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# Least squares Kirchhoff depth migration with anti-aliasing and preconditioning

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## Summary

Seismic data is generally incomplete and confined by limited aperture. These characteristics reduce the subsurface illumination and hinder the ability of the Kirchhoff migration operator to remove artifacts via destructive interference. Least squares migration (LSM) has been shown to be effective at suppressing many of the artifacts produced by the Kirchhoff operator. Preconditioning the LSM problem results in faster convergence and fewer artifacts in the common image gathers (CIGs). Large trace spacing is responsible for additional artifacts arising from operator aliasing and can be prevented through the use of anti-aliasing filters. This article reviews these issues and demonstrates how triangular-shaped filters can be used to efficiently suppress artifacts in the output image volume.

## Introduction

Kirchhoff migration is based on integration. The subsurface is treated as a body of scatterers and the seismic data consists of the part of the wavefield that is scattered back to the surface. The Kirchhoff operator integrates over all of the traces to collapse diffracted energy back to its correct location in the subsurface. Because the data is recorded at discrete locations integration is replaced with finite summation. This approximation can lead to artifacts in the output image for a number of reasons. First, because the spacing between successive traces is finite there is also a finite operator moveout. The moveout varies along the diffraction trajectory and limits the local maximum unaliased frequency that can be migrated. A number of methods have been developed for handling operator aliasing. Gray (1992) proposed a method in which the data is filtered using pass bands with different cutoffs. The migration operator then extracts amplitudes from the data set which has been filtered to match the local maximum unaliased frequency. Alternatively, one could interpolate new traces to reduce the amount of operator moveout (Yilmaz, 2001). Both of these approaches suffer from the drawback that they require generating and storing additional data. Lumley et al. (1994) solved this issue by utilizing a triangular filter that can be implemented on-the-fly at very little computational cost.

Additional artifacts arise from spreading the amplitudes from a finite number of traces over traveltime curves. Although much of the energy sums constructively at the location of real reflectors, some energy persists as artifacts. Least squares migration addresses this problem by searching for a model that minimizes the misfit between the recorded and synthetic data. A significant amount of work has been performed on this topic. Nemeth et al. (1999) performed least squares Kirchhoff migration and found that the image contained fewer artifacts compared with migration using the adjoint Kirchhoff operator. Küehl and Sacchi (2003) used LSM with a double square root operator to express reflectivity as a function of subsurface position and ray parameter.

This article considers least squares migration using a Kirchhoff operator. The issue of operator aliasing is reviewed along with a solution using a triangular anti-aliasing filter. Preconditioned least squares migration is discussed as a means for removing additional artifacts and improving the resolution of the image. Two synthetic examples are provided to illustrate the benefits of anti-aliasing and preconditioning in least squares migration.

## Theory

### Kirchhoff operator

Least squares migration requires both the forward and the adjoint operator. The forward operation is responsible for mapping the reflectivity volume $\mathbf{m}(\mathbf{x})$ to the seismic data $\mathbf{d}(\mathbf{s},\mathbf{g},\mathrm{t})$ and is given by

$$\mathbf{d}(\mathbf{s},\mathbf{g},\mathrm{t}) = \sum_{\mathbf{x}} \mathbf{m}(\mathbf{x})\mathbf{K}(\mathbf{x},\mathbf{s},\mathbf{g},\mathrm{t}) \,, \tag{1}$$

where $\mathbf{K}(\mathbf{x},\mathbf{s},\mathbf{g},\mathrm{t})$ are the Kirchhoff weights, $\mathbf{x}$ denotes position within the subsurface, $\mathbf{s}$ and $\mathbf{g}$ are the source and receiver position, respectively, and t is traveltime. The traveltimes and amplitudes can be computed by raytracing using the wavefront construction method (Vinje et al., 1993). The adjoint operation maps the seismic data to the reflectivity volume and is given by

$$\mathbf{m}(\mathbf{x}) = \sum_{N} \mathbf{d}(\mathbf{s},\mathbf{g},\mathrm{t})\mathbf{K}(\mathbf{x},\mathbf{s},\mathbf{g},\mathrm{t}) \,, \tag{2}$$

where N is the number of traces in the data. Equations (1) and (2) can be written more compactly as $\mathbf{d} = \mathbf{Lm}$ and $\mathbf{m} = \mathbf{L}^T\mathbf{d}$, where $\mathbf{L}$ and $\mathbf{L}^T$ are the forward and adjoint Kirchhoff operator, respectively.

### Anti-aliased Kirchhoff migration

Kirchhoff migration associates each scatterer in the model domain with a diffraction shape in the data domain. Seismic data contains a finite number of traces. Thus the diffraction will be sampled a finite number of times with a finite operator moveout between adjacent samples. In order to remain unaliased, the local operator moveout must satisfy the dip Nyquist criterion,

$$f_{max} = \frac{1}{2\Delta t} \,, \tag{3}$$

where $f_{max}$ is the maximum unaliased frequency allowed and $\Delta t$ is the local operator moveout. Lumley et al. (1994) proposed a clever method for efficiently implementing a triangular anti-aliasing filter on-the-fly. The filter can be expressed in the Z-domain as

$$g(Z) = \frac{-Z^{-k-1} + 2Z^0 - Z^{k+1}}{\alpha(1-Z^1)(1-Z^{-1})} \,, \tag{4}$$

where $\alpha = (k+1)^2$ and $k$ is the half-length of the triangular filter, given by

$$k = \frac{1}{f_{max}\Delta t} - 1 \,. \tag{5}$$

The $1/(1-Z^1)$ and $1/(1-Z^{-1})$ in Equation (4) represent causal and acausal integration, respectively. Thus the anti-aliasing filter can be implemented by replacing the Dirac delta-shaped diffraction trajectory with a three-point Laplacian shaped diffraction trajectory, followed (preceded) by causal and acausal integration for the forward (adjoint) operator.

### Least squares migration

The inverse Kirchhoff operator is generally complicated or impossible to compute. Instead it is common to use the adjoint for migration. The adjoint is capable of recovering the true location of reflectors, but generally with poor resolution and incorrect amplitudes. Least squares migration addresses these issues by solving an optimization problem where the goal is to recover a reflectivity model that minimizes the difference between the true data and the predicted synthetic data. Moreover, if the true data is given by the system of equations $\mathbf{d} = \mathbf{Lm} + \mathbf{n}$, where $\mathbf{n}$ is noise, then the least squares cost function to minimize is

$$J = ||\mathbf{Lm} - \mathbf{d}||_2^2 + \mu^2 R(\mathbf{m}) \,, \tag{6}$$

where $\mu$ is the tradeoff parameter and $R(\mathbf{m})$ is the regularization factor. Equation (6) is typically solved using iterative methods such as conjugate gradients (Scales, 1987).

## Preconditioned least squares migration

Common image gathers should consist of laterally continuous events if the migration velocity model is correct. It is therefore appropriate to utilize a cost function that penalizes sharp changes in model properties across the offset dimension. This can be written as

$$J = ||\mathbf{Lm} - \mathbf{d}||_2^2 + \mu^2 ||\mathbf{Dm}||_2^2 \,, \tag{7}$$

where $\mathbf{D}$ is the derivative operator applied along the offset dimension. $\mathbf{D}$ can also be thought of as a “bad pass" operator which emphasis features that should be minimized in the model. By making the change of variables $\mathbf{z} = \mathbf{Dm}$, Equation (7) can be rewritten as

$$J = ||\mathbf{LD}^{-1}\mathbf{z} - \mathbf{d}||_2^2 + \mu^2 ||\mathbf{z}||_2^2 \,. \tag{8}$$

If $\mathbf{D}$ acts as “bad pass" operator then its inverse, $\mathbf{D}^{-1}$, acts as a “good pass" operator (Wang et al., 2004). Thus $\mathbf{D}^{-1}$ can be replaced with an operator that promotes desired features in the model. A natural choice for promoting lateral continuity within CIGs is a triangular filter applied across offset. Triangular filters are inexpensive to implement and retain the advantage of placing more weight on the central sample. The preconditioner can be concatenated with the Kirchhoff operator and solved efficiently using conjugate gradients.

## Examples

## Anti-aliasing

We demonstrate the impact of operator aliasing by migrating synthetic zero-offset data based on a two-reflector model. In Figure 1 the input data are migrated using LSM (a) with an aliased Kirchhoff operator and (b) with an anti-aliased Kirchhoff operator. The aliased Kirchhoff operator attempts to migrate features whose frequencies are beyond the Nyquist frequency of the operator, resulting in precursor artifacts that precede the real reflectors. On the other hand, the anti-aliased operator suppresses frequencies beyond the Nyquist frequency and therefore eliminates the aliased energy from the image. Because the anti-aliased opertor automatically suppresses the aliased energy, least squares migration is capable of acheiving higher resolution and converging in fewer iterations, as shown in Figure 1 (c). This observation is important. Anti-aliasing not only improves the quality of the image, but also improves the convergence rate of conjugate gradients.

## Preconditioning

We investigate preconditioning using a 7-point triangular filter and synthetic prestack data. In Figure 2 the data are migrated using preconditioned LSM to form (a) a stacked image and (b) a common

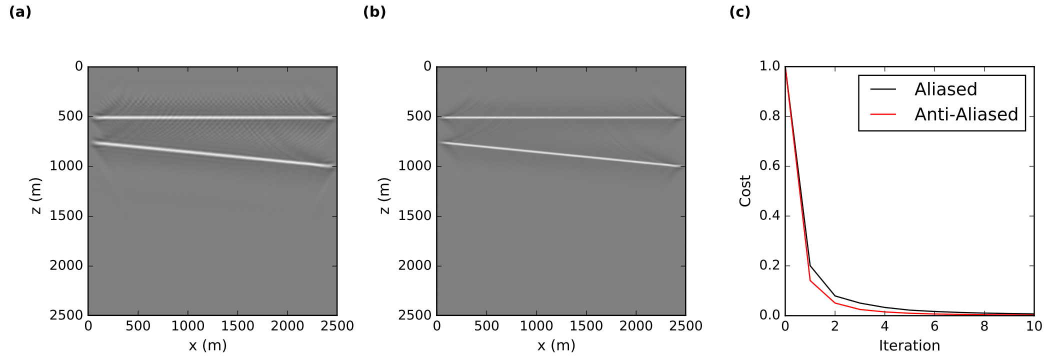


Figure 1: (a) LSM with an aliased Kirchhoff operator. (b) LSM with an anti-aliased Kirchhoff operator. (c) Convergence curve for (a) and (b).

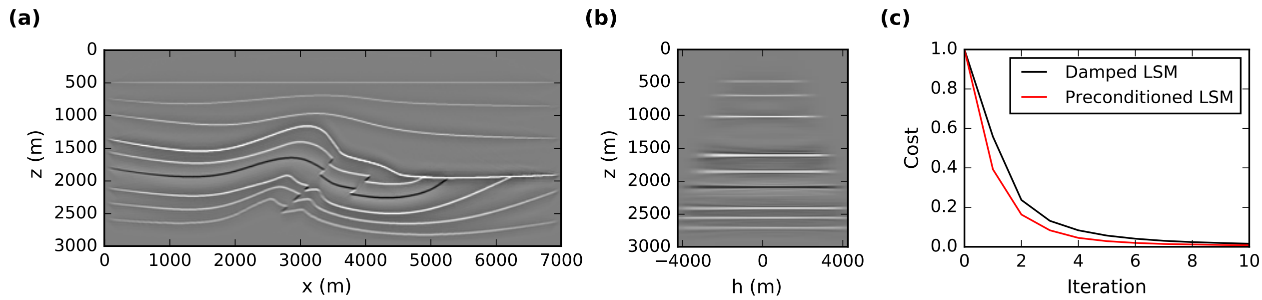


Figure 2: (a) Stacked image using preconditioned LSM. (b) CIG using preconditioned LSM. (c) Convergence curve for preconditioned and damped LSM.

image gather. Preconditioned LSM has resulted in an image with correct amplitudes, high resolution, and minimal artifacts. The common image gathers contain events that are laterally continuous, with minimal migration stretch and correct amplitude versus offset (AVO) signatures. Additionally, the preconditioner has helped to suppress the low-amplitude artifacts relating to incomplete destructive interference from the Kirchhoff operator. Although not shown here, the image volume is a substantial improvement over migration using the adjoint, which suffers from incorrect amplitudes, low resolution, and poor illumination in complex and deep parts of the model. The image volume is however remarkably similar to that obtained by damped LSM; the only noteworthy differences are improved lateral continuity of events and fewer artifacts within CIGs. The benefit of preconditioned LSM over damped LSM becomes apparent when one looks at the convergence curves shown in Figure 2 (c). Preconditioning has greatly improved the rate of convergence. In fact, preconditioned LSM required only 15 iterations of conjugate gradients to reach convergence while damped LSM required 33 iterations.

## Conclusions

This article discussed the problem of operator aliasing along with an solution using a triangular filter. Least squares migration was reviewed as a means for suppressing artifacts arising from incomplete seismic data. A preconditioner was introduced into the least squares formulation to promote lateral continuity within common image gathers. Synthetic tests revealed that the anti-aliasing filter is effective at suppressing the aliased energy in the image and can also reduce the number of iterations of conjugate gradients required to reach convergence. Preconditioned LSM was shown to be capable of recovering an image with correct amplitudes, high resolution, minimal artifats, and laterally continuous CIGs. Preconditioned LSM was also shown to converge much faster than damped LSM.


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## References

Gray, S. H., 1992, Frequency-selective design of the kirchhoff migration operator: Geophysical prospecting, **40**, 565–571.

Küehl, H., and M. D. Sacchi, 2003, Least-squares wave-equation migration for avp/ava inversion: Geophysics, **68**, 262–273.

Lumley, D. E., J. F. Claerbout, D. Bevc, et al., 1994, Anti-aliased kirchhoff 3-d migration: Exp. Abs. SEG National Convention, 1282–1285.

Nemeth, T., C. Wu, and G. T. Schuster, 1999, Least-squares migration of incomplete reflection data: Geophysics, **64**, 208–221.

Scales, J. A., 1987, Tomographic inversion via the conjugate gradient method: Geophysics, **52**, 179–185.

Vinje, V., E. Iversen, and H. Gjøystdal, 1993, Traveltime and amplitude estimation using wavefront construction: Geophysics, **58**, 1157–1166.

Wang, J., H. Kuehl, and M. D. Sacchi, 2004, Preconditioned least-squares wave-equation avp migration: Presented at the CSEG Annual Convention, CDROM.

Yilmaz, Ö., 2001, Seismic data analysis: Society of exploration geophysicists Tulsa, **1**.