Frequency-Domain Rank Reduction in Seismic Processing – An Overview

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Summary

Over the last fifteen years, a new family of matrix rank reduction methods has been developed to remove random noise from and interpolate seismic data. These methods are applied on constant-frequency slices, as a key theorem states that the signal should have low rank in this domain. These filters have been extended to multiple dimensions, robust filtering for erratic noise, coherent noise removal, source deblending, dealiasing, diffraction processing, migration, tensor rank reduction, automatic rank determination, and computational speedups. I briefly describe these developments and show many examples. I also list some open questions and avenues for future development.

Discussion

Over the last fifteen years, matrix rank reduction on constant-frequency slices has emerged as a powerful tool for seismic processing. This talk gives a whirlwind tour of its many applications. A fairly complete set of references, including many not cited in the text of this article, is given at the end.

The general method is as follows. Given a uniform multidimensional grid of traces, do the following:

```
Take the Discrete Fourier Transform (DFT) of every trace.
For every frequency of interest…
  {
    Form a complex-valued matrix from the constant-frequency slice.
    Reduce its rank.
    Place the elements of the matrix back into the frequency slice
       (repeated elements are averaged).
  }
Take the inverse DFT of every trace.
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The *Signal Preservation Property* holds for almost all filters considered here (Trickett, 2002, 2003, 2008):

If the traces are composed of the sum of plane waves having at most k distinct dips then rank-reduction filtering with rank k preserves the signal exactly.

Figure 1 demonstrates this property. It does not hold when filtering on constant-time slices, which is why we work in the temporal frequency domain. Decreasing the rank increases the strength of the filter, but also increases the likelihood that signal will be damaged.

Figure 2 shows how we can form the matrix. In one spatial dimension we form a Hankel matrix. This is called f-x Cadzow or Singular Spectrum Analysis (SSA) filtering (Trickett, 2002; Sacchi, 2009). In two spatial dimensions we have more options. We can treat the two-dimensional constant-frequency slice as an unstructured matrix (called f-xy eigenimage filtering), we can form a “Hankel matrix of Hankel matrices” (called f-xy Cadzow or multichannel SSA), or we can form a hybrid of the two (Figure 3). Unstructured matrices can preserve a wider class of signal, whereas Hankel matrices are more powerful noise suppressors and interpolators (Trickett, 2008; Trickett and Burroughs, 2009). Extending these filters to three or four spatial dimensions makes them more powerful still.

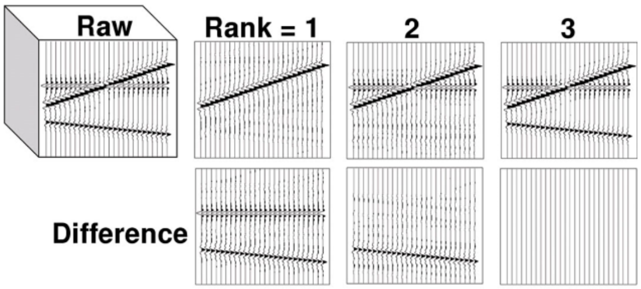

Figure 1: Noiseless synthetic data in two spatial dimensions made up of three plane waves. Rank-reduction filtering with rank three preserves the signal exactly.

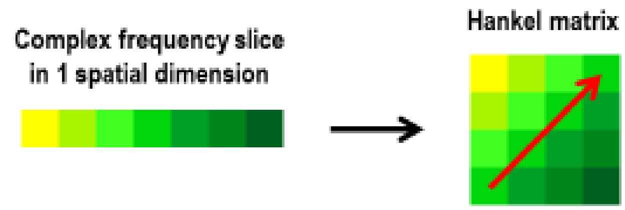

Figure 2: A constant-frequency slice in one spatial dimension can be embedded into a Hankel matrix, which is constant along each anti-diagonal

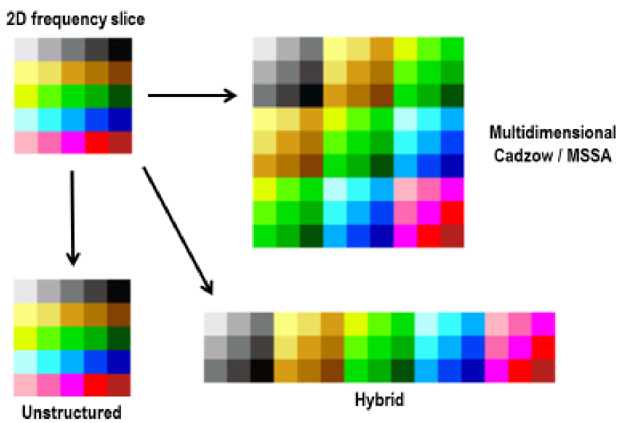

Figure 3: In two spatial dimensions there are more options for embedding a constant-frequency slice into a matrix.

Fast Algorithms

The gold standard for performing rank reduction is the truncated Singular Value Decomposition (TSVD). This finds the matrix of a set rank which minimizes the Frobenius (that is, least squares) norm between it and the original matrix. But the TSVD is expensive for large matrices. Fast approximations to the TSVD have been proposed based on Lanczos methods (Trickett, 2003; Gao et al., 2013; Lu et al., 2014), randomized matrices (Oropeza and Sacchi, 2009b), and QR decomposition (Cheng and Sacchi, 2015). I am not aware of any comparisons between these approaches in regards to speed and accuracy.

Random Noise Suppression

For stacked data, Cadzow / SSA filters are a more powerful alternative to f-x prediction filtering for removing random noise (Trickett, 2003, 2008; Sacchi, 2009). Denoising prestack data is more valuable but more difficult, as it raises the question of what domain to apply these filters. The source-receiver (for 2D) or cross-spread (for 3D) domains are particularly useful. Hybrid methods can be used when sources are not equally spaced (Trickett et al., 2003; Trickett and Burroughs, 2009).

Multifrequency Filtering

Normally one frequency slice goes into forming the matrix. Filtering a narrow band of frequencies at one time has been proposed, resulting in a stronger filter (Oropeza and Sacchi, 2009a; Falkovsky et al., 2011). Such filters do not obey the Signal Preservation Property, suggesting they could harm dipping events.

Robust Filtering for Erratic Noise

The rank reduction step is often carried out using a TSVD or some fast approximation to it. This is a least-squares solution, meaning that it is well suited for random noise which is Gaussian in the spatial directions. Prestack seismic traces, however, often contain erratic non-Gaussian noise, and so the TSVD can give poor results. Robust solutions have been proposed which can better handle erratic noise (Trickett et al., 2012; Chen and Sacchi, 2013; Cheng et al., 2015).

Automatic Rank Determination

The most critical parameter for rank-reduction filtering is the matrix rank. A small rank gives strong noise suppression that may damage signal, and is best suited for simple structures and noisy data. A large rank gives weak noise suppression that preserves signal well, and is best for complex structures and clean data. Conditions can change, however, with time, space, and frequency, making it impossible to select a single rank which is optimum throughout. Trickett (2015) proposed a method to automatically determine an appropriate rank for each matrix. Huang et al. (2015) discuss the related idea of damped SSA.

Interpolation

For a good migration, sources and receivers must be regularly and densely placed. Acquisition constraints, however, means that this is seldom the case. Many different interpolation methods have been proposed in up to four spatial dimensions to fill in missing traces prior to migration. One strategy is based on matrix completion, which fills in missing entries by assuming that the completed signal matrix has low rank (Trickett et al., 2010; Oropeza and Sacchi, 2011; Kumar et al., 2013; Ma, 2013; Yang et al., 2013). These interpolators differ from many others in that they are powerful noise attenuators.

Dealiasing Interpolation

Rank-reduction-based interpolation works well when missing traces are irregularly placed. It fails, however, when missing traces form a regular pattern, such as when every second trace is missing. This ambiguity can be overcome with dealiasing rank-reduction interpolation, which employs Spitz's strategy of using low frequencies to guide the interpolation of high frequencies (Naghizadeh and Sacchi, 2012; Martins et al., 2015).

Unstructured Tensor Interpolation

A tensor is a multi-way array; for example, a vector is a first-order tensor, a matrix is a second-order tensor, and a cube of data is a third-order tensor. A constant-frequency slice taken from a grid of traces in three or four spatial dimensions can be considered an unstructured third- or fourth-order tensor, and one can perform rank-reduction interpolation on it (Kreimer and Sacchi, 2012; Da Silva and Herrmann, 2013, 2014; Gao et al., 2015; Sacchi et al., 2015). One complication is that the concept of *rank* for higher-order tensors is not as simple and unambiguous as it is for matrices.

Nuclear Norm Interpolation

Given the difficulties of tensor rank reduction, nuclear norm minimization has been recommended in its stead (the nuclear norm of a matrix is the sum of its singular values). This approximates optimum tensor rank reduction, but is a more tractable problem (Kreimer et al., 2013; Ely et al., 2013; Kumar et. al., 2013).

Hankel Tensor Interpolation

Just like in matrices, the entries in tensors can be formed into Hankel structures. In the simplest such scheme, each spatial dimension results in two tensor orders. Thus four spatial dimensions results in an 8^{th}-order tensor (Trickett et al., 2013). Similar to matrices, unstructured tensors can model a wider range of signal, whereas Hankel tensors can remove more noise and interpolate across larger gaps. There have been no published studies as to which is more useful on real data.

Cleaning Up First Arrivals

Cleaning up first arrival data has the potential to improve the picking of first arrival times prior to estimating weathering statics. It's a difficult problem, however, because short-wavelength source and receiver statics must be preserved. Frequency-domain rank-reduction filtering of unstructured matrices on cross-spread gathers can preserve these statics (Dack et al., 2014). Selecting a single rank that works well for all frequencies, however, is tricky, and so automatic rank determination is advised.

Source Deblending

Simultaneous shooting, where a shot is fired before the previous shot is fully recorded, is becoming common. This presents the problem of removing external shot energy (cross-talk) from shot records. Rank reduction techniques have been used to solve this (Maraschini, 2012; Cheng and Sacchi, 2015; Kumar et al., 2015) These approaches exploit the fact that, when viewed in the correct domain, the cross-talk energy becomes incoherent. Robust methods are preferred since this energy appears erratic.

Coherent Noise Removal

Standard rank-reduction filtering is best suited for removing random noise. Anything coherent is considered signal and is preserved. It can be adapted for coherent noise removal, however, if you can determine which singular components represent noise rather than signal (Nagarajappa, 2012). This is tricky as there is no guarantee that it's the first component, and so such algorithms must guard against removing signal.

Diffraction Processing

Diffraction processing can help reveal structural discontinuities. The first step is to separate planar from diffracting energy. Liu et al. (2013) used rank-reduction filtering to do so.

Least-Squares Migration

Least-squares migration is an ill-conditioned linear inversion, requiring the selection of one solution from an infinite set of possibilities. Rank-reduction filtering can be used to constrain the solution to one that is spatially coherent, resulting in fewer migration artifacts (Li, Huang, et al., 2015).

Final Remarks

Rank-reduction filtering is finding wide application in seismic processing. This shows no sign of slowing down, as many interesting questions and avenues for development remain.

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