



## Internal multiple prediction in the time and offset domains

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### Summary

Internal multiples continue to be detrimental to the processing and interpretation of land seismic data. One method to predict internal multiples uses the inverse scattering series where the only required inputs are the data itself and a search limiting parameter epsilon ( $\epsilon$ ). The method was originally written such that the search limiting parameter was stationary due to the domain of calculation. The algorithm has been recently reformulated into the offset-time domain allowing for the selection of a nonstationary search limiting parameter. The impact of this reformulation into the offset-time domain using a nonstationary epsilon is displayed where a stationary epsilon is insufficient.

### Introduction

As a seismic wave propagates through a medium the source wavelet will be altered through mechanisms such as attenuation, short path multiples and other inelastic effects that will change the wavelet shape. Some of the energy that is traveling through the medium will also be reflected or scattered when there is a change in medium properties. If the wave is reflected off a single boundary and recorded this is termed a primary reflection. The seismic events that reflect off multiple interfaces prior to reaching the surface are termed multiples.

With increasing demands from seismic data including the prediction of AVO, inversion and rock physics parameters, the requirements for noise reduction also increase (Iverson, 2014). A method to predict internal multiples using the inverse scattering series was developed with a stationary epsilon (Weglein et al., 1997). It has been shown that reformulating the algorithm into the offset-time domain allows for the selection of a nonstationary search limiting parameter (Innanen, 2015). There are cases where a stationary epsilon is insufficient (Innanen, 2017; Innanen and Pan, 2014). This could be due to steeply dipping reflections, changes in frequency content or any other time variant wavelet changes due to the previously noted mechanisms. In certain 1.5D domains epsilon can vary in the transformed spatial dimension such as wavenumber (Innanen and Pan, 2014). These 1.5D domains allow for a variation of epsilon in the spatial dimension ( $k_g$ ) due to the method utilizing the 1D version of the algorithm over every spatial step. In 1D improvements in the prediction have been made through a nonstationary epsilon (Innanen, 2017). With the equation in offset-time there is now the flexibility to vary epsilon in both spatial and temporal dimensions.

### Theory and/or Method

The inverse scattering series was initially written in the wavenumber pseudodepth domain (Weglein et al., 1997). Giving equation (1) to predict interbed multiples from the seismic data and epsilon. The search limiting parameter is implemented in the integration bounds and sets a limit on the distance the multiple must have traveled ensuring the prediction of long path multiples. The combination of the pseudo-depth terms in the integration limits also ensures that the lower-higher-lower criteria is met and that no unwanted artifacts are in the prediction. The lower-higher-lower criteria defines the combination of events such that they are initially deeper in the subsurface, then shallower then deeper.

$$b_3(k_g, k_s, \omega) = \frac{1}{(2\pi)^2} \iint_{-\infty}^{\infty} dk_1 e^{-iq_1(\epsilon_g - \epsilon_s)} dk_2 e^{iq_2(\epsilon_g - \epsilon_s)} \int_{-\infty}^{\infty} dz_1 e^{i(q_g + q_1)z_1} b_1(k_g, -k_1, z_1) \\ \times \int_{-\infty}^{z_1 - \epsilon} dz_2 e^{-i(q_1 + q_2)z_2} b_1(k_1, -k_2, z_2) \int_{z_2 + \epsilon}^{\infty} dz_3 e^{i(q_2 + q_s)z_3} b_1(k_2, -k_s, z_3), \quad (1)$$

where in equation (1)

$$q_x = \frac{\omega}{c_0} \sqrt{1 - \frac{k_x^2 c_0^2}{\omega^2}}, \quad (2)$$

$b_3$  is the internal multiple prediction,  $b_1$  is the prepared input data,  $q_x$  is the vertical wavenumber and  $\epsilon$  is the depth below free surface of the source (s) and receiver (g),  $k$  is the Fourier conjugate variable,  $z_1, z_2$  and  $z_3$  are the depths chosen to satisfy the lower-higher-lower relationship and  $\epsilon$  is the search limiting parameter (Sun and Innanen, 2014). Multiples are predicted through multiplication in the Fourier domain, where in time this is equivalent to a combination of convolutions and correlations. It is shown schematically how two deeper events can be convolved relative to a shallower event, which can be correlated to mimic the equivalent multiple (Figure 1).

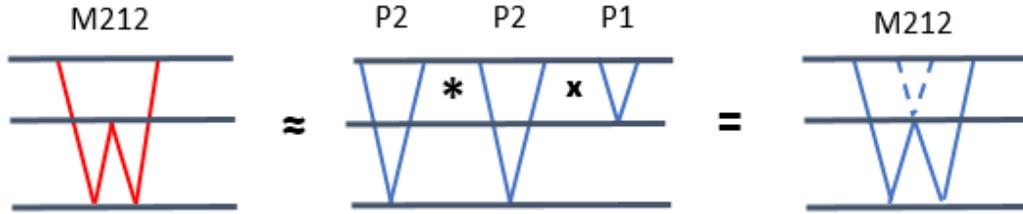


Figure. 1. Schematic displaying how a multiple can be replicated with a combination of primaries through a convolution (\*) and correlation (x)

Equation (1) can be simplified and reduced to a 1.5D domain by assuming a  $v(z)$  medium. This is done by assuming that the source and receiver wavenumbers are equivalent. Giving the 1.5D version of the algorithm giving equation (3)

$$b_3(k_g, \omega) = \int_{-\infty}^{\infty} dz_1 e^{ik_z z_1} b_1(k_g, z_1) \int_{-\infty}^{z_1 - \epsilon} dz_2 e^{-ik_z z_2} b_1(k_g, z_2) \int_{z_2 + \epsilon}^{\infty} dz_3 e^{ik_z z_3} b_1(k_g, z_3). \quad (3)$$

To write the algorithm in offset-time the first step is to replace  $b_1$  in terms of pseudo depth ( $z$ ) with  $S_1$  in terms of time ( $t$ ) and letting  $S_1(k_g, t)$  be the Fourier transform of the input data  $s_1(x, t)$  over the spatial dimension (Innanen, 2015) gives equation (4)

$$b_3(k_g, \omega) = \int_{-\infty}^{\infty} dt e^{i\omega t} S_1(k_g, t) \int_{-\infty}^{t - \epsilon} dt_2 e^{i\omega t_2} S_1(k_g, t_2) \int_{t_2 + \epsilon}^{\infty} dt' e^{i\omega t'} S_1(k_g, t'), \quad (4)$$

where the output domain is  $(k_g, \omega)$  as the convolutions and correlations are applied through multiplication in the frequency domain. This can be equivalently rewritten so that the convolutions and correlations are performed in the time domain. Giving the  $(k_g, t)$  version of the algorithm equation (5)

$$b_3(k_g, t) = \int_{-\infty}^{\infty} dt' S_1(k_g, t' - t) \int_{t' - (t - \epsilon)}^{t - \epsilon} dt'' S_1(k_g, t' - t'') S_1(k_g, t''). \quad (5)$$

Then by noting the remaining spatial convolutions applied in frequency this can be written fully in the time and space domain. Giving the  $(x, t)$  version of the algorithm below (Innanen, 2015)

$$B_3(x, t) = \int_{-\infty}^{\infty} dx' \int_{-\infty}^{\infty} dt' s_1(x - x', t' - t) \int_{-\infty}^{\infty} dx'' \int_{t' - (t - \epsilon)}^{t - \epsilon} dt'' s_1(x' - x'', t' - t'') s_1(x'', t''). \quad (6)$$

## Examples

To evaluate the  $(x, t)$  domain version of the algorithm a simple geologic model is used (Figure 2). The shot record is created with finite difference modeling and a 30Hz Ricker wavelet. The model is spatially sampled every 10m with a temporal sample rate of 0.002s. This geologic model has created a shot record with two primaries and a large first order internal multiple.

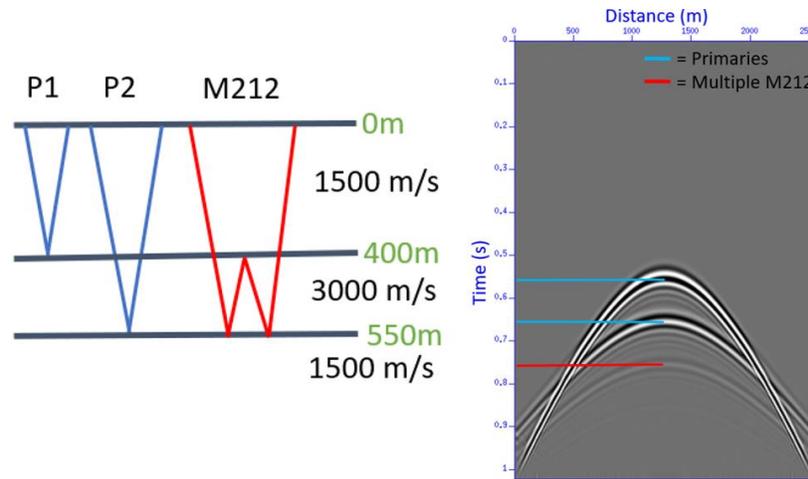


Figure. 2. (Left) Geologic model displaying velocities and depths used (Right) Shot record with two primaries and first order internal multiple defined

To use any version of the algorithm, at a minimum a stationary epsilon must be selected. Initially two values of epsilon are evaluated, a value of 30 and 70 (Figure 3).

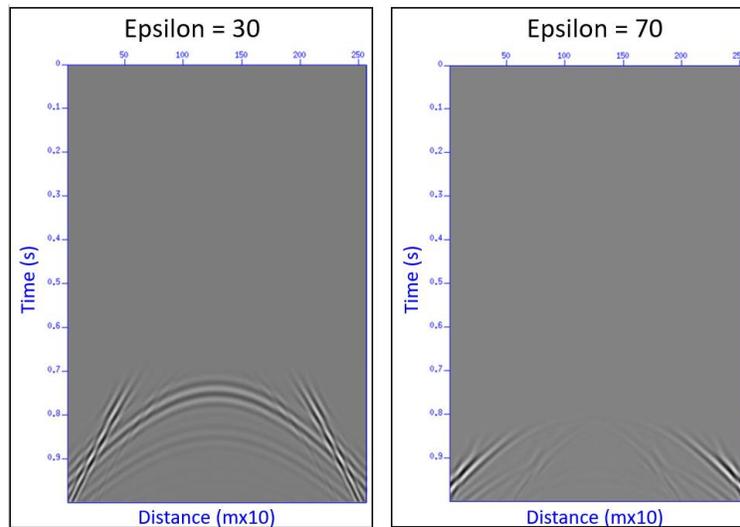


Figure. 3. (Left) multiple prediction with epsilon=30 (Right) multiple prediction with epsilon=70

For this shot record with an epsilon of 30 the first order internal multiple has been predicted along with higher order multiples. There is also a steeply dipping event predicted which is an unwanted artifact from the prediction. This issue has been previously noted when predicting in the  $(k_g, z)$  domain and was shown how a  $k_g$  varying epsilon could mitigate this issue (Innanen and Pan, 2014). The epsilon value of 70 has diminished this artifact but epsilon has also become sufficiently large that it impacts the prediction

of internal multiples. Next an epsilon schedule with the larger value (70) over the steeply dipping events and the lower value (30) elsewhere is utilized (Figure 4).

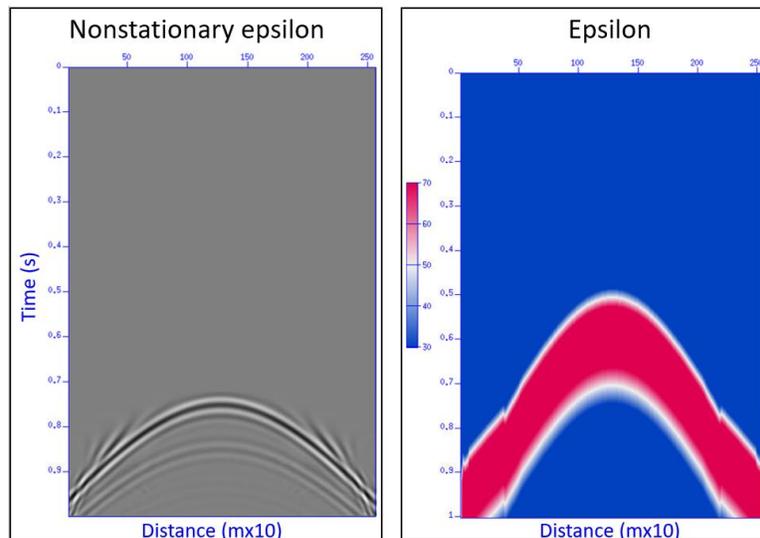


Figure 4. (Left) offset-time multiple prediction with spatially varying epsilon (Right) epsilon schedule used for prediction varying in both offset and time

This final schedule which varies in both offset and time is capable of both predicting the multiples in the data and minimizing the artifacts. Though there is still some residual artifact present from the steeply dipping event. With this full flexibility to vary epsilon in any dimension, this introduces future work to derive the optimum epsilon schedule for a given input model.

## Conclusions

Using the inverse scattering series for internal multiple prediction has been adapted to be computed in offset-time (Innanen, 2015). With this change in computational domain comes the ability to utilize a non-stationary epsilon. Displayed was an example of how varying epsilon can improve the prediction on a model where a stationary epsilon was insufficient. This now allows for the precise determination of epsilon in both dimensions which may allow for solving specific issues at various locations in a dataset. Now models can be tested with the offset-time algorithm and the flexibility in epsilon schedule creation.

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