SSA for multicomponent data

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Summary

Removing noise in seismic sections facilitates clearer interpretation by increasing the signal-to-noise ratio. Singular spectrum analysis (SSA) is one method for denoising seismic data. This method has been applied to single component data. Applying the algorithm to multicomponent geophones requires either repeating the algorithm for each component or to embed the multicomponent data into a special Hankel form. We present research that address the different ways in which one can embed multicomponent data in the SSA Hankel matrix.

Introduction

Originally, SSA utilized a trajectory (Hankel matrix) of time series in an eigen-spectra decomposition (Vautard and Ghil, 1989). The method is often applied to single component seismic data denoising (Oropeza and Sacchi, 2011). SSA has also been employed in geophysics for studying climatic records (Read, 1993; Plaut and Vautard, 1994, Hsieh and Wu, 2002; Ghil et al., 2002), filtering digital terrain models (Golyandina et al., 2007), and to estimate time series for signal reconstruction (Golyandina and Stepanov, 2005). However, our focus is its application to seismic data enhancement. We concentrate our research into understanding ways of utilizing SSA for vector field data.

Theory

The SSA denoising method can be summarized by the following steps

1. Perform Fourier Transform on data to bring it to the frequency domain.
2. Form a Hankel matrix from spatial samples at one frequency. Samples are represented by S_i. A Hankel matrix is of the form
$$M = \begin{bmatrix} S_1 & S_2 & \cdots & S_K \\ S_2 & S_3 & & S_{K+1} \\ \vdots & & \ddots & \vdots \\ S_K & S_{K+1} & \cdots & S_N \end{bmatrix} .$$
3. Execute a Singular Value Decomposition (SVD) on the Hankel matrix to reduce its rank.
4. Average along the anti-diagonals of the reduced-rank matrix to estimate the denoised spatial signal.
5. After performing SSA on all frequencies, we inverse Fourier Transform the data.

Method

We investigate two primary methods for simultaneous denoising. The second method is separated into parts a and b. The methods involve modifying the Hankel matrix to embed two-component data. Otherwise, the same steps are followed as in SSA. The two components are represented by V_i for the vertical component and H_i for the horizontal component.

Method 1, alternate the vertical and horizontal components along the anti-diagonals given by the following matrix

$$
\begin{bmatrix}
V_1 & H_1 & V_2 & H_2 & & V_K & H_K \\
H_1 & V_2 & H_2 & V_3 & \cdots & H_K & V_{K+1} \\
V_2 & H_2 & V_3 & H_3 & & V_{K+1} & H_{K+1} \\
 & \vdots & & & \ddots & & \vdots \\
H_L & V_{L+1} & H_{L+1} & V_{L+2} & \cdots & V_N & H_N
\end{bmatrix}.
$$

Method 2a, vertical and horizontal Hankel matrices which are originally square and concatenated into one matrix given by

$$
\begin{bmatrix}
V_1 & V_2 & V_3 & \cdots & V_K & H_1 & H_2 & H_3 & \cdots & H_K \\
V_2 & V_3 & V_4 & \cdots & V_{K+1} & H_2 & H_3 & H_4 & \cdots & H_{K+1} \\
V_3 & V_4 & V_5 & \cdots & V_{K+2} & H_3 & H_4 & H_5 & \cdots & H_{K+2} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
V_K & V_{K+1} & V_{K+2} & \cdots & V_N & H_K & H_{K+1} & H_{K+2} & \cdots & H_N
\end{bmatrix}.
$$

Method 2b, vertical and horizontal Hankel matrices concatenated into a matrix that is close to square given by

$$
\begin{bmatrix}
V_1 & V_2 & \cdots & V_K & H_1 & H_2 & \cdots & H_K \\
V_2 & V_3 & \cdots & V_{K+1} & H_2 & H_3 & \cdots & H_{K+1} \\
V_3 & V_4 & \cdots & V_{K+2} & H_3 & H_4 & \cdots & H_{K+2} \\
V_4 & V_5 & \cdots & V_{K+3} & H_4 & H_5 & \cdots & H_{K+3} \\
V_5 & V_6 & \cdots & V_{K+4} & H_5 & H_6 & \cdots & H_{K+4} \\
V_6 & V_7 & \cdots & V_{K+5} & H_6 & H_7 & \cdots & H_{K+5} \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
V_L & V_{L+1} & \cdots & V_N & H_L & H_{L+1} & \cdots & H_N
\end{bmatrix}.
$$

Examples

We tested the new methods on synthetic and real examples of two-component seismic data. Figures 1-3 show results of the synthetic example. Figure 1 replicates a seismic gather. The vertical component measures the P-wave and the horizontal component the S-wave. The denoised result for the methods are shown in Figures 2 and 3. Table 1 gives the time of each method and the corresponding Q factor given by the following equation:

$$
Q = 10 \log_{10} \frac{||\mathbf{r}||^2}{||\mathbf{d} - \mathbf{r}||^2}
$$

Where **r** is the noise free data and **d** is the denoised data. The brackets refer to the Frobenius norm.

For all synthetic examples, we use a rank of 6 which is double the number of linear events.

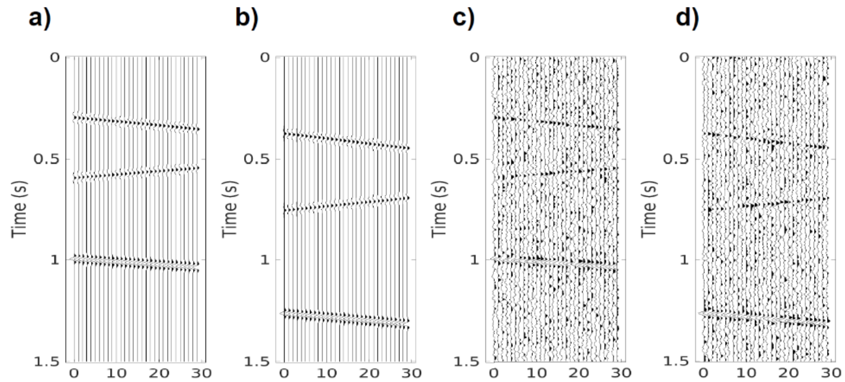

Figure 1: Synthetic example for three events. (a) Vertical data without noise. (b) Horizontal data without noise. (c) Vertical data with randomly generated with Gaussian noise. (d) Horizontal data with added noise.

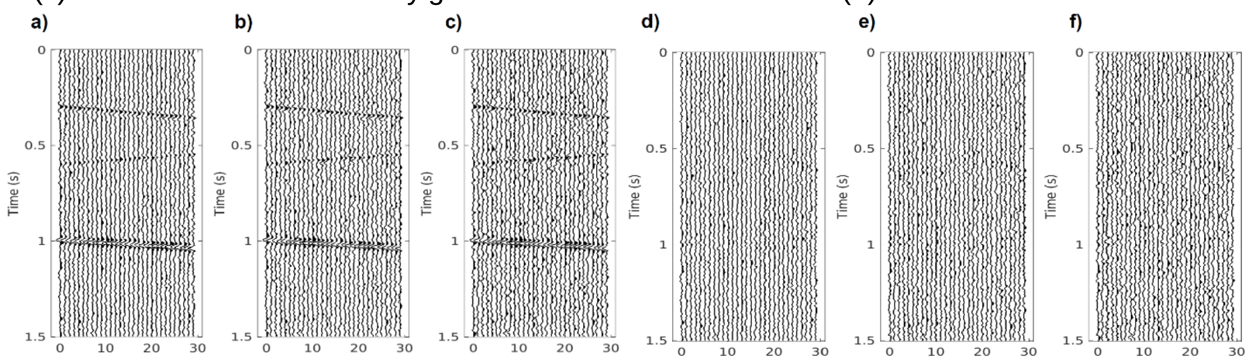

Figure 2: New denoising methods performed on the vertical component. Denoised data using methods (a) 1, (b) 2a, (c) 2b. Parts d to f represent the error between the denoised data and the original non-noisy data (figure 1 a) and b)). Error from methods 1, 2a, and 2b are shown in parts d), e), and f) respectively.

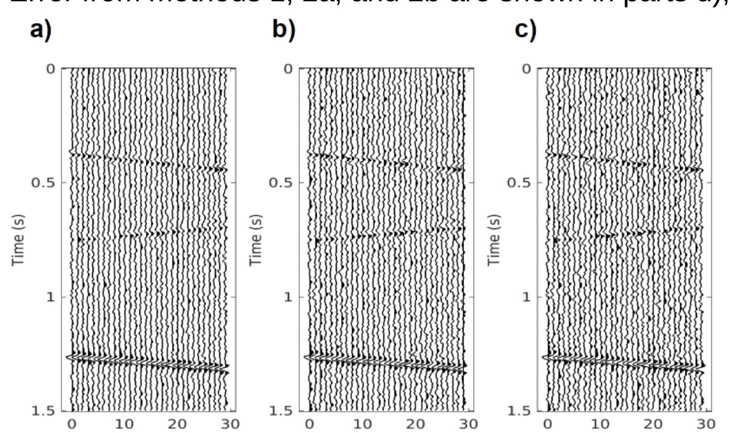

Figure 3: Denoised horizontal component for methods (a) 1, (b) 2a, (c) 2b.

Method	Original SSA	1	2a	2b
Vertical Q	1.0144	1.0118	0.7000	0.7528
Horizontal Q	0.8544	0.8435	0.5236	0.5728

Table 1: The Q value and processing time for each of the new methods. Q values are calculated separately for the vertical and horizontal components. There is negligible improvement with method 1 and a worsening with method 2.

The following field data example applies method 1 to a real data set. The data is from the Western Canadian Sedimentary Basin (WCSB) and is a near offset section. Figure 4 only shows the result from method 1 because the Q factor is the highest. Twelve eigenvalues were retained.

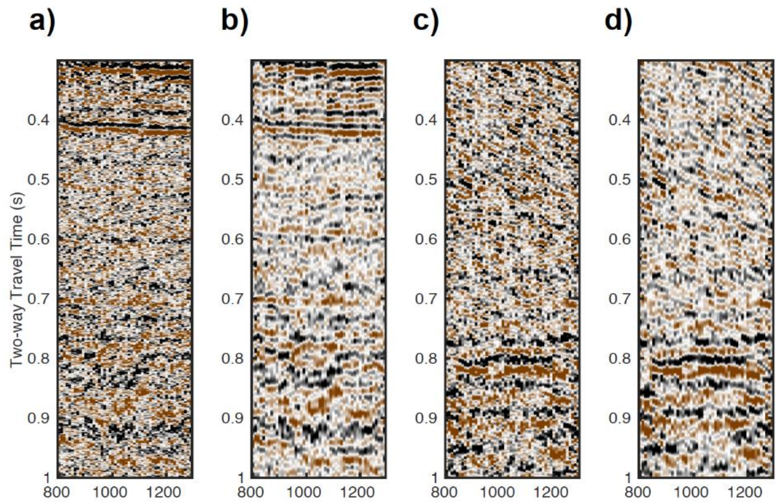

Figure 4: Real data example from the WCSB. (a) Original vertical data with added noise. (b) Denoised vertical data using method 1. (c) Noisy horizontal data. (d) Denoised horizontal data.

Conclusions

We investigated different ways of denoising multicomponent seismic data simultaneously via Singular Spectrum Analysis. The first method involves alternating the vertical and horizontal components along the anti-diagonal of the Hankel matrix. The second method concatenates vertical and horizontal Hankel matrices. After calculating Q (our factor of quality), we found method 1 to be superior to method 2. However, method 1 is negligibly better than performing SSA on separate components. We applied method 1 to a real data example from the WCSB with added noise. These new algorithms are viable alternatives for multicomponent data denoising.

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References

Golyandina, N., K. Usevich, and I. Florinsky, 2007, Filtering of digital terrain models by two-dimensional singular spectrum analysis: International Journal of Ecology & Development, 8, 81–94.

Golyandina, N. E., and D. Stepanov, 2005, SSA-based approaches to analysis and forecast of multidimensional time series: Presented at the 5th St. Petersburg Workshop on Simulation.

Ghil, M., M. Allen, M. Dettinger, K. Ide, D. Kondrashov, M. Mann, A. Robertson, A. Saunders, Y. Tian, F. Varadi, and P. Yiou, 2002, Advanced spectral methods for climatic time series: Reviews of Geophysics, 40, 1003, no. 1, doi:10.1029/2000RG000092.

Hsieh, W., and A. Wu, 2002, Nonlinear multichannel singular spectrum analysis of the tropical Pacific climate variability using a neural network approach: Journal of Geophysical Research, 107, no. C7, 3076, doi:10.1029/2001JC000957.

Oropeza, V., and M. Sacchi, "Simultaneous seismic data denoising and reconstruction via multichannel singular spectrum analysis." Geophysics 76.3 (2011): V25-V32.

Plaut, G., and R. Vautard, 1994, Spells of low-frequency oscillations and weather regimes in the northern hemisphere: Journal of the Atmospheric Sciences, 51, 210–236, doi:10.1175/1520-0469(1994)051<0210: SOLFOA>2.0.CO;2.

Read, P.,1993, Phase portrait reconstruction using multivariate singular systems analysis: Physica D: Nonlinear Phenomena, 69, 353–365, doi:10.1016/0167-2789(93)90099-M.

Vautard, R., and M. Ghil, "Singular spectrum analysis in nonlinear dynamics, with applications to paleoclimatic time series." Physica D: Nonlinear Phenomena 35.3 (1989): 395-424.

Sacchi, M. "FX singular spectrum analysis." CSPG CSEG CWLS Convention. 2009.