Deriving Geotechnical Parameters from Density and Incomplete Seismic Datasets

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Summary

A three-parameter configuration framework provides a comprehensive visualization for the relationship between density, seismic velocities, and elastic modulis such as Bulk Modulus, Young’s Modulus, and Poisson’s Ratio. However, these constants cannot be obtained with the absence of shear wave velocity measurements. With the datasets missing shear wave measurements, the task is to carry out an error analysis and conduct a relative range for elastic modulis, which are essential for geotechnical purposes. We achieve this by adding a variable Vp/Vs range to the three-parameter plot.

Introduction

It is important to analyze whether geophysical parameters can represent some of the mineralogical and geotechnical parameters in order to minimize risk for the follow-up exploration progress. Density (d), resistivity, and compressional wave velocity (Vp) data were obtained from boreholes in the Athabasca Basin. However, the obtained data provides no information for the shear wave velocity. Without information from the shear wave velocity, it is impossible to calculate the elastic properties precisely since they all depend on this variable. In order to provide an estimate of the elastic properties of the host rocks in the Athabasca Basin with the absence of the shear wave velocity, we conducted error analysis on these elastic moduli. The significance of estimating these elastic properties is to provide essential information on whether the follow-up exploration process would be attainable.

Theory and Method

Wave velocities and density can be used to calculate some of the elastic properties of the rock. Young’s Modulus, which describes the tensile elasticity or the stiffness of a rock, are derived from wave velocity and density. Other properties such as Shear Modulus, Bulk Modulus, and Poisson’s Ratio can be derived from the same parameters.

<table>
<thead>
<tr>
<th>Elastic Moduli</th>
<th>Unit</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Poisson’s Ratio (c0)</td>
<td>N/A</td>
<td>σ = ρ (Vp^2 - 2Vs^2) / 2(Vp^2 - Vs^2)</td>
</tr>
<tr>
<td>Young’s Modulus (E)</td>
<td>GPa</td>
<td>E = ρ Vp^2 / Vp^2 - Vs^2</td>
</tr>
<tr>
<td>Shear Modulus (μ)</td>
<td>GPa</td>
<td>μ = d Vp^2</td>
</tr>
<tr>
<td>Bulk Modulus (κ)</td>
<td>GPa</td>
<td>κ = d (Vp^2 - 2Vs^2)</td>
</tr>
<tr>
<td>Seismic Parameter (sp)</td>
<td>km²/s²</td>
<td>sp = Vp^2 - 4Vs^2</td>
</tr>
</tbody>
</table>

Table 1. Equations for elastic modulis such as Poisson’s Ratio, Yong’s Modulus, Shear Modulus Bulk Modulus, and Seismic Parameter
To visualize and interpret the geophysical data, we used a three-parameter configuration framework shown in a 3D plot. A surface representing the bulk modulus is calculated using the empirical relationships between density, seismic velocities and $\kappa$. The benefit of using such a framework is that the data obtained from all types of minerals, including the high-density ones such as iron-oxides and massive sulfides, all fall on this theoretically surface.

An estimate for the Vs values is derived from the empirical ratio of compressional wave velocity and shear wave velocity. Typically, the $V_p/V_s$ ratio ranges from 1.5 to 3.0, but it can range up to 8.0 for very soft materials. Using the estimated Vs, the data can be plotted onto the surface, with an error bar representing the range of the estimated elastic modulus.

**Examples**

Some common igneous, metamorphic rocks and some common sulfides (Schmitt et. al., 2003) are projected onto a theoretical surface in a three-parameter framework ($d$-$sp$-$\kappa$) shown in Figure 1.
Figure 1. 3D framework for density, seismic parameter and bulk modulus with some common igneous and metamorphic rocks (green), and common sulfides (blue) plotted. All data points fall onto the surface. Values are from Schmitt et. al. (2003).

Error bars are plotted according to the error percentage from the estimated Vs values. It shows the trends for the resulting change in $\kappa$ for each data points. The color code represents the corresponding values of the Young’s Modulus along these trends.

Figure 2. Added real data (yellow) and error bars for all data points. Error percentage is obtained from setting Vp/Vs to range from 1.7 to 2.1, and is approximately 10%. The color bar represents the Young’s Modulus of the estimated data for the error trends.
Conclusions
The three-parameter framework is advantageous in visualizing rock characterization for various types of minerals. The error analysis in bulk modulus and Young’s modulus due to uncertainties in shear wave velocities provides a proxy for the realistic ranges of the attainable values in these elastic constants for different rock types. Both the bulk modulus and Young’s modulus are positively related to the seismic parameter. Errors in these elastic parameters due to uncertainties in Vs are more sensitive to high density and seismic parameter materials and less sensitive to low density and seismic parameter materials. The results provide useful information for its practicability in mineral exploration purposes.

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References

Milkereit, Bernd, and Maria Wu. "The Velocity – Density Relationship Revisited."

