A twelve-parameter source model: Force, Moment and Torque

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Summary

The conventional model of a seismic source as a force or a symmetric moment tensor is unified and augmented with a torque source. The force+moment+torque (FMT) source model requires 12 parameters for its description. A paramerization for such a generalized source model is suggested. The ray theory Green function for particle displacements is given and radiation patterns are shown for basic and selected composite sources. Downhole and surface source field data examples are shown where force and torque sources are indicated. The model of induced microseismic sources caused by pumps at the surface may require the addition of force and torque to accurately represent field data. Only if force and torque are included in the source model can they be ruled out through inversion.

Introduction

Seismic sources that occur naturally within the earth (indigenous) involve no external forces or reaction, the net force or torque is zero and the source is represented by a symmetric moment tensor. A source due to external (nonindigenous) forces on the earth may require unbalanced forces (and possibly torques) for its representation. An example of the former is an earthquake at a plate boundary due to tectonic forces; an example of the latter is a meteorite or asteroid impact. Early theoretical papers included a torque source to study Sh wave propagation (e.g. Miller and Pursey 1954, Pekeris *et al.* 1963) and a few textbooks have included it (e.g. Ben-Menahem and Singh 1981). A torque source is represented by the asymmetric part of a general moment tensor, and Aldridge and Symons (2001) included it in their study of seismic reciprocity. The possibility of torque sources has largely been ignored.

Regarding composite sources, moment plus force have been used to represent natural phenomena, for example volcanic eruptions (Chouet *et al*. 2003), where the force was found to be in the direction of the ejecta. A vibroseis source on the surface is an example of an externally applied force, but unintentional horizontal forces and torques resulting from a “vertical vibe”, while studied (Wei 2010), have received little attention. Whether torque or forces need to be added to nonindigenous microseismic sources for their accurate representation remains an open question. In this paper we lay the foundation for future investigations into nonindigenous sources, suggest a parameterization for an FMT source and show radiation patterns and field data examples where force and force+torque sources are indicated.

Theory

The vector displacement $\mathbf{u}$ at a receiver location $\mathbf{x}_R$ for an impulsive, point source at $\mathbf{x}_S$ with the ray approximation is

$$\mathbf{u}(t, \mathbf{x}_R; \mathbf{x}_S) \cong \sum_{\text{rays}} \frac{\mathrm{Re}\left(\Delta(t - T_{ray}) \prod_l T_l \, e^{-i\pi\sigma/2}\right)}{4\pi(\rho_S \rho_R V_S V_R S)^{1/2} c_S} \hat{\mathbf{u}}_R * \left[c_S \hat{\mathbf{u}}_S \cdot \mathbf{f}(t) + \mathbf{E}^S : \mathbf{M}^S(t) - \mathbf{E}^A : \mathbf{M}^A(t)\right] \tag{1}$$

where T_{ray} is the arrival time of the chosen ray (e.g. direct qP, qSh, qSv), T_l are transmission coefficients along the ray for interface l , σ is the KMAH index counting caustics, ρ is density, V is group velocity, S is geometrical spreading, c is phase velocity and $\hat{\mathbf{u}}$ are normalized polarization vectors. $\mathbf{f}$ is the force vector and the superscripts ^{S} and ^{A} on moment tensors $\mathbf{M}$ signify symmetric and asymmetric parts, respectively.

The symmetric and asymmetric ray strain tensors at the source are $\mathbf{E}^{\mathrm{S}} = (\hat{\mathbf{u}}\hat{\mathbf{p}} + \hat{\mathbf{p}}\hat{\mathbf{u}})/2$ and $\mathbf{E}^{\mathrm{A}} = (\hat{\mathbf{u}}\hat{\mathbf{p}} - \hat{\mathbf{p}}\hat{\mathbf{u}})/2$, where $\hat{\mathbf{p}}$ is the phase direction vector. Symmetric and asymmetric moment tensors are obtained from the general moment tensor, $\mathbf{M}$, as $\mathbf{M}^{\mathrm{S}} = (\mathbf{M} + \mathbf{M}^{\mathrm{T}})/2$ and $\mathbf{M}^{\mathrm{A}} = (\mathbf{M} - \mathbf{M}^{\mathrm{T}})/2$ where the superscript $^{\mathrm{T}}$ means transpose. The symbol : represents the scalar product between second-order tensors and * is temporal convolution. Each of the source terms in (1) may have different time dependencies, but presumably would be causally related. Expression (1) is based on a combination of the well-known Green functions for force and moment sources, generalized to include an anti-symmetric moment tensor (torque) source.

The terms in the square brackets in (1) together comprise the radiated amplitude. There are 12 parameters required – 3 for the force (F), 6 for the symmetric moment tensor (M) and 3 more for the asymmetric moment tensor or torque (T) source. The units of the force vector, $\mathbf{f}$, are $[\mathrm{MLT}^{-1}]$ or Newton-seconds and when mutiplied by the phase velocity at the source provide units consistent with the moment tensor, $[\mathrm{ML}^2\mathrm{T}^{-2}]$. This suggests that a parameterization for the relative magnitudes of the FMT source should include a velocity multiplying the simple force source. We have chosen the horizontal Sh velocity in a VTI medium, $\mathrm{v} = \sqrt{\mathrm{A}_{66}}$, to scale the force term, but other choices are possible. The scalar moment, M, for the FMT source is thus defined as:

$$M = \mathrm{F}\|\mathrm{v}\mathbf{f}\| + \mathrm{M}\left\|\mathbf{M}^{\mathrm{S}}\right\|/\sqrt{2} + \mathrm{T}\left\|\mathbf{M}^{\mathrm{A}}\right\| \tag{2}$$

where the factor of $\sqrt{2}$ is included to honor convention for the scalar moment (Silver and Jordan 1982). A set of positive numbers $(\mathrm{F}, \mathrm{M}, \mathrm{T})$ which sum to unity define relative sizes of the FMT components. Given a target scalar moment, equation (2) with normalized sources is multiplied by the appropriate scalar to determine the sizes of the three source terms.

For convenience, the following vectors are used to describe the FMT source: $\mathbf{f} = (\mathrm{f}_1, \mathrm{f}_2, \mathrm{f}_3)^{\mathrm{T}}$, $\mathbf{m} = (\mathrm{M^S}_{11}, \mathrm{M^S}_{22}, \mathrm{M^S}_{33}, \mathrm{M^S}_{23}, \mathrm{M^S}_{13}, \mathrm{M^S}_{12})^{\mathrm{T}}$, $\mathbf{t} = (\mathrm{M^A}_{23}, \mathrm{M^A}_{31}, \mathrm{M^A}_{12})^{\mathrm{T}}$. Following the approach described in Leaney (2014, section 2.6), the symmetric moment tensor is constructed using four angles defined in the east north up (ENU) coordinate system and labelled SDRO. S means strike measured from north, D means co-dip so a vertical fracture has D=0 (as opposed to 90 as is common), R means rake measured counterclockwise from horizontal as viewed from the hanging wall of a fault, O means the opening angle of a tensile event (Dufumier and Rivera 1997) and a parameter ν quantifies the amount of isotropic expansion due to a pressure change (Chapman and Leaney 2012). These five parameters plus magnitude allow for the construction of the potency tensor, which is contracted with the stiffness tensor to construct the symmetric moment tensor. The force and torque axis vectors are defined either in component form as unit vectors or by specifying two angles each in the ENU coordinate system. After the FMT source is defined, (2) is normalized to a target scalar moment and the force vector is divided by v prior to calculation using (1). With this theoretical background and parameter definitions we will now consider some example radiation patterns.

Radiation patterns

A vertical-up force source with vector $\mathbf{f} = (0,0,1)^{\mathrm{T}}$ produces P and S radiation as shown in Figure 1 (left, top). Since polarizations are the same for opposite ray directions, there is anti-point symmetry. A torque source with $\mathbf{t} = (0,0,1)^{\mathrm{T}}$ produces the P and S radiation patterns shown in Figure 1 (left, bottom). The force source produces shear polarization in the vertical plane (Sv), and the torque source produces horizontally polarized shear (Sh) and no P radiation. Radiation patterns for the basic sources of a symmetric moment tensor can be found in standard texts (e.g. Chapman 2004); they will not be shown here. Figure 1, right, shows Sh radiation patterns on the focal sphere in an equal area display with vertical-up in the center for selected torque, force and symmetric moment sources: $\mathbf{t} = (0,0,1)^{\mathrm{T}}$, $\mathbf{f} = (1,0,0)^{\mathrm{T}}$, $\mathbf{m} = (0,0,0,0,0,-1)^{\mathrm{T}}$. Also shown in Figure 1, right, bottom row, are two composite sources using the aforementioned source vectors. In one case the horizontal force is combined with some torque in the proportions $\mathrm{FMT} = (0.8,0,0.2)$ and in the other case a double couple (strike slip on a vertical E-W fault) is combined with an eastward-pointing horizontal force in the proportions $\mathrm{FMT} = (0.2, 0.8, 0)$. The first case may represent a downhole vibroseis source activated to vibrate horizontally but with unintentional rotational motion and hence torque. The second case may represent slip on a vertical fracture striking in the direction of an additive (perhaps causal) force. Note the lack of point symmetry in both cases due to the added force.

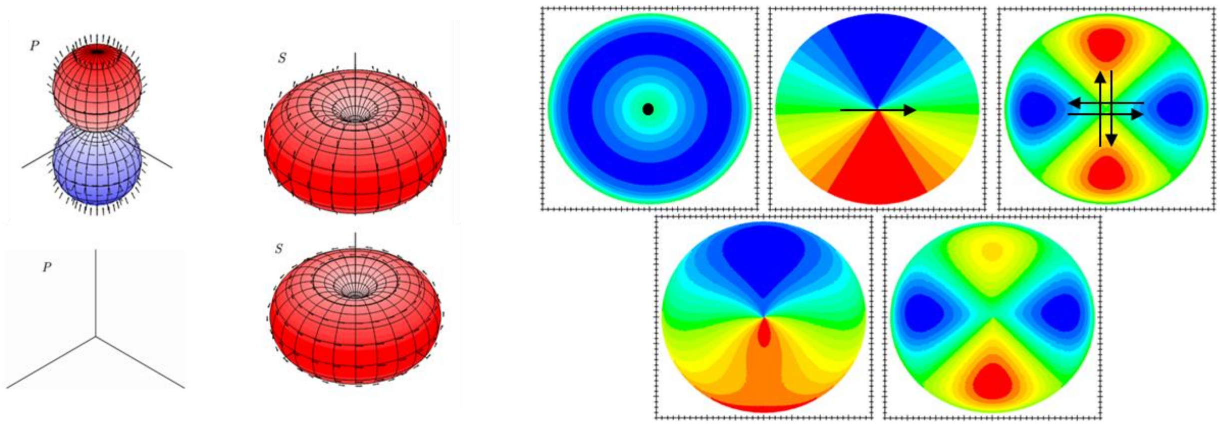

Figure 1. Left: top row, P and S radiation for a vertical-up force with polarization vectors shown with small arrows. The S polarizations are in the vertical plane (Sv). Both radiations exhibit anti-point symmetry since the polarization vectors are the same in opposite ray directions. Bottom row: P (null) and S radiation for a torque source with vertical-up axis. S polarizations are toroidal (Sh). Right: Focal sphere equal-area projections of Sh radiation patterns displayed with vertical-up in the center and vertical-down around the perimeter. Top row: $\mathbf{t} = (0,0,1)$, $\mathbf{f} = (1,0,0)$, $\mathbf{m} = (0,0,0,0,0,-1)$. Bottom row: composite sources force+torque with $\mathrm{FMT} = (0.8, 0.0, 0.2)$ (left) and force+moment with $\mathrm{FMT} = (0.2, 0.8, 0.0)$.

Field data examples

The source produced by sliding-sleeve well completion due to "ball drops" (Maxwell *et al.* 2011) shows more P-wave directionality than a simple dipole source and a force source inversion (FSI) fits the data better than a symmetric moment tensor inversion (Leaney 2014, p.137). Figure 2 compares a microseismic event with a ball drop event recorded by two vertical monitor wells to the north and south. After projection onto ray trace polarization vectors at the receiver it is observed that the P-wave first motion is of opposite sign in the two arrays for the sleeve source compared to the microseismic event. The sleeve source also shows very weak Sh compared to the microseismic event. Many of the sleeve events in this data set were well fit by a horizontal force pointing towards the heel of the well.

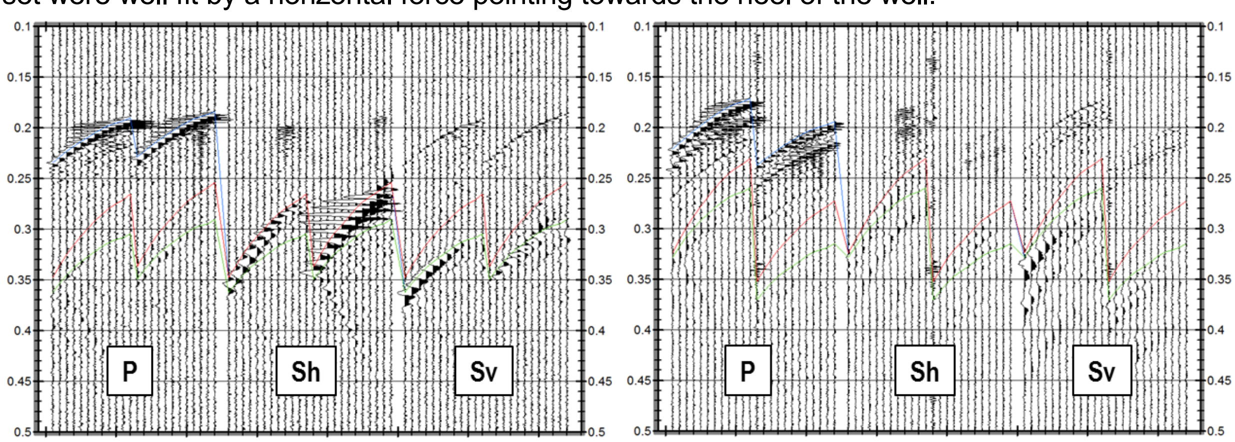

Figure 2. Microseismic event (left) and sliding sleeve source (right) recorded by dual vertical receiver arrays to the north and south. The oriented ENU waveforms have been projected to P, Sh, Sv based on ray-traced receiver polarization vectors. The event shows consistent P first motion polarity on both arrays whereas the sleeve source shows opposite P first motion polarity. The blue, red and green curves show the P, Sh and Sv arrival times of a calibrated VTI model (Leaney *et al.* 2014).

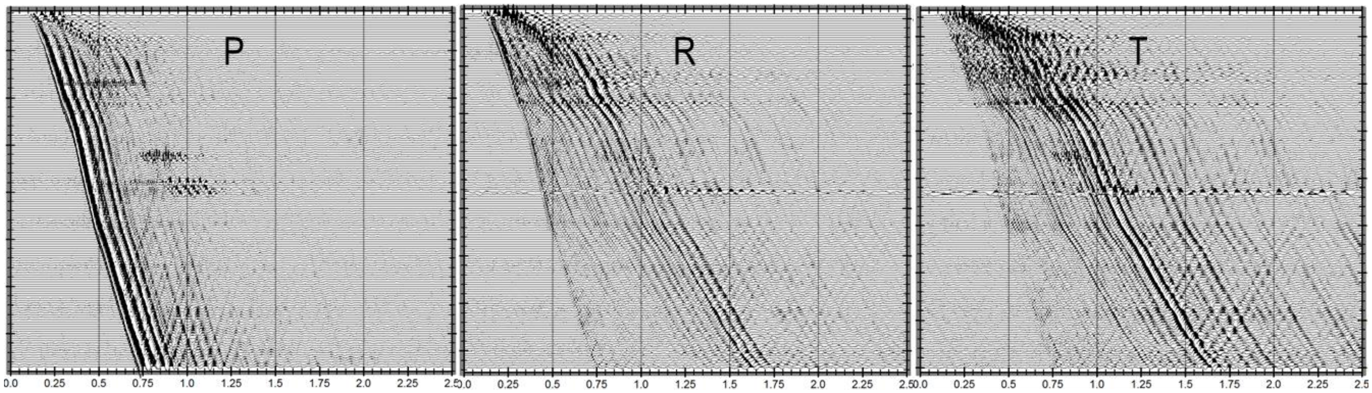

Figure 3. Rotated 3C VSP waveforms from a vertical vibroseis source at an offset of 250 ft from the wellhead. Shown from left to right are the maximum direct P, horizontal radial (R) and horizontal transverse (T) components. Plots are true relative amplitude as normalization is by receiver using the 3C envelope.

Theory says that a vertical force acting on the earth's surface such as is assumed to be the case for a P-wave vibroseis should generate no S waves in the vertical direction, and only at nonvertical directions are S waves polarized in the vertical plane expected (e.g. Kahler and Meissner 1983). However, significant direct S arrivals are commonly observed for small offset VSP data sets acquired with a single vibroseis source (e.g. Armstrong *et al.* 2001). The presence of velocity in the denominator in (1) means that small amounts of horizontal force or torque may lead to relatively large amplitude shear displacements. Hardage and Wagner (2014) studied the direct shear in VSP data but attributed the shear on the transverse component as stemming from the direct Sv. Zhao *et al.* (2005) observed strong direct Sh arrivals in near-offset VSP data and speculated that an elliptical motion of the base plate could produce such energy. We conjecture that asymmetric weight distribution in a vibroseis truck could set up periodic horizontal and rotational forces at the base plate, producing a significant shear source. Indeed, this was observed in a study by Wei (2010), who measured horizontal base plate motion during a "vertical" vibe acquisition. Figure 3 shows 3C VSP waveforms from a P-vibroseis source at an offset of 250ft from a vertical well. The R and T components show a strong arrival that can be traced up to the surface, indicating that it comes directly from the source. A source model of force+torque on the free surface could explain these recordings.

Conclusions

Nonindigenous sources may require unbalanced forces and torques for their representation. We presented theory for a unified FMT source and proposed a 12-parameter description for such a source. The torque source is represented by an asymmetric moment tensor. Radiation patterns were presented for basic force, moment and torque sources and examples were shown for composite sources combining force and torque and combining symmetric moment and force. Field data examples indicate force and torque sources. In a downhole microseismic field data example comparing a microseismic event and a sliding sleeve source, the sleeve event is better represented by a simple force than a symmetric moment tensor. A vibroseis VSP data set was shown in support of significant horizontal and/or rotational forces being generated by a "vertical" P vibroseis. Whether microseismic events require additional force or torque sources in addition to a symmetric moment tensor remains an open question. The source model described here provides a foundation for an inversion and future investigations into nonindigenous seismic sources. Only if force and torque are included in the source model can they be ruled out through inversion.

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