Fast waveform inversion: a low-cost solution for the full waveform inversion

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Summary

The Fast Waveform Inversion (FastWI) is a linear solution of the Full Waveform Inversion. It is fast, as it is applied on post-stack data, and does not require any forward modeling. The gradient is calibrated by available sonic logs. We tested the inversion to apply an acoustic inversion on the vertical component of the processed Hussar data. The inversion is guided and compared to three sonic logs acquired over the 2D line. The inverted P-wave velocities show consistency in the well location and with the stacked section, with the clear limitations of the acoustic inversion on elastic data. Also, we noticed that the calibration is only effective in areas with high signal-to-noise ratio in the stacked section.

Introduction

Seismic inversion techniques are the ones that use intrinsic information contained in the data to determine rock properties by matching a model that "explains" the data. Some examples are the variation of amplitude per offset, or AVO (Shuey, 1985; Fatti et al., 1994), the traveltime differences between traces, named traveltime tomography (Langan et al., 1984; Bishop and Spongberg, 1984; Cutler et al., 1984), or even by matching synthetic data to the observed data, as it is done in full waveform inversion (Tarantola, 1984; Virieux and Operto, 2009; Margrave et al., 2010; Pratt et al., 1998), among others. These inversions can compute rock parameters as P and S waves velocities, density, viscosity and others, but we are interested on the inversion of the P wave velocity.

FWI is a least-square based inversion, which objective is to find the model parameters that minimizes the difference between observed (acquired) and synthetic shots (Margrave et al., 2011), or the residuals. This is accomplished in an iterative fit method by linearizing a non-linear problem. It was proposed in the early 80’s (Pratt et al., 1998) but the technique was considered too expensive in computational terms. Lailly (1983) and Tarantola (1984) simplified the methodology by using the steepest-descent method (or gradient method) in the time domain to minimize the objective function without calculate, explicitly, the partial derivatives. They compute the gradient by a reverse-time migration (RTM) of the residuals. Pratt et al. (1998) develop a matrix formulation for the full waveform inversion in the frequency domain and present more efficient ways to compute the gradient and the inverse of the Hessian matrix (the sensitive matrix) the Gauss-Newton or the Newton approximations. The FWI is shown to be more efficient if applied in a multi-scale method, where lower frequencies are inverted first and is increased as more iterations are done (Pratt et al., 1998; Virieux and Operto, 2009; Margrave et al., 2010). An overview of the FWI theory and studies are compiled by Virieux and Operto (2009). Lindseth (1979) showed that an impedance inversion from seismic data is not effective due to the lack of low frequencies during the acquisition but could be compensated by the match with a sonic-log profile. Warner and Guasch (2014) use the deviation of the Weiner filters of the real and estimated data as the object function with great results. Margrave et al. (2010) and Romahn and Innanen (2016) calibrate the gradient by matching it with sonic logs, estimating a more precise step length and avoiding cycle skipping. They also use the Gazdag and Sguazzero (1984) PSPI migration algorithm with a deconvolution imaging condition (Margrave et al., 2011; Wenyong et al., 2013) to migrate the residuals faster than the RTM, but preserving the gradient’s resolution. Guarido et al. (2015) use the PSPI migration to migrate each frequency content of the residuals independently, generating a pseudo-gradient for each frequency and then averaging stacking them, using the step length as weight. It resulted on a highly detailed model, but the computation costs are high. Guarido et al. (2016) apply an impedance inversion in the gradient to improve the resolution of the inverted model. Guarido et al. (2016, 2017b) propose a simpler approximation for the gradient that does not require any forward modeling, by linearizing the problem. They just apply a PSPI migration on the acquired data and compare the result with the current model. The methodology was also extended for a post-stack depth migration, the Fast Waveform Inversion, or FastWI. However, two forward modeling are required to estimate the step length. Guarido et al. (2017a) proposes to combine the forward modeling-free gradient method with the well calibration of Margrave et al. (2010) and Romahn and Innanen (2016) to obtain a fast waveform inversion that doesn't require any forward modeling and source estimation, it only requires the PSPI migration of the acquired data (that can be pre or post stack). Guarido et al. (2017c) compare the use of the RTM and PSPI migrations in the estimation of the gradient for the FastWI, showing that the choice depends on how complex is the velocity model and the computational power available.

The FastWI is applied to the Hussar data (Margrave et al., 2012) to obtain the P-wave velocity of the subsurface. Three sonic logs are used to create the initial model (an interpolation of their linear trend), and to guide the direction and calibration of the gradient. As the stacked section shows to be composed mostly by horizontally flat reflectors, we use a zero-offset PSPI migration to estimate the gradient, as it is cheaper but preserves the resolution when compared to the RTM (Guarido et al., 2017c).

Theory

The FastWI is based on the steepest-descent approximation of the FWI, which minimizes L2-norm of the residuals **$\Delta\mathbf{d}(\mathbf{m})$**, that is the difference between observed data **$\mathbf{d}_0$** and synthetic data **$\mathbf{d}(\mathbf{m})$**, when the model **$\mathbf{m}$** is changed:

$$C(\mathbf{m}) = ||\mathbf{d}_0 - \mathbf{d}(\mathbf{m})||^2 = ||\Delta\mathbf{d}(\mathbf{m})||^2 \tag{1}$$

The minimization can be done by the steepest-descent method (Pratt et al.,1998):

$$\mathbf{m}_{n+1} = \mathbf{m}_n - \alpha_n \mathbf{g}_n \tag{2}$$

where **α** is the step length, **$\mathbf{g}$** is the gradient and **$\mathbf{n}$** is the n-th iteration. The gradient is computed by a RTM of the residuals (Tarantola, 1984; Pratt et al., 1998; Virieux and Operto, 2009), but we decided to use PSPI migration. Understanding all the gradient estimation steps as seismic pocessing tools, equation 2 can be rewritten in terms of the migration **M**, stacking **S** and impedance inversion **I** operators:

$$\mathbf{m}_{n+1} = \mathbf{m}_n - \alpha_n I\left\{S\left[M\left(\mathbf{d}_0 - \mathbf{d}_\mathbf{n}\right)\right]\right\} \tag{3}$$

where **$\mathbf{d}_\mathbf{n}$** is the synthetic shot. Guarido et al. (2016) assume all three operators are linear (true for migration and stack and approximate for impedance inversion) and the gradient can be interpreted as a residual difference of the processed acquired data and the processed synthetic. The processed synthetic shot is the current model itself. This explanation is better visualized by looking to equation 4:

$$\begin{aligned}\mathbf{m}_{n+1} &= \mathbf{m}_n - \alpha_n\Big(I\left\{S\left[M\left(\mathbf{d}_0\right)\right]\right\} - \underbrace{I\left\{S\left[M\left(\mathbf{d}_\mathbf{n}\right)\right]\right\}}_{\text{Current model}}\Big)\\ &= \mathbf{m}_n - \alpha_n\left(I\left\{S\left[M\left(\mathbf{d}_0\right)\right]\right\} - \mathbf{m}_n\right)\end{aligned} \tag{4}$$

Interpreting the gradient as the residual difference of the processed acquired data and the current model saves us to actually compute a synthetic data at each shot position. Source estimation is also not required. The order of the migration and stacking operators on equation 4 can be inverted, resulting on a post-stack solution for the FWI (equation 5).

$$\mathbf{m}_{n+1} = \mathbf{m}_n - \alpha_n\left(I\left\{M\left[S\left(\mathbf{d}_0\right)\right]\right\} - \mathbf{m}_n\right) \tag{5}$$

The step length **α** can be replaced by a well calibration (Margrave et al., 2010; Romahn and Innanen, 2016), leading to a methodology applied on post-stack data and that is 100% forward modeling-free, named *Fast Waveform Inversion*.

FastWI Applied to Hussar Data

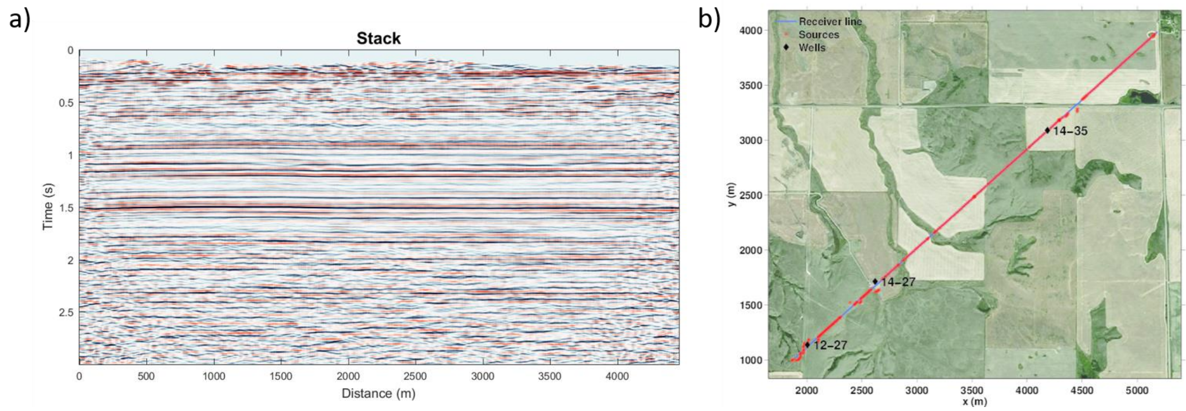

Figure 1: a) 2D stacked section and b) survey location, showing the wells 12-27, 14-27 and 14-35.

Figure 1a shows the processed and stacked section of the Hussar data, survey acquired east of Calgary (figure 1b). The shots were acquired with the use of dynamite. Three sonic logs (figures 1, 2 and 3) were acquired across the line and are used to calibrate the gradient. The initial model (figures 2a and 3a) is the interpolation of the linear trend of the sonic logs. It is used in the first iteration to migrate the stacked section with a zero-offset PSPI algorithm to estimate the gradient. The gradient is calibrated by creating a matching filter at each well location that minimize the difference between the gradient to the sonic log, and then they are interpolated for all CDP locations. The matching filters are then convolved trace-by-trace with the gradient, to estimate the model update, and the routine is repeated each iteration.

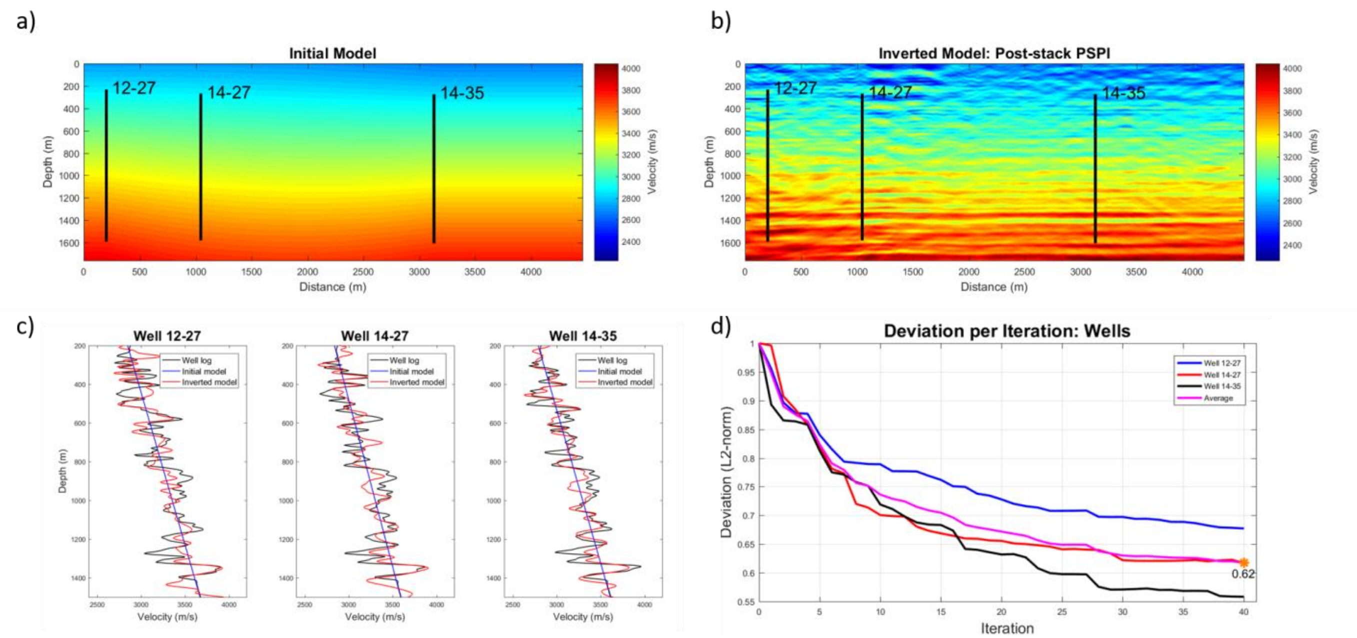

Figure 2: a) Initial model, b) inverted model, c) sonic log (black) compared with the initial (blue) and inverted (red) models and d) the deviation at each well location and their average (magenta) when matching the 3 wells.

Figure 2b is the inverted model when the 3 wells are used to match the gradient. Figures 2c and 2d show the convergence of the inverted model at the wells locations, which is expected, as the gradient is driven to match the sonic logs. To make sure that the inversion is consist in between the wells, we applied a serie of tests where each one of the wells were used as blind (not used to match) and also using each well individually and we came with an interesting observation. The well 12-27 is located close to the border of the survey, where the stacked section of figure 1a does not have full-fold (resulting on lower signal-to-noise ratio). As this region is noiser, the matching filter generated at this location was trying to minimize the noise. In the end, including this well to the inversion was “damaging” the final model. On figure 3b is the inverted model when the well 12-27 is not used to generate a matching filter, and the it improved the continuity of the flat layers at the left side of the model. Figures 3c and 3d show that the inverted model is still converging at the wells locations.

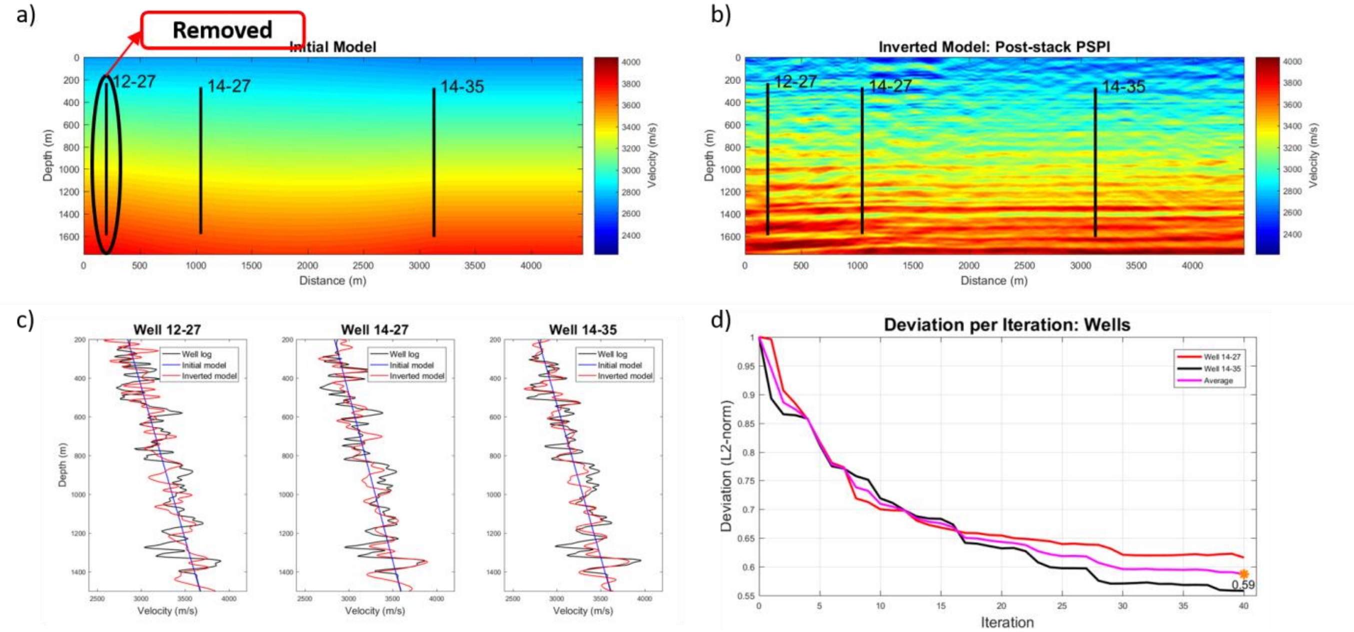

Figure 3: a) Initial model, b) inverted model, c) sonic log (black) compared with the initial (blue) and inverted (red) models and d) the deviation at each well location and their average (magenta) when the well 12-27 is removed.

Conclusions

The Fast Waveform Inversion (FastWI) is a linear solution of the full waveform inversion, leading to a forward modeling-free method, and it can be performed with post-stack migration. The optimization is driven by calibrating the gradient with sonic logs.

It was applied to the Hussar data and the inverted model is consistent with the sonic logs and the stacked section, showing good resolution, considering it is an acoustic inversion applied to field data. The method showed to be stable, as the deviation (L2-norm) at the wells locations shows to converge, and it is a low cost method that requires fewer preliminary information when compared to the FWI, for example, the wavelet is not required for the inversion. By selecting different wells as a blind spot, to check how the inversion’s behavior outside the wells locations, we came to the conclusion that the calibration of the gradient will work better if the sonic log used to obtain the match filter is located in an area with high signal-to-noise ratio. If this requirement is satisfied, the FastWI showed to provide a reasonable inversion at the blind well spot, leading us to assume that the same is reached at any area of the model.

For future work, we will take advantage of its low computational cost to extend it for 3D surveys.

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