

5D reconstruction via robust tensor completion

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Summary

Tensor completion techniques (including tensor denoising) can be used to solve the ubiquitous multi-dimensional data reconstruction problem. We present a robust tensor reconstruction method that can tolerate the presence of erratic noise. The method is derived by minimizing a robust cost function with the addition of low rank constraints. Our presentation is based on the Parallel Matrix Factorization (PMF) tensor completion method that we modify to cope with erratic noise.

Introduction

Seismic data reconstruction via reduced-rank methods offers an interesting solution to the multi-dimensional seismic reconstruction problem (Trickett and Burroughs, 2009; Burroughs and Trickett, 2009; Oropeza and Sacchi, 2011). However, reduced-rank methods might fail when the data are contaminated by erratic non-Gaussian noise. This problem has mainly been studied in the context of Cadzow reconstruction algorithms (Trickett et al., 2012; Chen and Sacchi, 2015). Recently, tensor completion methods (Kreimer and Sacchi, 2012; Kreimer et al., 2013) have been proposed as an alternative to Cadzow reconstruction algorithms. Tensor completion operates directly on multi-linear arrays formed from binned patches of seismic data in the frequency-space domain and, unlike Cadzow reconstruction methods, they do not require to embed multidimensional data in block Hankel matrices.

We propose to adapt the PMF tensor completion method (Xu et al., 2013) to cope with erratic noise and to design a robust algorithm for 5D seismic interpolation (Gao et al., 2015, 2016). The PMF tensor reconstruction method is implemented in midpoint-offset frequency domain. The data are represented by $D(\omega, x, y, h_x, h_y)$, where x, y, h_x and h_y indicate the spatial coordinates in the inline midpoint, cross-line midpoint, in-line offset and cross-line offset. After binning the data into a midpoint-offset grid, a frequency slide can be denoted as $D(\omega, x, y, h_x, h_y)$. The latter can be represented by a 4th-order tensor D with elements D_{i_1, i_2, i_3, i_4} , where i_1, i_2, i_3, i_4 are bin indices for the spatial coordinates x, y, h_x and h_y , respectively. We will remove the dependency on ω to simplify the notation. Clearly, we could have also adopted a reconstruction in terms of x, y, h and A where h is absolute offset $h = \sqrt{h_x^2 + h_y^2}$ and A is azimuth $A = \arctan(h_x/h_y)$.

Theory

We assume the following model for our multi-dimensional data tensor

$$D = P \circ Z + E. \quad (1)$$

We will assume tensors of order N but bear in mind that our final application entails reconstructing tensors of order $N = 4$ to solve the 5D reconstruction problem. The data tensor denoted by D contains the observed seismic data, Z is the ideal complete and uncorrupted data tensor, P is the sampling operator and E indicates additive noise. The symbol \circ indicates element-to-element product. The reconstructed data are obtained by minimizing the following cost function

$$\Phi = \Phi_M + \mu \Phi_C, \quad (2)$$

where Φ_M is the data misfit term and Φ_C is a low-rank constraint term

$$\Phi_C = \sum_{k=1}^N \|\mathbf{X}_{(k)} \mathbf{Y}_{(k)} - \mathbf{Z}_{(k)}\|_F^2. \quad (3)$$

The matrix $\mathbf{Z}_{(k)}$ is the mode- k unfolding matrix of the ideal data Z . Figure 2 portrays the process of unfolding and folding arbitrary tensor X of order $N = 3$. A low-rank matrix factorization is applied to each mode unfolding of Z by seeking matrices $\mathbf{X}_{(k)} \in \mathbb{C}^{I_k \times r_k}$ and $\mathbf{Y}_{(k)} \in \mathbb{C}^{r_k \times I_1 \dots I_{k-1} I_{k+1} \dots I_N}$ such that $\mathbf{Z}_{(k)} \approx \mathbf{X}_{(k)} \mathbf{Y}_{(k)}$ for $k = 1, \dots, N$, where r_k is the rank of the unfolding matrix $\mathbf{Z}_{(k)}$. In our original algorithm (Gao et al., 2015) the data misfit is given by

$$\begin{aligned} \Phi_M &= \|(P \circ Z - D)\|_2^2 \\ &= \sum_{i_1, i_2, \dots, i_N} |P_{i_1, i_2, \dots, i_N} Z_{i_1, i_2, \dots, i_N} - D_{i_1, i_2, \dots, i_N}|^2. \end{aligned} \quad (4)$$

The latter can cope with Gaussian noise and it is inadequate for data containing erratic noise. In this presentation, we adopted the l_1/l_2 norm (Bube and Langan, 1997; Lee et al., 2006) to develop an algorithm that can tolerate outliers

$$\begin{aligned} \Phi_M &= \|(P \circ Z - D)\|_{l_1/l_2} \\ &= \sum_{i_1, i_2, \dots, i_N} \sqrt{|P_{i_1, i_2, \dots, i_N} Z_{i_1, i_2, \dots, i_N} - D_{i_1, i_2, \dots, i_N}|^2 + \varepsilon^2}. \end{aligned} \quad (5)$$

In order to solve for $\mathbf{X}_{(k)}$, $\mathbf{Y}_{(k)}$ and Z , we minimize the cost function Φ via an alternating least-squares algorithm. If we adopt the robust misfit (equation 5), we end up with the following iterative algorithm

$$\mathbf{X}_{(k)}^{i+1} = \mathbf{Z}_{(k)}^i (\mathbf{Y}_{(k)}^i)^H, \quad k = 1, \dots, N, \quad (6a)$$

$$\mathbf{Y}_{(k)}^{i+1} = ((\mathbf{X}_{(k)}^{i+1})^H \mathbf{X}_{(k)}^{i+1})^\dagger (\mathbf{X}_{(k)}^{i+1})^H \mathbf{Z}_{(k)}^i, \quad k = 1, \dots, N, \quad (6b)$$

$$\mathbf{Z}^{i+1} = A \circ D + (I - A \circ P) \circ C, \quad (6c)$$

where I is the N th order tensor with all entries equal to 1 and C is given by

$$C = \frac{1}{N} \sum_{k=1}^N \text{fold}_k[\mathbf{X}_{(k)}^{i+1} \mathbf{Y}_{(k)}^{i+1}]. \quad (7)$$

The index k is used to indicate mode and i indicates iteration number. The tensor A denotes the weights induced by the l_1/l_2 norm

$$A_{i_1, i_2, \dots, i_N} = \frac{1}{1 + N\mu \sqrt{|P_{i_1, i_2, \dots, i_N} Z_{i_1, i_2, \dots, i_N} - D_{i_1, i_2, \dots, i_N}|^2 + \varepsilon^2}}. \quad (8)$$

It is easy to show that if one adopts a quadratic (l_2) misfit (equation 4), the expression in equation 6(c) needs to be replaced by $\mathbf{Z}^{i+1} = aD + (I - aP) \circ C$ where $a = (1 + N\mu)^{-1}$. In other words, the tensor of weights A reduces to a scalar (Gao et al., 2015). Figure 2a portrays the robust l_1/l_2 norm and Figure 2b shows the function of weights A for two values of the parameter μ . The shape of the weighting function is controlled by the trade-off parameters and ε . One must tune these two parameters to reject outliers. The problem also involves tuning the rank parameter r_k in the constraint.

Examples

We have designed a synthetic example that consists of a 5D patch of size $256 \times 18 \times 10 \times 10 \times 10$. We are providing a view of one slice of the patch in Figure 3a. The 5D patch was contaminated with Gaussian noise and erratic noise. The maximum amplitude of the clean signal is 1. The signal was contaminated with white Gaussian additive noise with standard deviation $\sigma = 0.2$ and then, we have

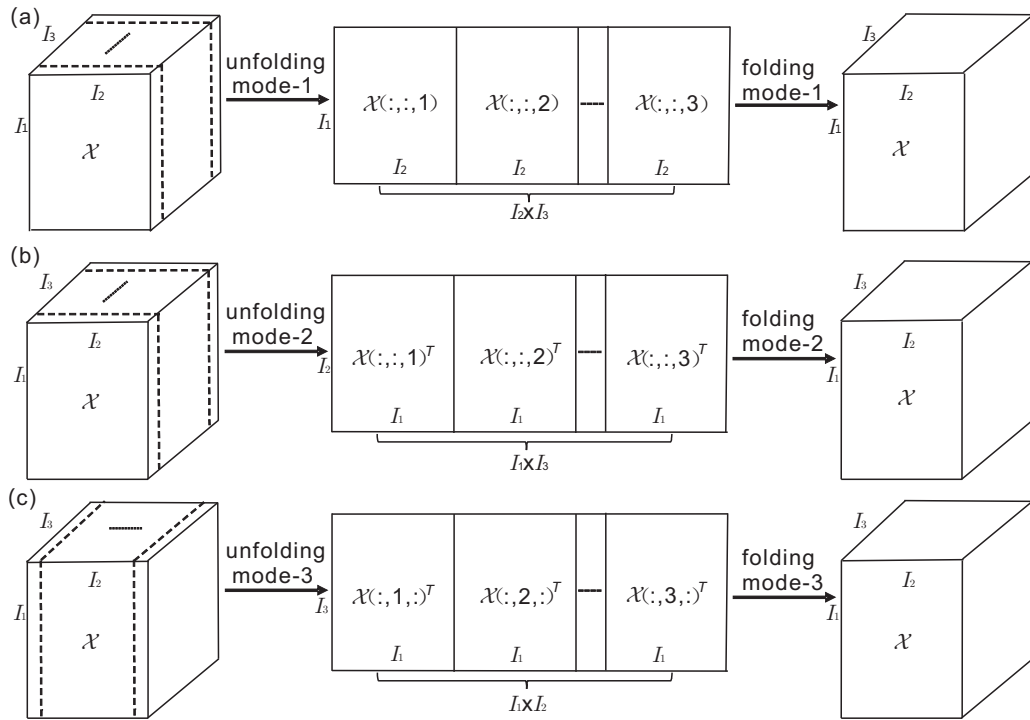


Figure 1: Illustration showing the 3 modes in which a tensor of order N can be unfolded. The PMF algorithm applies rank reduction to matrices obtained by unfolding the original tensor.

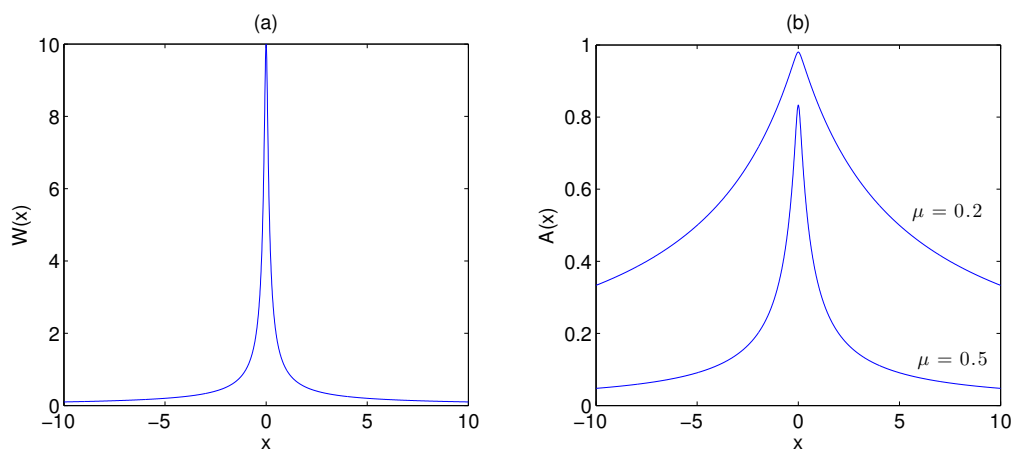


Figure 2: a) Functional form of the l_2/l_1 norm $W(x) = \sqrt{|x|^2 + \epsilon^2}$. b) Functional form of the weights $A(x) = (1 + M\mu\sqrt{|x|^2 + \epsilon^2})^{-1}$ for trade-off parameters $\mu = 0.2$ and 0.5

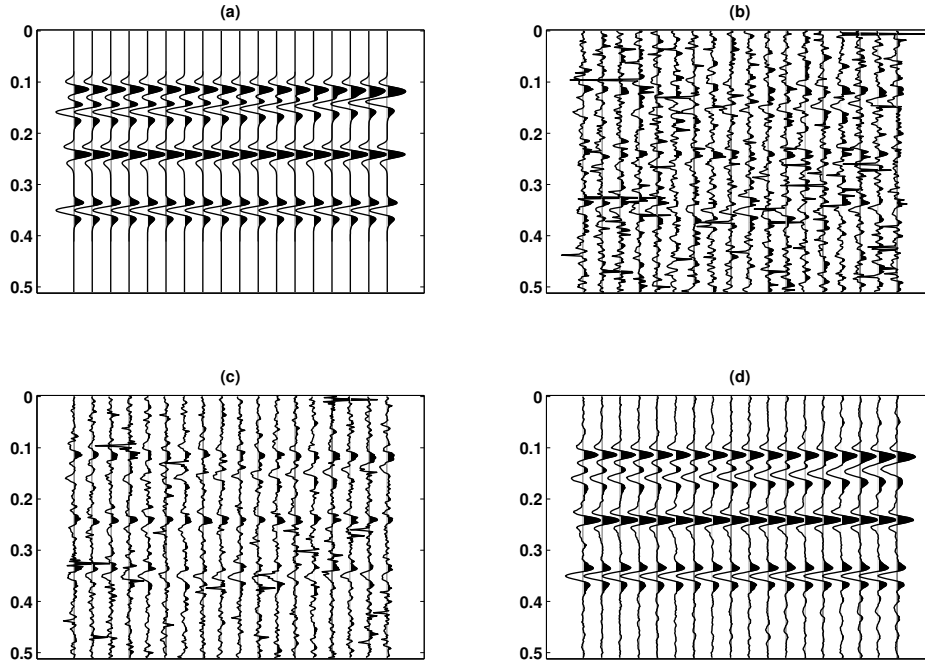


Figure 3: Denoising of a 5D volume of size $256 \times 18 \times 10 \times 10 \times 10$. a) Slice of the uncorrupted volume. b) Data contaminated with Gaussian and erratic noise. c) Tensor reconstruction assuming data contaminated with Gaussian noise. d) Robust tensor reconstruction.

also added erratic noise in the form of spikes with maximum amplitude ± 25 . The data contaminated by Gaussian noise plus erratic spiky noise is portrayed in Figure 3b. In this case the sampling operator $P = 1$ for all grid point because we are considering a denoising problem with a fully sampled 5D tensor. Figure 3c is the resulting data after applying our original algorithm using the quadratic l_2 misfit given by equation 4. The method has failed to properly denoise the data. To continue with our analysis, we have run the proposed algorithm with the robust misfit function (equation 5) with result portrayed in Figure 3d. The new method can cope with the presence of erratic noise. Our second example involves simultaneous denoising and reconstruction of a 5D cube. The cube size is $256 \times 18 \times 10 \times 10 \times 10$ and 60% of the data has been eliminated via the sampling operator to simulate a patch of a sparse orthogonal survey after binning the data in mid-point offset. The reconstruction via the l_2 misfit is portrayed in Figure 4c. Finally, we also show the reconstruction with the proposed method that uses a robust l_1/l_2 misfit in Figure 4d. The proposed method can also cope with the reconstruction of 5D volumes in the presence of erratic noise. In both examples we have used $\mu = 0.5$ and $\varepsilon = 0.01$.

Conclusion

The PMF algorithm has been proposed to reconstruct 5D volumes (Gao et al., 2015, 2016). One shortcoming of our original PMF algorithm was its inability to reconstruct 5D data when erratic noise corrupts the observations. A simple modification has permitted us to obtain an algorithm that down-weights the re-insertion of samples containing erratic noise.

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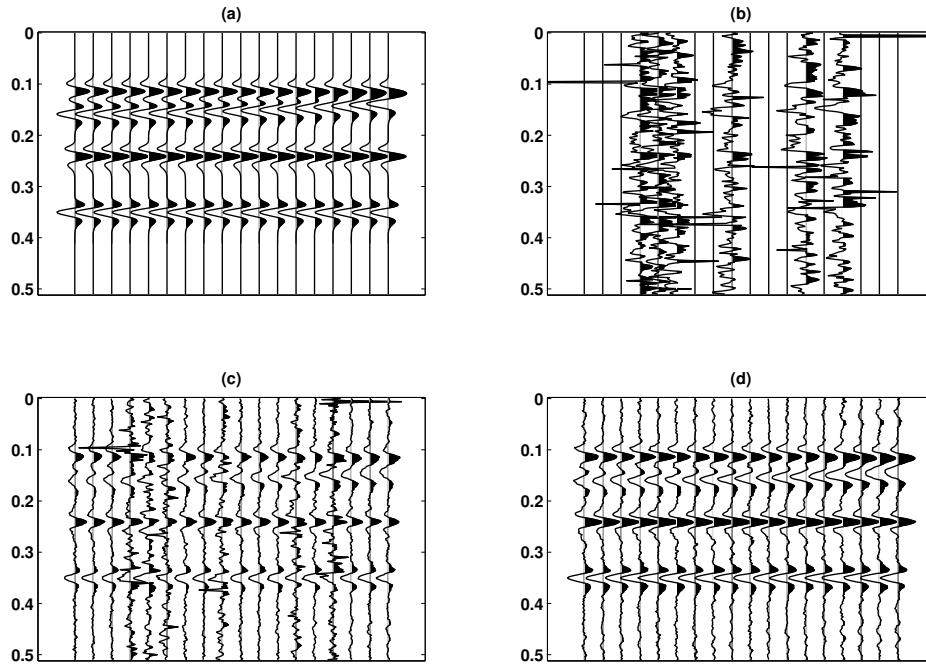


Figure 4: Reconstruction of a 5D volume of size $256 \times 18 \times 10 \times 10 \times 10$. a) Slice of the uncorrupted volume. b) Data contaminated with Gaussian and erratic noise. c) Tensor reconstruction assuming data contaminated with Gaussian noise. d) Robust tensor reconstruction.

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