geoconvention
Calgary • Canada • May 15-19 2017

Passive seismic source location using group sparsity constrained two-way waveform inversion

Wenlei Gao, Mauricio D. Sacchi and Zhenhua Li

Signal Analysis and Imaging Group (SAIG), Department of Physics, University of Alberta

Summary

Passive seismic source location plays an important role in the seismological community for it is commonly used in retrieving the epicenter of earthquakes and evaluating the effectiveness of hydraulic fracturing. Time reversal imaging is one of the most popular methods for source imaging. This method back-propagates seismic recordings into medium and focuses energy at source locations. However, events often can't be fully focused leading to inadequate resolution of sources from a limited observing aperture. To solve this problem, this paper illustrate a source location method that adopts group sparsity constrained waveform inversion. This method can retrieve source locations and origin time simultaneously and the adopted group sparsity constraint is capable of yielding spatially sparse results while the source function is smooth in time axis .

Introduction

Source imaging methods can be divided into two categories, namely ray based methods and waveform based methods (Castellanos and van der Baan, 2015). Compared with ray based methods, waveform based methods (Artman et al., 2010) are more suitable for low signal-to-noise ratio data. They use the complete waveform information, including not only P- and S-wave travel time but also their phases and magnitudes. In the group of waveform methods, Artman et al. (2010) applied reverse time imaging on P- and S-wave respectively, followed by the application of an autocorrelation imaging condition to image source locations. Similarly, Li et al. (2015) propose to back-propagate acoustic seismic data using a representation-theorem-based reverse time imaging method. In a series of papers (Rodriguez et al., 2012a,b), it has been proposed to use group sparsity for the inversion of seismic moment tensor. The latter adopted analytical Green functions for constant velocity media. This article expands the aforementioned research and adopts Green functions that are computed by solving two-way wave equation via finite difference method. At present time, our formulation is in terms of acoustic Green functions for heterogeneous media but the proposed method can be easily generalized to elastic media. Our method solves a least-squares two-way wave equation imaging problem with group sparsity constraints. In essence, the method is similar to least-squares RTM migration (Yao and Jakubowicz, 2012; Dai et al., 2012) but instead of imaging scatterers, we directly image unknown sources.

Theory

The $2D$ velocity-stress acoustic wave equation can be written as

$$\begin{aligned} \rho\frac{\partial v_z}{\partial t} &= \frac{\partial p}{\partial z}, \\ \rho\frac{\partial v_x}{\partial t} &= \frac{\partial p}{\partial x}, \\ \frac{\partial p}{\partial t} &= \lambda(\frac{\partial v_x}{\partial x}+\frac{\partial v_z}{\partial z})+f(z,x,t), \end{aligned} \tag{1}$$

where z,x denote spatial coordinate and t is time. v_z and v_x are used to represent vertical and horizontal particle velocity, respectively, p is stress. ρ is density, λ is $lame$ constant. In acoustic case, the shear modulus is assumed to be 0, it does not appears in our partial differential equations. $f(z,x,t)$ is source signature. To simplify the explanation of our method, we assume a simulation with $nt = 4$ forward propagation and the length of source signature is $nt' = 2$. Then, the forward propagation is equivalent to matrix-vector operation (Ji, 2009)

$$\begin{pmatrix}\mathbf{d_1}\\\mathbf{d_2}\\\mathbf{d_3}\\\mathbf{d_4}\end{pmatrix} = \begin{pmatrix}\mathbf{S} & & & \\ & \mathbf{S} & & \\ & & \mathbf{S} & \\ & & & \mathbf{S}\end{pmatrix}\begin{pmatrix}\mathbf{I} & & \\ & \mathbf{I} & \\ & & \mathbf{I}\\ & & \mathbf{L}\end{pmatrix}\begin{pmatrix}\mathbf{I} & \\ & \mathbf{I}\\ & \mathbf{L}\end{pmatrix}\begin{pmatrix}\mathbf{I} & \\ \mathbf{L} & \mathbf{I}\end{pmatrix}\begin{pmatrix}\mathbf{W} & \\ & \mathbf{W}\end{pmatrix}\begin{pmatrix}\mathbf{m_1}\\\mathbf{m_2}\end{pmatrix}, \tag{2}$$

where $\mathbf{d_i}$ is the i^{th} time step wave field at all receiver's location. The length of $\mathbf{d_i}$ is equal to the number of receivers, $\mathbf{S}$ indicates sampling matrix, which is specified by recording geometry. The operator L is the one-step forward propagator, which is a highly sparse matrix. The matrix $\mathbf{W}$ denotes a weighting matrix, which is designed to shrinking the size of model parameters. $\mathbf{I}$ is identity matrix with size of $4nz \cdot nx \times 4nz \cdot nx$, nz and nx are the grid size of physical model (including absorbing boundary layers) in the z-axis and x-axis. The vector $\mathbf{m_i}$ represents the source signatures at i^{th} time step,

$$\mathbf{m_i} = \mathbf{vec}(f(z,x,t_i)), \tag{3}$$

where $\mathbf{vec}$ indicate the vectorization of a $2D$ array. An important assumption is embed in equation 2, the length of source time function nt' is shorter than that of recordings nt $(nt' < nt)$. The forward modelling process can be further simplified as

$$\mathbf{d} = \mathbf{Gm}, \tag{4}$$

where $\mathbf{d}$ is seismic recordings, $\mathbf{G}$ represent forward modelling operator which includes source injection, finite difference propagation and data acquisition. This source location problem is solved by least square method, the cost function is given as

$$\mathbf{m} = \underset{m}{\operatorname{argmin}} \quad ||\mathbf{d} - \mathbf{Gm}||_\mathbf{2}^\mathbf{2} + \lambda||\mathbf{M}||_{\mathbf{1,1,2}}, \tag{5}$$

where $\mathbf{M}$ is a $3D$ array obtained by reshaping $\mathbf{m}$ via

$$\mathbf{M} = \text{reshape}(\mathbf{m}, nz', nx', nt'). \tag{6}$$

We can easily spot that $\mathbf{M}$ is sparse in the first and second dimensions. The latter corresponds to making the sparse assumption in the spatial domain. However the source time function is smooth. In another words, the reshaped model parameter $\mathbf{M}$ is smooth in the third dimension. We introduce the group sparsity constraint (Kowalski and Torrésani, 2009) to simultaneously capture spatial sparse and smoothness in time. Therefore, we adopt the constraint defined as

$$||\mathbf{M}||_{1,1,2} = \sum_{iz=1}^{nz'}\sum_{ix=1}^{nx'}\left(\sum_{it=1}^{nt'} M_{iz,ix,it}^2\right)^{\frac{1}{2}}. \tag{7}$$

This cost function is solved in the framework of the FISTA algorithm (Beck and Teboulle, 2009) which requires the adjoint of forward modelling operators. We provide the adjoint operator by transposing the blocks matrix in equation 2 .

Example

A smoothed Marmousi velocity model (Figure 1a) is used to demonstrate the effectiveness of our method. The triangles in Figure 1a are used to represent receiver array and stars indicate the location of sources. The grid size in both x-axis and z-axis is $20m$, time step size is $0.002s$. $15Hz$ Ricker wavelet

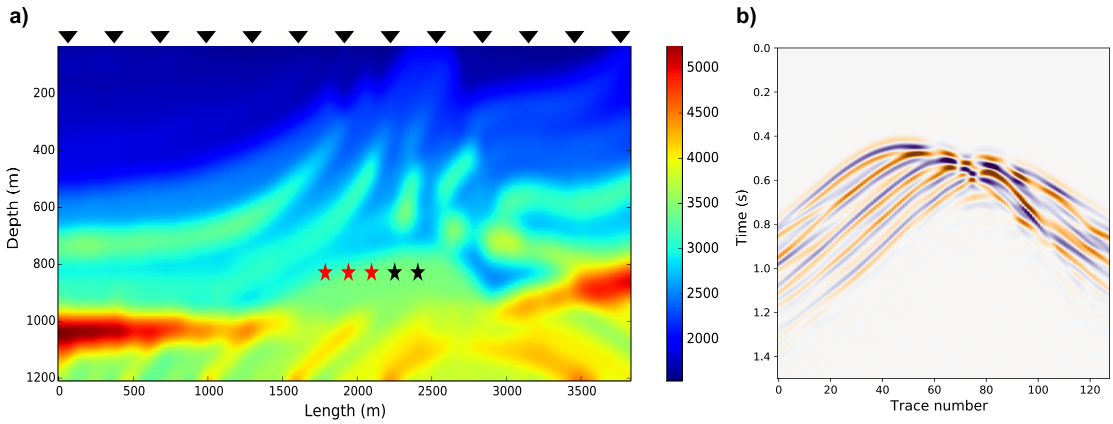

Figure 1: a) Smoothed Marmousi velocity model, the black triangles are receivers, the stars indicate the locations of sources, red color denotes the sources with positive amplitude, black color with negative amplitude. b) Multi-channel recordings.

is implemented as source time function. The amplitude of the source indicated by red star is equal to 1 while the other one is -1. The multi-channel recordings are displayed in Figure 1b. Figure 2 shows the inverted results which include source images (Fiugre 2a) and time function (Figure 2b). Figure 2c display the true source time functions. We can see that the spatial distribution of sources and time function are correctly retrieved. We plot 2 time slices (Figure 3b, c) to further examine the inverted result, Figure 3b is corresponding to the time slice indicated by AA' and BB' is shown in Figure 3c.

Conclusion

We have developed a source imaging method via group sparsity constrained waveform inversion. This method are capable of estimating source location and retrieving origin time simultaneously. The size of unknowns can be reduced by implementing a weighting matrix, which is designed based on proper prior information about the area containing passive seismic events. A group sparsity constraint is used to provide results which are sparse in spatial domain while smooth in time axis.

Acknowledgements

The authors are grateful to the sponsors of Signal Analysis and Imaging Group (SAIG) at the University of Alberta for their continued support.

References

Artman, B., I. Podladtchikov, and B. Witten, 2010, Source location using time-reverse imaging: Geophysical Prospecting, **58**, 861–873.
Beck, A., and M. Teboulle, 2009, A fast iterative shrinkage-thresholding algorithm for linear inverse problems: SIAM journal on imaging sciences, **2**, 183–202.
Castellanos, F., and M. van der Baan, 2015, Waveform similarity for quality control of event locations, time picking, and moment tensor solutions: Geophysics, **80**, WC99–WC106.
Dai, W., P. Fowler, and G. T. Schuster, 2012, Multi-source least-squares reverse time migration: Geophysical Prospecting, **60**, 681–695.
Ji, J., 2009, An exact adjoint operation pair in time extrapolation and its application in least-squares reverse-time migration: Geophysics, **74**, H27–H33.
Kowalski, M., and B. Torrésani, 2009, Sparsity and persistence: mixed norms provide simple signal models with dependent coefficients: Signal, image and video processing, **3**, 251–264.
Li, Z., M. van der Baan, et al., 2015, Enhanced microseismic event localization by reverse time extrapolation: Presented at the 2015 SEG Annual Meeting, Society of Exploration Geophysicists.
Rodriguez, I. V., M. Sacchi, and Y. J. Gu, 2012a, A compressive sensing framework for seismic source parameter estimation: Geophysical Journal International, **191**, 1226–1236.
———, 2012b, Simultaneous recovery of origin time, hypocentre location and seismic moment tensor using sparse representation theory: Geophysical Journal International, **188**, 1188–1202.
Yao, G., and H. Jakubowicz, 2012, Least-squares reverse-time migration: Presented at the 74th EAGE Conference and Exhibition incorporating EUROPEC 2012.

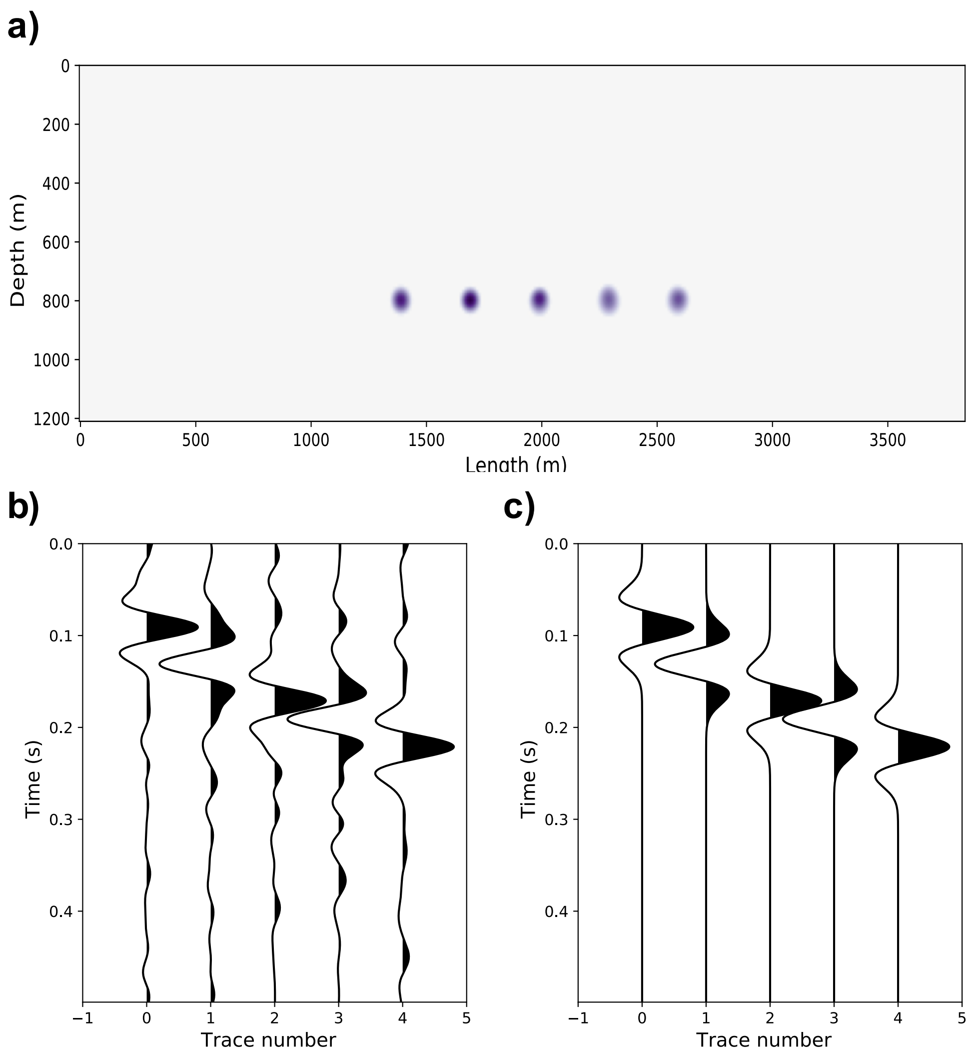

Figure 2: a) Inverted source image. b) Inverted source time function. c) true source time function.

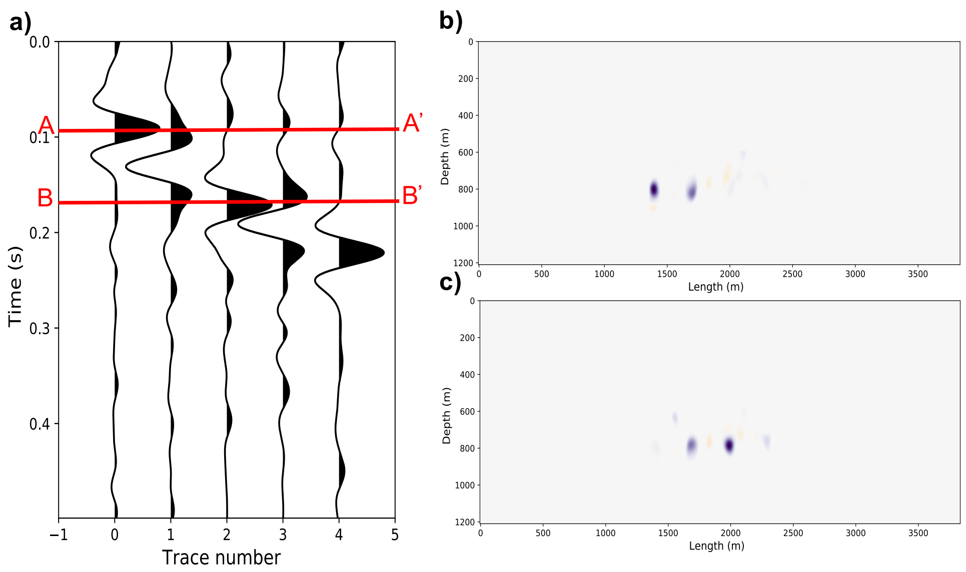

Figure 3: a) Inverted source image. b) Inverted source time function. c) true source time function.