Reducing cross-talk with multi-resolution QFWI

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Summary

Full waveform inversion typically defines many more variables at each iteration than can be expected to be independently recovered. This prevents usage of more powerful optimization techniques, where large numbers of variables become very expensive. Without these techniques, it can be extremely difficult to distinguish between velocity and attenuation in a QFWI problem. Here, QFWI is performed with a model resolution defined based on the data frequencies considered at each iteration. This allows for Newton optimization to be employed in the recovery of the low frequency part of the model, considerably reducing cross-talk on long wavelength scales.

Introduction

The objective of full waveform inversion (FWI) is to reconstruct a complete subsurface model which accurately reproduces measured seismic data (Lailly, 1983; Tarantola, 1984; Virieux and Operto, 2009). Great advances have been made in, for instance, constructing high resolution velocity models through use of FWI (Virieux and Operto, 2009). Most advances in FWI have been made in the single parameter problem however, where only P-wave velocity is considered, and this results in the neglect of important effects. In particular attenuation and dispersion, which have a considerable impact on seismic data, are typically neglected. While extensions of FWI to attenuation and dispersion (QFWI) are relatively simple to formulate, they typically perform very poorly compared to single parameter FWI. This is due in large part to the problems arising from parameter cross-talk, where different physical properties are confused in the inversion, a serious concern in most multiparameter FWI formulations (e.g., Operto et al., 2013; Pan et al., 2016). Parameter cross-talk in FWI can be greatly reduced by using powerful optimization techniques. These techniques often have extremely large computational or memory requirements however, and are not typically feasible in the FWI problem. The inefficiencies of these approaches are a significant barrier to the implementation of effective QFWI.

Many authors have experimented with different ways of improving the efficiency of the FWI algorithm. One approach that has been investigated is to use powerful global optimization techniques to recover a coarse approximation of the subsurface, followed by traditional FWI (Datta and Sen, 2016). This allows for a relatively accurate model at large scales, helping to improve the results of the inversion that builds on this model. While this approach is promising, the extreme cost of performing a global optimization with a large number of parameters severely limits the scales at which it can be employed. Many techniques intermediate between steepest descent optimization and global optimization are also too computationally intensive for the very large number of variables considered. Notably, Newton optimization, which has been shown to be very effective in eliminating cross-talk in multi-parameter FWI, falls into this category. If Newton optimization can be applied at an affordable scale, it may provide useful large-scale information that helps to prevent cross-talk in the FWI result.

While FWI typically considers a fixed number of model variables, this may not be efficient or necessary. In multi-scale FWI early iterations are performed using only the low-frequency content of the data, while later iterations progressively introduce higher frequencies (Bunks et al., 1995). This naturally suggests that the achievable resolution of the recovered model will change as the inversion progresses. Here, we investigate the idea of applying Newton optimization in QFWI with a multi-resolution approach, wherein the inverted model is re-parameterized as the inversion progresses to reflect the frequency content of the data being considered.

Theory

Multi-parameter FWI, where several physical properties are recovered in the inversion, is often prone to parameter cross-talk, where data residuals introduced by one physical property mistakenly influence the estimate of another. This problem is not sufficiently treated when employing very simple numerical optimization techniques, such as the steepest descent method. On the other hand, it is well understood to be powerfully mitigated by considering second derivative information in the optimization procedure. A Newton optimization approach, where this second order information is fully employed, has the capacity to greatly reduce cross-talk, an important consideration in multiparameter FWI.

By considering second derivatives, Newton optimization helps to prevent one variable from being modified as a result of a data residual that can be entirely explained by a change in another variable. So, for instance, greater resolution can be obtained in a Newton update, as variables representing the same physical property at different locations are not confused for one another. Similarly, cross-talk between physically distinct properties can be reduced.

Exact Gauss-Newton optimization is typically unachievable in FWI due to the extremely large associated computational costs and memory requirements. The memory requirements consist chiefly of storing the Hessian matrix H, which for a model of **N** elements contains $\mathbf{N}^2$ entries. The computational cost is driven by the solution for the Newton update:

$\Delta p = H^{-1} g$ (1)

where Δp is the descent direction and g is the gradient. The cost of solving this system is on the order of approximately $\mathbf{N}^3$ operations. Achieving high resolution models of the subsurface with FWI necessitates choosing a very large value for **N**, causing this approach to typically exceed computational feasibility.

Methods which approximate the Newton update, such as truncated Newton and l-BFGS offer reduced cost, but can also struggle to eliminate cross-talk in QFWI updates. The large computational demands that can be exacted by even these methods motivate investigation of more efficient strategies for eliminating cross-talk.

Much of the information present in a Newton update may be useful, but perhaps not crucial to the inversion procedure. For instance the part of the Hessian which allows for better resolution forms a considerable fraction of the size and cost, but does not form the key motivation for using the Hessian. If cross-talk reduction is a major objective, Newton optimization with a coarse set of parameters may be desirable, as it mitigates cross-talk at a large scale, but reduces the number of parameters considered, and so the cost.

Often, a multiscale inversion approach is used in FWI to mitigate nonlinearity in the objective function. In this approach, low frequencies are inverted at early iterations, and successively higher frequencies are gradually introduced as the inversion proceeds. In effect, this allows for a relatively simple optimization problem to be solved initially, and for more nonlinear optimization steps to be approached with better starting models, simplifying the solution of nonlinear optimization. Given that seismic resolution is determined by the frequencies considered, this approach means that a model update obtained at early in the procedure will have significantly coarser achievable resolution than one obtained later. Typically, however, the same finite difference grid is used at all iterations in FWI, and this can result in an inverted model being defined on a grid which may have orders of magnitude more variables than can be accurately recovered. As it is the large number of variables that makes powerful optimization techniques prohibitively costly, it is reasonable to consider whether there are advantages to more conservatively parameterized models. We investigate here the idea of changing in tandem the grid on which the inverted model is defined, in the hopes of achieving efficiency improvements. We refer to this approach as ’multi-resolution FWI’.

In the multi-resolution approach we suggest, the grid on which the model update is defined is changed as the maximum frequency considered increases. This is done in such a way that the grid resolution of the model updates remain approximately equal to $\lambda/8$, where λ is the smallest wavelength in the reference medium. This approach allows for powerful optimization techniques to be employed at low frequencies. As frequencies increase and grid resolution increases, powerful optimization techniques are replaced with less demanding ones to allow for computational feasibility. In this way a largely cross-talk free model can be obtained at long wavelength scales, and better resolution can introduced at higher frequencies. Here we focus on applying this method using Newton optimization at low frequencies and steepest descent at higher frequencies, but multi-resolution QFWI could be applied using other optimization strategies.

Examples

The numerical examples below highlight the advantages of multi-resolution FWI for the visco-acoustic problem. The simple model considered features two velocity anomalies, the lower obscured by a region of anomalous Q. This model is shown in figure 1. The inversions were started using the background values of velocity and Q. 25 Sources and 50 receivers were placed along the top of the model, and frequencies from 1 to 25 Hz were assumed to be available. Figure 2 shows the result of visco-acoustic FWI using a truncated Newton optimization strategy which allows for consideration of significant second order derivative information. Despite a large computational cost, this approach still allows a significant degree of parameter cross-talk. The result of exact Newton optimization in a multi-resolution approach, using frequencies up to 7 Hz, is shown in figure 3. As expected, this approach results in significantly reduced spatial resolution, but also largely eliminates cross-talk on the model scales considered. This result can be further improved by performing steepest descent optimization at full resolution and higher frequencies. This allows for resolution improvement, as shown in figure 4.

Conclusions

Multiresolution FWI offers the potential for more powerful optimization techniques to be brought to bear on early stages of the FWI procedure. Gauss-Newton optimization in a multiresolution context, offers the potential to reduce large scale cross-talk in multiparameter FWI. This could help to make characterization of attenuation through FWI more feasible. More efficient implementations based on the same idea may be achievable by applying the multiresolution concept to the finite difference grid as well.

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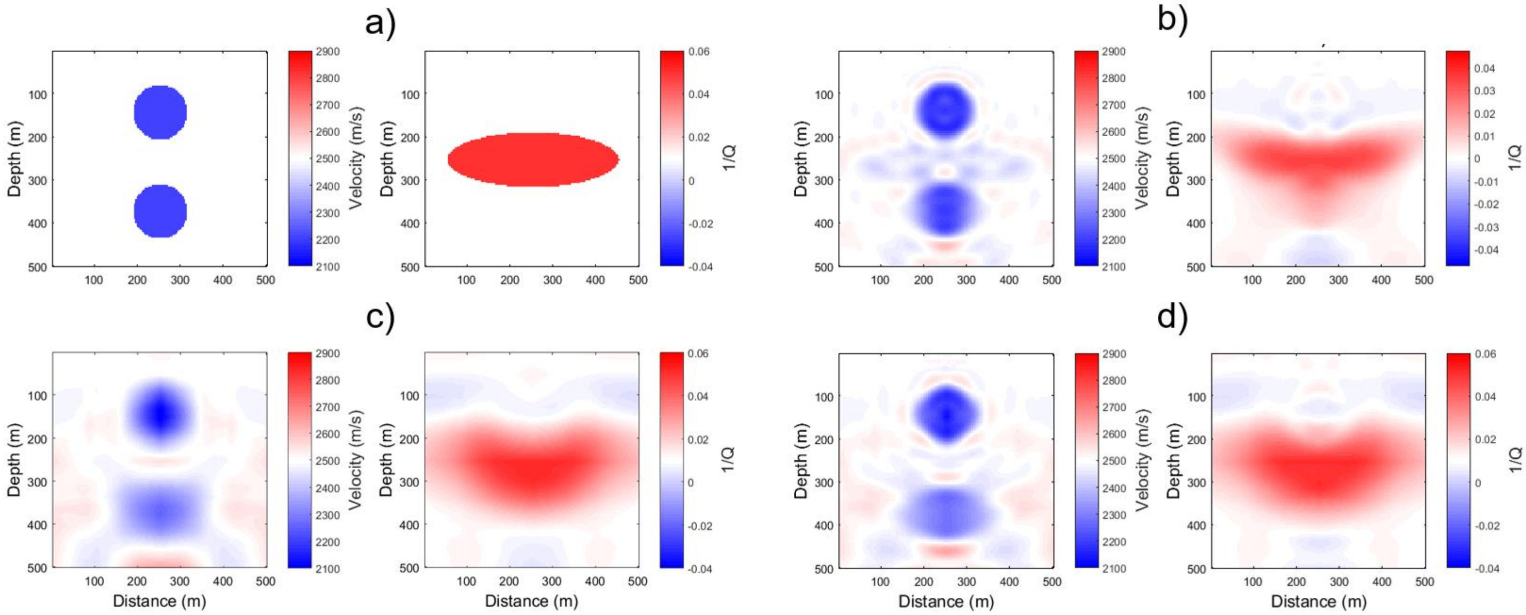

Figure 1: a) Velocity at 25 Hz (left) and reciprocal Q (right) of the true model. b) Inversion result using truncated Newton optimization up to 25 Hz. Cross-talk is apparent in the velocity updates at the Q anomaly location and vice versa. c) Inversion result using only frequencies up to 7 Hz in an exact Gauss-Newton multiresolution inversion. Note the reduction in cross-talk and resolution in comparison to c). d) Inversion result after applying steepest descent optimization up to 25 Hz to the result of c). Note the increase in resolution and relatively small change in cross-talk.

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