geoconvention
Calgary • Canada • May 13-17 2019

SSA reconstruction without binning

Fernanda Carozzi and Mauricio D. Sacchi

Department of Physics, University of Alberta

Summary

Hankel-matrix rank reduction principles lead to effective and flexible reconstruction techniques in seismic processing. The method relies on the predictability of the seismic signal for equally spaced channels. However, given the logistics of the seismic acquisition operation, sampling the wavefield following a regular survey can be challenging. We introduce an inversion approach that reconstructs and denoises the observed data with irregular coordinates to a desired regular grid. The objective function considers a low-rank constraint. We propose using the Projected Gradient method to optimize the objective function.

We present synthetic and field test. From the experiments, we find that IMSSA outperforms the classical MSSA reconstruction method with the reassignment of traces. Such improvement is assigned to the use of the real, observed coordinates, instead of an ideal geometry, to reconstruct the dataset.

Introduction

Seismic reconstruction is a primary step in seismic data processing workflows. There is a variety of algorithms available in the industry, utilizing concepts ranging from the wave equation to signal processing tools. Methods considering signal processing tools are most frequently used by industry. They can be classified into three sets. First, those algorithms using transforms, such as Fourier (Zwartjes and Gisolf, 2006; Trad, 2009), Radon (Verschuur and Kabir, 1995) or Curvelet (Hennenfent et al., 2010; Naghizadeh and Sacchi, 2010). Second, techniques considering prediction error filters (Spitz, 1991; Naghizadeh and Sacchi, 2007). Finally, methods that use rank reduction of matrices (Trickett, 2008; Oropeza and Sacchi, 2011; Chen and Sacchi, 2015) or tensors (Kreimer and Sacchi, 2012; Kreimer et al., 2013).

Rank-reduction reconstruction algorithms assume that adequately sampled, noise-free data can be represented via a low-rank array. Missing data and additive noise increase the rank of the matrix or tensor describing the data. In particular, MSSA assumes that given a seismic volume with k dipping events, the Hankel matrix of each frequency slice presents k linearly independent vectors. To compute the rank-reduced array, standard MSSA resorts to the Eckart- Young theorem by using truncated Singular Value Decomposition. To obtain the reconstructed seismic data, one should average along the anti-diagonals of the low-rank matrix. A drawback to this method is that the input data should be regularly distributed. Then, the process starts by reading seismic traces and allocating them to a regular grid, before doing the MSSA reconstruction. As a result, the traces provided to the MSSA algorithm contain coordinates that have been moved from their observed position to the center of a cell of the desired output geometry.

This abstract discusses a modification to the MSSA method to cope with irregular data, avoiding the introduction of spurious errors by binning. We propose an iterative rank reduction algorithm based on a Projected Descent method to approximate the reconstructed traces by minimizing the error between observed and approximated data. Finally, we show synthetic and field data examples.

Theory

In seismic processing, the reconstruction problem aims to recover a regularly and completely sampled volume D_r, from a set of irregularly distributed and incompletely sampled dataset D_{obs}. That is, one intends to describe the wavefield such that

$$\Phi D_r \approx D_{obs} \ . \tag{1}$$

If the observed data were regularly gridded, the operator Φ would represent the adjoint of the sampling matrix. In reality, given the complexity of the seismic acquisition, the observed dataset is not regularly gridded. Then, one needs to analyze the operator Φ in greater detail.

The observed seismic data can be described as a combination of a vector of measurements and a set of irregular coordinates. However, the reconstructed traces present a regular geometry. Then, we determine a transformation of the regular grid to the exact field position. Such transformation is the operator Φ in Equation 1. In this abstract, we present $2D$ reconstruction and the transformation is a linear interpolator, but one could opt for other interpolation techniques.

The reconstruction problem described in Equation 1 is underdetermined. To find a stable solution one poses the problem as the constrained optimization

$$\min_{D_r} J = \|\Phi D_r - D_{obs}\|_F^2 \ \text{,s.t.}\ D_r = S\{D_r\} \ , \tag{2}$$

where D_r is the approximated volume or unknown, D_{obs} is the observed data, and F is the Frobenius norm. The result is constrained to those solutions whose Hankel matrix are low rank. We enforce the low-rank constrain via the SSA rank-reduction filtering method

$$S\{D_r\} = ARH\{D_r\} \ , \tag{3}$$

where H is the Hankelization operator, R is a rank reduction operator, and A represents the anti-diagonal averaging of the approximated low-rank matrix.

To solve for D_r, we propose a Projected Gradient algorithm (Cheng and Sacchi, 2016; Bolduc et al., 2017). Then, the iterative scheme to solve Equation 2 is

$$\hat{D}_r = D_r^{i-1} - s\Phi^* \left(\Phi D_r^{i-1} - D_{obs}\right) \tag{4}$$

$$D_r^i = S\{\hat{D}_r^i\} \ , \tag{5}$$

where s is a step size, Φ is the interpolator operator, and Φ^* its adjoint.The interpolator operator calculates the estimated value at the observed position as a weighted linear combination of the approximated values in the regular nods.

To conclude, the proposed algorithm evades the storage of a binned dataset, which includes a significant percentage of zero-traces. Besides, instead of modifying the input data, IMSSA reconstructs directly from the observed traces. Finally, as the regularization of the data is calculated iteratively, the algorithm completes the unfilled positions gradually, avoiding the introduction of significant amplitude and phase errors.

Examples

We test the method with a synthetic example. We model 40 traces, we introduce jittering in the coordinates, and randomly decimate 50% of the traces. Finally, we include random Gaussian noise with a SNR$= 1.2$ dB. The reconstruction is calculated via IMSSA in the frequency range $[1\text{-}60]$ Hz, step size $s = 0.5$, and ideal rank 2. Figure 1(a), shows the ideal, regular traces. Figure 1(b) shows the decimated, irregular traces. These traces represent the observed traces. Finally, Figure 1(c) depicts the result of the reconstruction via IMSSA.

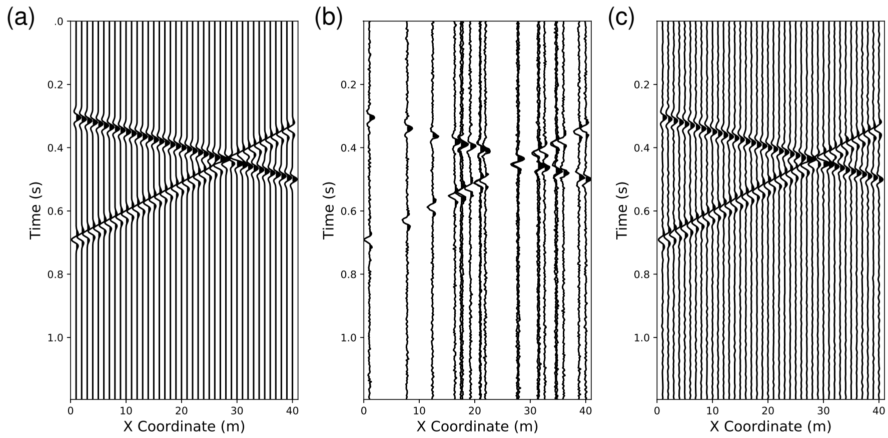

Figure 1: Reconstruction of linear synthetic events. (a) Original data. (b) Randomly decimated with erratic noise. (c) Reconstructed data via the proposed IMSSA method.

Next, we compare the reconstruction quality between IMSSA and the standard MSSA with re-assignment of traces. The comparison is based on the output signal-to-noise ratio of the reconstructions. Figure 2 shows the results for both methods for different percentages of available traces. We conclude that IMSSA consistently outperforms MSSA.

We also tested the algorithm with field data from a survey on a heavy oil field in Alberta, Canada. We extract one receiver line from a given shot gather and randomly decimate 40% of the available traces. Following, we reconstruct via IMSSA.

Figure 3(a) shows the observed data. Figure 3(b) depicts the randomly decimated traces. Figure 3(c) shows the results after reconstruction via IMSSA. The results show that the method does not create spurious reflections. Also, the data show more continuous events and increase the signal-to-noise ratio.

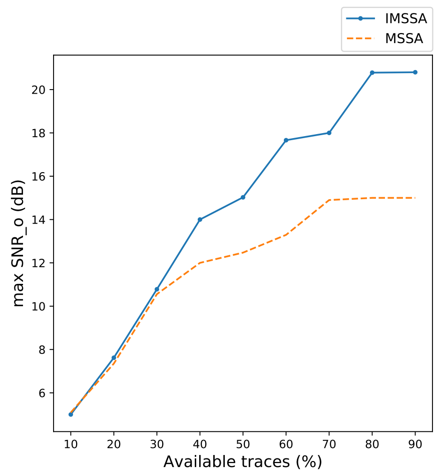

Figure 2: Signal-to-noise ratio for MSSA and IMSSA.

Conclusion

Many seismic reconstruction techniques require regularly binned input data, which introduces spurious errors to the observed traces. In this abstract, we propose reconstructing seismic data from the irregular coordinates to a regular grid, without binning.

We present an objective function including an interpolator operator that approximates the value of the reconstructed volume to the observed position. The optimization problem is constrained to those solutions that present a low-rank Hankel matrix. We implement a Projected gradient algorithm to approximate the result. Finally, we show synthetic and field examples. We conclude that the proposed method successfully reconstructs seismic data, outperforming the standard SSA algorithm.

Acknowledgements

The authors are grateful to the sponsors of Signal Analysis and Imaging Group (SAIG) at the University of Alberta for their continued support. We also acknowledge support from the Natural Sciences and Engineering Research Council of Canada (NSERC) via a Discovery Grant to MDS.

References

Bolduc, E., G. C. Knee, E. M. Gauger, and J. Leach, 2017, Projected gradient descent algorithms for quantum state tomography: npj Quantum information, **3**, 44.

Chen, K., and M. D. Sacchi, 2015, Robust reduced-rank filtering for erratic seismic noise attenuation: Geophysics, **80**, no. 1, V1–V11.

Cheng, J., and M. D. Sacchi, 2016, Fast dual-domain reduced-rank algorithm for 3D deblending via randomized QR decomposition: Geophysics, **81**, V89–V101.

Hennenfent, G., L. Fenelon, and F. J. Herrmann, 2010, Nonequispaced Curvelet transform for seismic data reconstruction: A sparsity-promoting approach: Geophysics, **75**, WB203–WB210.

Kreimer, N., and M. D. Sacchi, 2012, A tensor higher-order singular value decomposition for prestack seismic data noise reduction and interpolation: Geophysics, **77**, V113–V122.

Kreimer, N., A. Stanton, and M. D. Sacchi, 2013, Tensor completion based on nuclear norm minimization for 5D seismic data reconstruction: Geophysics, **78**, V273–V284.

Naghizadeh, M., and M. D. Sacchi, 2007, Multistep autoregressive reconstruction of seismic records: Geophysics, **72**, no. 6, V111–V118.

———, 2010, Beyond alias hierarchical scale curvelet interpolation of regularly and irregularly sampled seismic data: Geophysics, **75**, no. 6, WB189–WB202.

Oropeza, V., and M. D. Sacchi, 2011, Simultaneous seismic data denoising and reconstruction via multichannel singular spectrum analysis: Geophysics, **76**, V25–V32.

Spitz, S., 1991, Seismic trace interpolation in the *F-X* domain: Geophysics, **56**, no. 6, 785–794.

Trad, D., 2009, Five-dimensional interpolation: Recovering from acquisition constraints: Geophysics, **74**, no. 6, V123–V132.

Trickett, S., 2008, *F-XY* Cadzow noise suppression: SEG, Expanded Abstracts, **27**, no. 1, 2586–2590.

Verschuur, D. J., and M. M. N. Kabir, 1995, Restoration of missing offsets by parabolic radon transform: Geophysical Prospecting, **43**, no. 3, 347–368.

Zwartjes, P., and A. Gisolf, 2006, Fourier reconstruction of marine-streamer data in four spatial coordinates: Geophysics, **71**, no. 6, V171–V186.

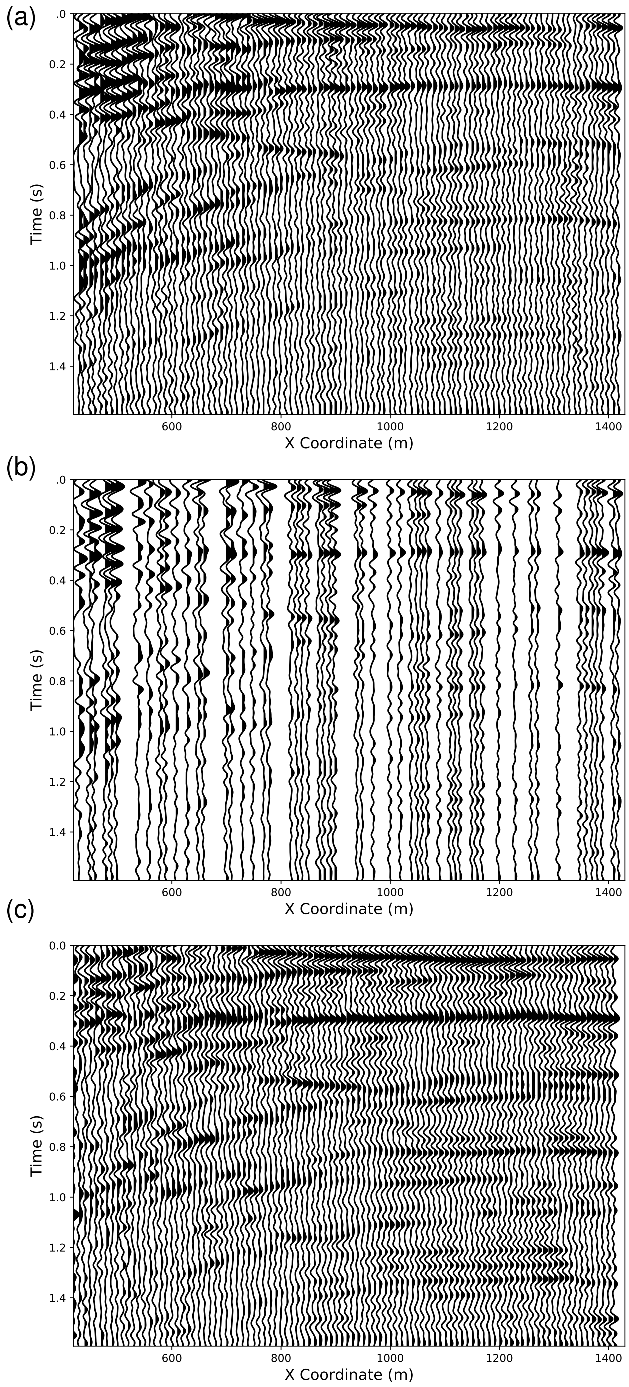

Figure 3: Reconstruction of field data. (a) Observed data. (b) Randomly decimated traces. (c) Reconstructed data via the proposed IMSSA method.