \mathbf{W}_d (\mathbf{S}^{obs} - \mathbf{S}) \right\|^2 + \lambda^2 \left\| \mathbf{W}_m (\mathbf{L} - \mathbf{L}_{ref}) \right\|^2 + \beta^2 \left\| \mathbf{R}_m (\mathbf{L}) \right\|^2, \quad (2)$$

The least-square solution to this is as follows

$$\mathbf{L} = \mathbf{A}^{-1} \mathbf{B} \quad (3)$$

where,

$$\mathbf{A} = \overline{\mathbf{G}}^T \overline{\mathbf{W}}_d^T \overline{\mathbf{W}}_d \overline{\mathbf{G}} + \lambda^2 (\overline{\mathbf{W}}_m^T \overline{\mathbf{W}}_m) + \beta^2 (\overline{\mathbf{R}}_m^T \overline{\mathbf{R}}_m), \mathbf{B} = \overline{\mathbf{G}}^T \overline{\mathbf{W}}_d^T \overline{\mathbf{W}}_d \overline{\mathbf{S}}^{obs} + \lambda^2 (\overline{\mathbf{W}}_m^T \overline{\mathbf{W}}_m \mathbf{L}_{ref}),$$

\mathbf{S} is a column wise representation of m traces, \mathbf{w}_d is a diagonal matrix of the data error standard deviation (σ_d) of n traces. In this work, \mathbf{w}_d was estimated by calculating the standard deviation, trace-to-trace, over a particular sample interval. We note this is not a true sample standard deviation, though it is useful of estimating the relative variability in a given trace. With this algorithm, we can easily explore the effects of this error term by changing our estimate of \mathbf{w}_d , this is useful as a true error model is rarely known. \mathbf{w}_m is a diagonal matrix of the model standard deviation (σ_m). In the absence of a measure of the model standard deviation, this is set to be the identity matrix which is equivalent to setting the standard deviation equal to 1, \mathbf{L} is a concatenated vector of 1D model (l_j) that represents the 2D model space, λ and β are regularization parameters, and \mathbf{R}_m is an approximation to the second derivative in the horizontal direction. We note that using a discretized second order derivative means that the lateral constraint is applied between 3 traces, one to the left of the trace being estimated and one to the right of the trace being estimated. This means that the constraint is a relatively local constraint. The effect of derivative matrix is to minimize the differences between adjacent models. We solve equation 3 using the preconditioned conjugated gradient (Barrett et al., 1994).

Synthetic data

Marmousi Model

Figure 1a shows the Marmousi model (Versteeg, 1994), and the corresponding dataset in figure 1b. The model, based on a prospect from offshore Western Africa, shows two anticlines with the upper anticline sitting virtually on the top of the lower one and contains numerous faults, steep dips and strong velocity variations in both the lateral and vertical direction. Data were simulated at 298 shot locations, with a shot spacing of 25 meters. The data presented in figure 1b are depth migrated with 300 samples per trace and a depth interval of 10m. In addition to the geologic complexity in the Marmousi model, we use this dataset to explore the sensitivity of our algorithm to noise.

Noise test

Figure 2a shows the inversion results using the 1D-LUI for the case of a reference model obtained from trace number 200 in the model (Figure 1a) and 5% Gaussian noise; Figure 2b is the same scenario with using the 1D-LCI algorithm. Figure 2c shows the inversion results using the 1D-LUI for the case of a reference model obtained from trace number 200 in the model (Figure 1a) with 10% Gaussian noise; Figure 2d is the same scenario as in Figure 5c using the 1D-LCI algorithm. The inversion results using 1D-LUI are noisy and suffer from edge effects. It can be seen that the layer boundaries, faults and reservoir are hard to recognize and to track. In contrast, the inversion results produced by 1D-LCI show that the layer boundaries, faults and the reservoir can be mapped with reasonable accuracy. 1D-LCI handles noise well with minimal effect on the resulting model due to increasing the noise level from 5% to 10%.

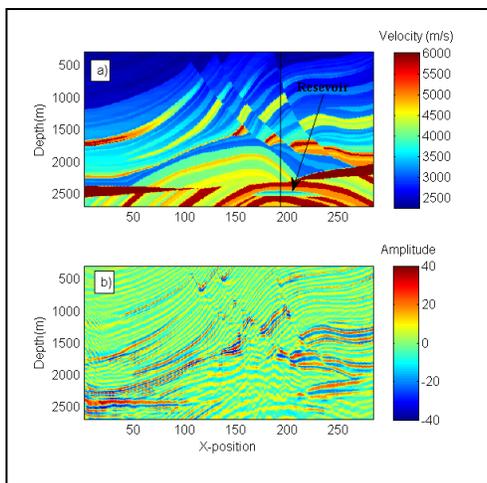


Figure 1, (a) Marmousi true model, (b) Synthetic data produced using Marmousi true model. The black line is the trace number 200 which is used as a reference model true model.

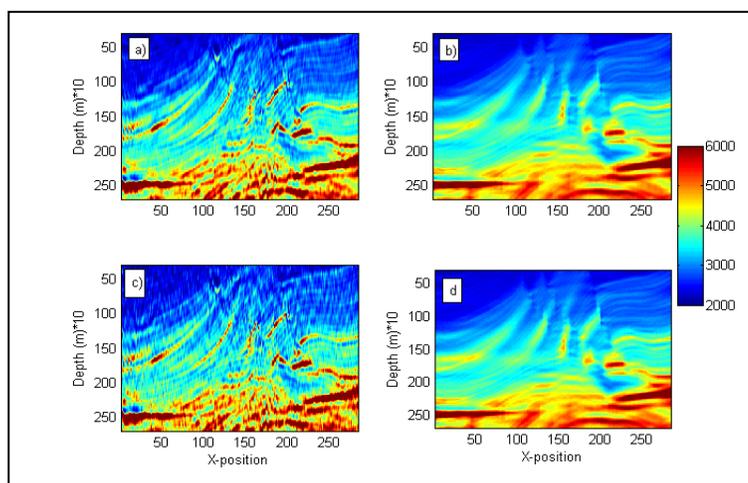


Figure 2, (a) predicted model using 1D-LUI, 5% noise added to the data (b) predicted model with 1D-LCI, 5% noise added to the data, (c) predicted model with 1D-LUI, 10% noise added to the data and (d) predicted model with LCI, 10% noise added to the data.

Field Data

This dataset is known as Husky structural data set and it is owned by Husky Oil and Talisman Energy. This dataset have been used in many studies (e.g. Lines et al., 1996). The dataset contains a seismic line, 9 well logs and picks of the tops of the geological formations of these wells. The 2D seismic line is located in the Western Canadian Sedimentary Basin in the foothills zone, North West of Calgary, Alberta. The total length of this line is about 14.5 km. This data contains 751 samples at 4 ms sampling rate. There are 143 shots with up to 300 channels per shot. The shot and receiver spacing are variable because of the rugged terrain. The average receiver group interval is 20 m, the average shot interval is approximately 100 m and the nearest offset is about 60 m. This field dataset has been chosen because the 2D seismic data has good geological control from several nearby wells and minimal 3D effects, and it captures complex geologic structures. The seismic data were interpreted using synthetic data to correlate key seismic event with the formation tops. The synthetic data were generated from the nearby wells based on sonic and density logs where available. Generally, the hydrocarbon targets in the foothills consist of the structural traps in the Mississippian and Devonian (Paleozoic) carbonate rocks.

Inversion results

The above inversion algorithms were implemented in Matlab, and were used to perform the inversion using a reference model (well log E). The inversion results will be band limited and low frequency data will be added to the results. Figure 3 shows the 2D model obtained with 1D-LUI and 1D-LCI. In 1D-LUI,

the output 1D models were simply combined together resulting in abrupt jumps between neighboring model and a noisy looking image where it is shown jagged appearance of the boundaries between layers. The inversion results using LCI produce well defined horizontal layer interference, that allows us clearly map and distinguish distinct horizontal layers and formation boundaries. 1D-LCI does not suffer from the noisy look common to 1D-LUI because of the constraints that are applied between neighboring models during the inversion.

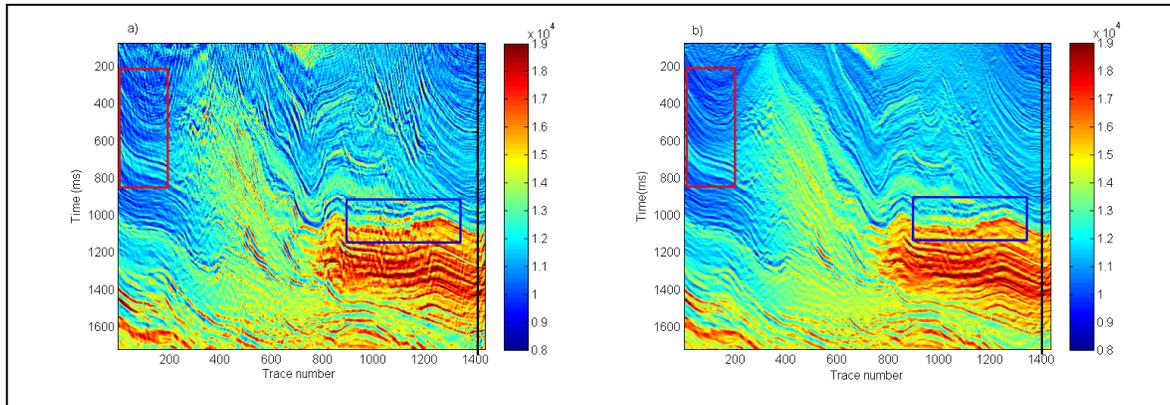


Figure 3. The inverted results of P-impedance; a) result of P-impedance inversion with 1D-LUI and b) result of P-impedance inversion with 1D-LCI. The black line is the reference well used in the inversion. The red boxes show that the noise level of LUI is much higher than LCI. The blue rectangular on 1D-LCI results highlights well define formation boundaries compared to 1D-LUI.

Conclusions

In the 1D-LCI approach, we apply lateral constraints to the estimated model in order to produce a smoother model that, in certain scenarios, may be more geologically realistic when compared to 1D-LUI. 1D-LCI has been tested on complex velocity models (both real and synthetic). 1D-LCI produces well-defined horizontal layer interfaces when compared with 1D-LUI. The 1D-LCI takes advantage of the prior knowledge and lateral constraints to enhance the resolution of the deeper layers. The synthetic models suggest that, while 1D-LCI can offer improvements over an unconstrained inversion; caution must be exercised when using this approach in regions with highly dipping structures (> 20 degrees), as the lateral constraint may overly smooth that impedance model. For our field example, the 1D-LCI approach was able to recover the subsurface model and produce low noise images with a more laterally continuous impedance field when compared to 1D-LUI.

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