



Upscaling of Anisotropic Reservoirs Using Quad-Tree Decomposition

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Summary

The Quad-Tree decomposition method is extended to unstructured oblong grid blocks for upscaling of absolute permeability in 2D heterogeneous thin stratified reservoirs with anisotropic permeability. The simulation results before and after upscaling are then compared in terms of accuracy and computational efficiency.

Introduction

The main datasets required for dynamic simulation of oil and gas reservoirs are geological models [1-3]. However, using fine scale geological and petrophysical realizations increases the computational time of the dynamic fluid flow simulations and requires high-performance computing platforms [4-9]. A popular solution involves decreasing the number of computational nodes by upscaling the geological models [10-13]. The upscaled model preserves the main features of the original model, but contains less grid blocks [10,12,14,15]. The upscaling process is performed through a two-step procedure. In the first step, called upgridding or homogenization, all neighboring fine blocks with similar homogeneity indicator parameters are combined and construct coarser computational grid blocks [9,16,17]. The homogeneity indicator can be a dynamic parameter such as fluid velocity, a petrophysical variable such as absolute permeability, or a combination of both. In the second step, called property reassignment, the permeability and porosity maps are propagated through the upscaled model [9,16,17]. The upscaling techniques can be classified based on their indicator parameters [18-21], or according to the topology of target models [22-26]. Most of the available upscaling techniques consider geological models with square grid blocks, and thus the grid blocks of the upscaled models are also square-shaped. However, in thin stratified reservoirs permeability variations in the vertical direction is sharper than the horizontal direction, and hence non-square grid blocks are required for adequate representation of heterogeneity. Here, an upscaling technique is proposed to combine the Quad-Tree decomposition [9, 28] and King's renormalization [29] techniques and upscale the fine scale geological models with non-square grid blocks.

Theory

The quad-tree decomposition is a 2D data structuring technique based on the successive subdivision of an image array into quadrants [28]. The original 2D model is considered as a parent node and is partitioned into four equal-sized quadrants. Then the homogeneity of each quadrant is examined using a homogeneity criterion to decide whether the decomposition is a valid one or not. This procedure is continued until all the constructed sub-quadrants satisfy the homogeneity criterion. Each partitioning stage is called a stage level or stage depth. To assess the homogeneity or heterogeneity of the quadrants, a proper homogeneity criterion must be applied [28]. An important advantage of the quad-tree decomposition algorithm is its flexibility in using different homogeneity criteria and indicator parameters such as standard deviation, variance, etc. Here, a threshold value is used to examine the homogeneity in permeability distribution. At each step if the permeability variation in every four quadrants is higher than a threshold value, the quadrants are not homogeneous and must remain partitioned. Otherwise, those are combined to construct coarser blocks [16,17].

After upgridding, the effective permeability of the reconstructed coarse grid blocks is calculated using the renormalization formula proposed by King [29]. Several other methods are also available through the literature [5], including but not limited to harmonic average, geometric average, arithmetic average, and pressure solver [30]. Using the renormalization method for every four rectangular neighboring grid blocks, **Fig.1**, the effective permeability of the corresponding coarse grid block is calculated as follows [16,17],

$$\bar{k} = \frac{N}{D} \quad (1)$$

where:

$$N = 4(k_1 + k_3)(k_2 + k_4)(k_1 k_3 (k_2 + k_4) + k_2 k_4 (k_1 + k_3)) \quad (2)$$

$$D = (k_1 + k_2 + k_3 + k_4)(k_1 k_3 (k_2 + k_4) + k_2 k_4 (k_1 + k_3)) + 3(k_1 + k_2)(k_3 + k_4)(k_1 + k_3)(k_2 + k_4) \quad (3)$$

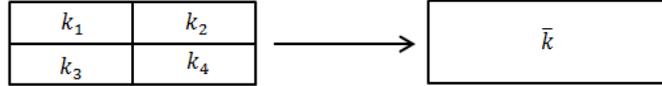


Fig. 1 Demonstration of how four fine square grid blocks construct a coarser rectangular grid block.

The most popular way to validate the workflow is to simulate a fluid flow process through both fine and coarse models and compare the simulation results. In this work, a waterflooding process is simulated. For a waterflooding process, in which water displaces oil, assuming that both of the phases are slightly compressible, the flow equations for oil and water phases are as follows [31],

$$\vec{\nabla} \cdot \left(\frac{k_{ro}}{\mu_o} k \vec{\nabla} P_o \right) - B_o Q_{o,sc} = -\varphi \frac{\partial S_w}{\partial t} + \varphi (1 - S_w) (c_r + c_o) \frac{\partial P_o}{\partial t} \quad (4)$$

$$\vec{\nabla} \cdot \left(\frac{k_{rw}}{\mu_w} k \vec{\nabla} (P_o - P_{cow}) \right) - B_w Q_{w,sc} = \varphi \frac{\partial S_w}{\partial t} + \varphi S_w (c_r + c_w) \frac{\partial P_o}{\partial t} \quad (5)$$

where P_o , $Q_{o,sc}$, B_o , μ_o and k_{ro} denote pressure, standard flow rate, formation volume factor, viscosity and relative permeability of oil, respectively. $Q_{w,sc}$, B_w , μ_w and k_{rw} represent the analogous parameters for water. S_w denotes water saturation and P_{cow} represents oil-water capillary pressure. c_o , c_w and c_r represent oil, water and rock compressibility factors, and φ and k are porosity and absolute permeability, respectively. There are two unknown parameters of S_w and P_o that must be calculated by simultaneous solving of the above PDE equations. A popular approach is to use IMPES formulation [32], where the coupled PDE set of **Eq.4-5** is separated into two pressure and saturation equations. The oil pressure is achieved implicitly, and subsequently the parabolic saturation equation is solved using explicit time approximations [32]. Combining **Eq.4** with **Eq.5** gives the pressure equation [16,17],

$$\vec{\nabla} \cdot \left(\frac{k_{ro}}{\mu_o} k \vec{\nabla} P_o \right) + \vec{\nabla} \cdot \left(\frac{k_{rw}}{\mu_w} k \vec{\nabla} (P_o - P_{cow}) \right) = B_o Q_{o,sc} + B_w Q_{w,sc} + \varphi c_t \frac{\partial P_o}{\partial t} \quad (6)$$

where c_t is the total compressibility and can be calculated as,

$$c_t = c_r + c_w S_w + c_o S_o \quad (7)$$

Once **Eq.6** is solved implicitly and P_o distribution in the medium is computed, **Eq.4** is solved explicitly to compute S_w distribution map. Discretization of an equation using the finite volume method involves integration of the equation over a control volume [20,33]. By integrating **Eq.6**,

$$\oiint \left[\vec{\nabla} \cdot \left(\frac{k_{ro}}{\mu_o} k \vec{\nabla} P_o \right) + \vec{\nabla} \cdot \left(\frac{k_{rw}}{\mu_w} k \vec{\nabla} (P_o - P_{cow}) \right) \right] dA = \oiint \left[\varphi c_t \frac{\partial P_o}{\partial t} + B_o Q_{o,sc} + B_w Q_{w,sc} \right] dA \quad (8)$$

We replace the right side of the above equation with an average value. Moreover, using the divergence theorem [34] the surface integral on the left side of the above equation can be simplified to a line integral,

$$\int \vec{n} \cdot \left(\frac{k_{ro}}{\mu_o} k \vec{\nabla} P_o + \frac{k_{rw}}{\mu_w} k \vec{\nabla} (P_o - P_{cow}) \right) ds = \left[B_o Q_{o,sc} + B_w Q_{w,sc} + \varphi c_t \frac{P_o^{n+1} - P_o^n}{\Delta t} \right] \Delta A \quad (9)$$

where ΔA and Δt denote the area of the grid block and time step, respectively. P_o^{n+1} and P_o^n represent oil phase pressure at times t_{n+1} and t_n , respectively. \vec{n} is the outward normal vector to the control volume boundaries. The last equation is essentially the mass conservation equation in which the left-hand side

terms represent the net input of oil and water into each grid block, and the right-hand side terms represent the accumulation of the oil and water phases in each grid block. Thus, we have,

$$\int (f_o + f_w) ds = \left[B_o Q_{o,sc} + B_w Q_{w,sc} + \varphi c_t \frac{P_o^{n+1} - P_o^n}{\Delta t} \right] \Delta A \quad (10)$$

where f_o and f_w represent the net flux of oil and water phases to each grid block, respectively. For the original fine model that is comprised of equal-sized grid blocks, calculation of f_o and f_w is straightforward and can be accomplished using the harmonic average. For example, f_o across the boundary of a grid block P and its neighbor, W , can be obtained by [16,17],

$$f_{ow} = \left(\frac{k_{ro}}{\mu_o} \right)_w \frac{2k_{xx,W}k_{xx,P}}{k_{xx,W} + k_{xx,P}} \frac{1}{\zeta} \left(\frac{P_{o,P} - P_{o,W}}{l} \right) \quad (11)$$

where f_{ow} is the oil flux across the boundary between P and W , and $(k_{ro}/\mu_o)_w$ denotes the oil mobility. l represents the distance between the center of two grid blocks, and ζ is the aspect ratio of grid blocks (aspect ratio = length of grid block / width of grid block). The above mentioned procedure is called Two-Point Flux Approximation (TPFA) [5]. In upscaled models with unequal-sized grid blocks, the phase fluxes across the grid boundaries cannot be approximated by TPFA and another approximation called Multi-Point Flux Approximation (MPFA) must be applied [5]. It should be mentioned that in upscaled models obtained by Quad-Tree Decomposition, each grid block may have 1, 2, 3, or more adjacent grid blocks at each direction, making it challenging to implement the flow governing equations. To overcome this problem, after the upscaling process, using a technique called tree balancing [22], all the grid blocks are scanned and in case the number of neighboring grid blocks is larger than 2, the grid block under-consideration is partitioned into 4 finer grid blocks. The most widely used approach to approximate fluxes across the boundary of unstructured grid blocks is Edwards approach [35]. For the unstructured configuration shown in **Fig.2**, using Edwards approach, the oil flux (f_o) across the boundary between P and WN can be approximated by,

$$f_{own} = \left(\frac{k_{ro}}{\mu_o} \right)_{wn} \frac{4k_{xx,P}k_{xx,WN}k_{xx,WS}}{4k_{xx,WN}k_{xx,WS} + k_{xx,P}(k_{xx,WN} + k_{xx,WS})} \frac{1}{\zeta} \left(P_{o,P} - \frac{P_{o,WN} + P_{o,WS}}{2} \right) \frac{1}{l} \quad (12)$$

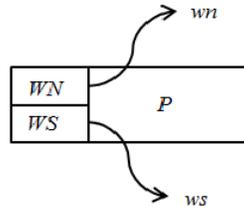


Fig. 2 Three adjacent unstructured grid blocks.

where $(k_{ro}/\mu_o)_{wn}$ represents oil mobility in the boundary between grid blocks P and WN . One of the well-known methods to calculate the mobility is TVD (total variation diminishing) [36-38],

$$\lambda_{o,boundary} = \lambda_{o,up} + \frac{\psi}{2} (\lambda_{o,down} - \lambda_{o,up}) \quad (13)$$

where $\lambda_{o,up}$, $\lambda_{o,down}$ and $\lambda_{o,boundary}$ represent oil mobility at the upstream grid block, downstream grid block, and the boundary between them, respectively, and parameter ψ is the mobility limiter [27]. Similarly, for other boundaries of the grid block P , the flux of oil and water phases are approximated. Assuming that the upscaled model has g grid blocks, the above-mentioned approach establishes a linear set of equations with g unknowns (i.e. P_o) and g equations. The final solution gives the value of P_o in all grid blocks. According to the IMPES procedure, the next step is to calculate the value of S_w in all grid blocks using the explicit time approximation of **Eq.4** (saturation equation). The discretized form of the saturation equation can be expressed as [16, 17],

$$S_w^{n+1} = S_w^n + \frac{\Delta t}{\varphi \Delta A} \left[B_o Q_{o,sc} \Delta A + \varphi (1 - S_w) (c_r + c_o) \frac{P_o^{n+1} - P_o^n}{\Delta t} \Delta A - \int f_o ds \right] \quad (16)$$

where S_w^{n+1} and S_w^n are water saturation at t_{n+1} and t_n , respectively. S_w^{n+1} in each grid block is independent of S_w^{n+1} in its neighboring grid blocks and therefore, S_w^{n+1} in each grid block can be calculated explicitly.

Results and Discussion

To demonstrate the efficiency and accuracy of the proposed workflow, a synthetic thin stratified reservoir is considered in which due to the low variation of absolute permeability in the horizontal direction and high variability in the vertical direction, the permeability distribution should be described with oblong grid blocks. The reservoir is upscaled, and a two-phase waterflooding process is simulated in both original and upscaled models. The original permeability map, upscaled permeability map, and its corresponding computational grid blocks are illustrated in **Fig.3**. The original permeability map is comprised of 84×72 oblong grid blocks with a width of 8 ft and an aspect ratio of 3. A visual comparison between the upscaled and original maps confirms that in the regions of the original map with minor permeability variations, the grid blocks of the upscaled model are coarser and in the other area with sharp permeability variations, the grid blocks of the upscaled map remain fine to preserve the original heterogeneity of the structure. The initial oil pressure and the initial water saturation in the reservoir are equal to 3600 psi and 0.22, respectively. The porosity is assumed constant and equal to 0.16. The formation volume factor and viscosity of both phases are assumed pressure-dependent. The simulated flow process is a two-spot waterflooding in which water with a constant flow rate of 40 STB/day/ft (2000 STB/day) is injected via a well located at the left-side of the reservoir and both oil and water are produced with a constant bottom-hole pressure of 2400 psi from a well located at the right-side of the reservoir, **Fig.4**. The radius of both injection and production wells is 0.3 ft, and the reservoir area is equal to 387072 ft². The oil and water flow rates and the injection bottom hole pressure are illustrated in **Fig.5** for both fine and upscaled models. With an increase in the threshold value the upscaling error increases. The values of other flow parameters at the end of 250 days for both original and upscaled models are listed in **Table.1**. Moreover, the number of grid blocks and the dynamic simulation time of the original and upscaled models with different thresholds are listed in **Table.2**. By applying the proposed upscaling workflow, one can successfully reduce the computational time and complexity of dynamic anisotropic reservoir simulation models while preserving main characteristics of fine-scale geological models.

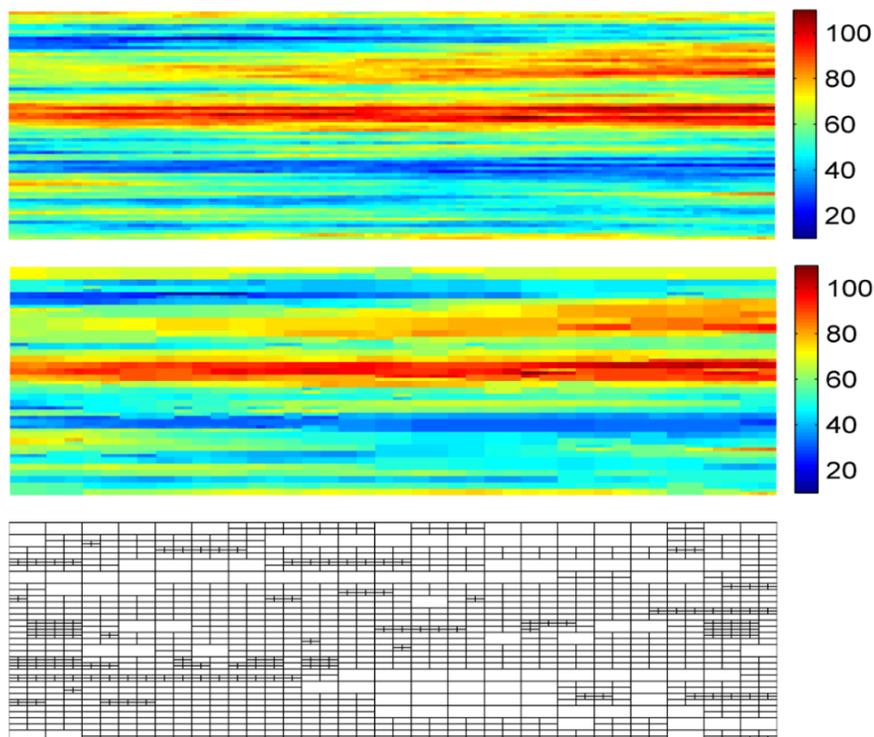


Fig. 3 Original permeability map in md (top), upscaled permeability map for a threshold of 21 (middle) and its corresponding computational grid blocks (bottom)

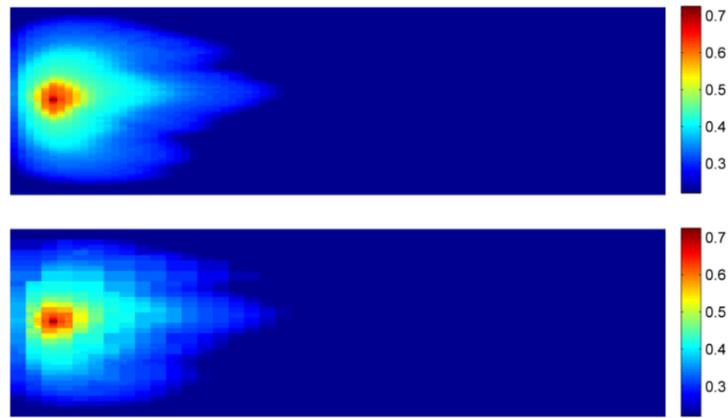


Fig. 4 Water saturation distribution in the original (top) and the upscaled (bottom) reservoirs after 30 days.

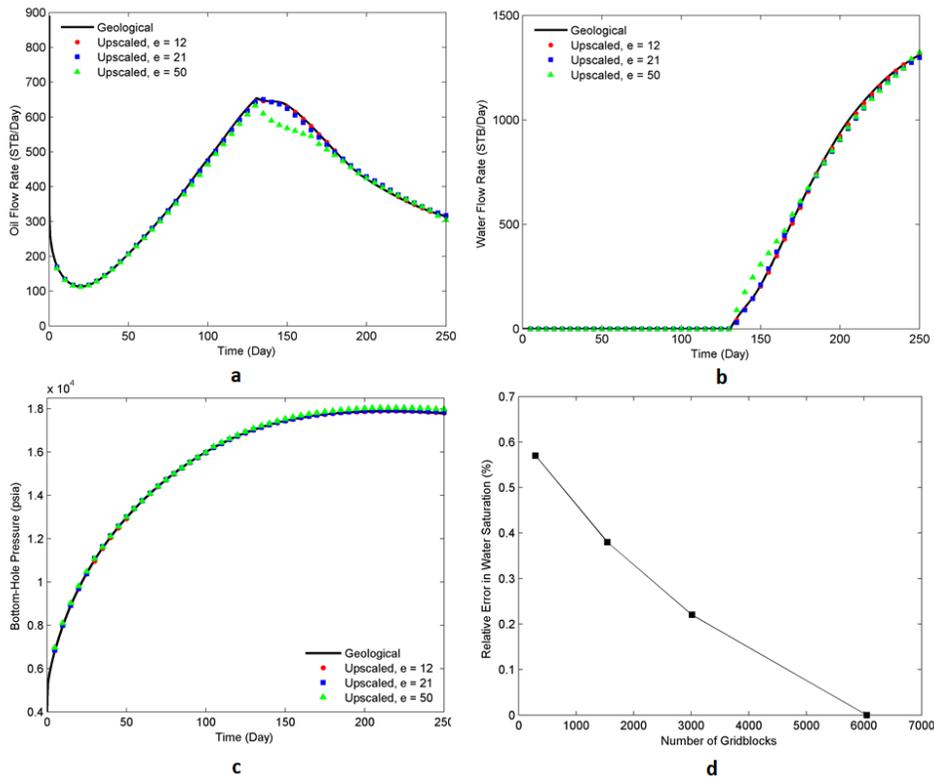


Fig. 5 Comparison between (a) Oil production rate, (b) water production rate, and (c) bottom-hole pressure of the injection well before and after upscaling. (d) relative error in water saturation vs. number of grid blocks.

Table 1. Number of the grid blocks, runtime, size reduction factor, speed-up factor of the upscaled and original models.

Parameter	Number of Grid blocks	Model Size Reduction Factor	Run Time (s)	Speed-Up Factor
Original Model	6048	-	113109	-
Upscaled Model (e = 12)	3015	2.0	20632	5.5
Upscaled Model (e = 21)	1542	3.9	5616	20.1
Upscaled Model (e = 50)	294	20.6	658	171.9

Table 2. Summary of the simulation results.

Parameter	Average Water Saturation (fraction)	Error (%)	Average Oil Pressure (psi)	Error (%)
Original Model	0.4051	-	11858	-
Upscaled Model (e = 12)	0.4042	0.22	11862	0.03
Upscaled Model (e = 21)	0.4035	0.38	11870	0.10
Upscaled Model (e = 50)	0.4028	0.57	11984	1.06

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