



Forecasting Gutenberg-Richter parameters in probabilistic seismic hazard analysis for induced seismicity

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Summary

One of the major challenges in seismic hazard analysis for induced seismicity is the prediction of future seismicity rates, which are described by the Gutenberg-Richter parameters. In this abstract, we study two methodologies in order to predict the Gutenberg-Richter parameters related to induced seismicity: the Seismogenic Index and the Hydromechanical nucleation model. We apply both methods in one recent case of induced seismicity: the Horn River Basin, Northeast B.C. We compare the predictions of both models with the observed seismicity. Our preliminary results show that the predictions follow the observed induced seismicity patterns. However, by comparing the Gutenberg-Richter parameters, the predictions may both under- and over- estimate the hazard, due to the complexity in the evolution of the seismicity, the stochastic nature of earthquakes, and the assumption of constant b -values. More work has to be done in order to properly test the predictions as well as predict more accurate Gutenberg-Richter parameters.

Theory and Method

Non-stationary seismicity and time-dependent Gutenberg-Richter Parameters

Reyes Canales and van der Baan (2018) derived analytical expressions required in Probabilistic Seismic hazard Analysis (PSHA) for non-stationary seismic sources, as well as modifications in the Monte Carlo simulation method to generate non-stationary synthetic earthquake catalogs. To account for non-stationarity, we define time-dependent Gutenberg Richter (GR) parameters ($a(t)$ -and $b(t)$ -values). Some of the derived non-stationary expressions include the total expected number of earthquakes $N(M_{min} \leq m \leq M_{max}; t)$ per time unit in the range $M = [M_{min}, M_{max}]$, which is given by:

$$N(M_{min} \leq m \leq M_{max}; t) = 10^{a(t)-b(t)M_{min}} - 10^{a(t)-b(t)M_{max}}, \quad (1)$$

Where the $b(t)$ -value indicates the ratio of small and large magnitude events, the $a(t)$ -value is related to the cumulative number $N_0(t)$ of earthquakes with a non-negative magnitude up to time t , $N_0(t) = 10^{a(t)}$. M_{min} and M_{max} are the minimum and maximum magnitude, respectively. Also, by assuming non-stationary Poisson distribution, the number of events in a certain time interval is given by:

$$P[N = n; t_a, t_b] = \frac{m_\lambda^n(t_a, t_b)(t_b - t_a)^n e^{-m_\lambda(t_a, t_b)(t_b - t_a)}}{n!}, \quad (2)$$

where $m_\lambda(t_a; t_b)$ is the mean of the time-varying rate of occurrence $\lambda(t)$ in the time interval $t = [t_a, t_b]$.

Forecasting Gutenberg Richter parameters: Physics-based models

One of the challenges in the seismic hazard analysis for induced seismicity is the prediction of the Gutenberg-Richter parameters. In this paper we describe two physics-based models in order to address this issue: The Seismogenic Index and the Hydromechanical nucleation model.

Seismogenic Index: Shapiro et al. (2010) modify the classical Gutenberg-Richter recurrence law (Gutenberg and Richter, 1944) in order to include fluid injection-induced earthquakes at hydrocarbon and geothermal reservoirs:

$$\text{Log}(N) = \text{Log}(Q_c(t)) + \Sigma - bm = a'_c(t) - bm, \quad (3)$$

Where N is the number of earthquakes with a magnitude greater than m . $Q_c(t)$ is the cumulative volume injected up to time t , Σ is the Seismogenic Index, and $a'_c(t)$ -value is related to the cumulative number $N_{0c}(t)$ of earthquakes with a non-negative magnitude up to time t , $N_{0c}(t) = 10^{a'_c(t)}$. The Seismogenic Index Σ incorporates the volume concentration of pre-existing faults and the state of stress in one area (Shapiro et al.2010). In practice, the Seismogenic Index is obtained by calculating the cumulative a_c -value from a catalog with induced earthquakes and the cumulative volume injected in that time, as follows:

$$\Sigma = a_c - \text{Log}(Q_c(t)). \quad (4)$$

Once the Seismogenic Index is calculated for one area, it is possible to predict the changes in the cumulative $a'_c(t)$ -value by adding the Log of the future volume to inject to the Seismogenic Index:

$$a'_c(t) = \text{Log}(Q_c(t)) + \Sigma. \quad (5)$$

Notice that the validity of these equations also relies on a constant Seismogenic Index and b -value.

Hydromechanical nucleation model: Dieterich (1994) and Segall and Lu (2015) developed an empirical seismicity rate model that relates changes in the Coulomb stress with changes in the seismicity rates. The temporal evolution of seismicity rate can be described by using the following ordinary differential equation:

$$R'(t) = \left(\frac{R(t)}{t_c} \right) \left(\left(\frac{\dot{s}}{\dot{s}_0} \right) - R(t) \right), \quad (6)$$

where $R(t)$ is the ratio between the seismicity rate $r(t)$ resulting from the injection, and the background seismicity rate r_0 , thus $R(t) = r(t)/r_0$. The stressing rate \dot{s} is the Coulomb stressing rate on the faults, and \dot{s}_0 represents the tectonic stressing rates. Finally, the parameter t_c is the characteristic decay time and is defined by: $t_c = (\bar{a} \bar{\sigma}) / \dot{s}_0$, where \bar{a} is the direct-effect parameter in the rate-and-state friction formulation, and $\bar{\sigma}$ is the effective normal stress. In order to solve this ordinary differential equation, a model to describe the changes in the Coulomb stressing rate \dot{s} is required. Norbeck and Rubinstein (2018) assume that the changes in the Coulomb stressing rate \dot{s} is approximately equivalent to the pressurization rate \dot{p} . The pressurization rate \dot{p} in response to injection is moderated by the compressibility of the system:

$$\dot{s} \approx \dot{p} = Q(t)/(V\varphi\beta), \quad (7)$$

where $Q(t)$ is the injection rate per time unit, V is the reservoir bulk volume, φ is the rock porosity and β is the total reservoir compressibility. By solving equation (5) we can model the Coulomb stressing rate \dot{s} necessary to solve equation (4) and finally obtain the parameter $R(t)$, which ultimately reflects the change in the a -value, as follows:

$$a'(t) = \log(R(t)) + a. \quad (8)$$

In other words, the $a'(t)$ -value can be forecasted by knowing the $R(t)$ parameter, which depend on the injection rates and several stress and tectonic parameters from the site, as well as the background a -value.

Case example: Horn River Basin induced seismicity

We apply both methods in one area with recent induced seismicity: The Horn River Basin, Northeast B.C. The detected seismicity in the area was very low prior 2006, but with an important increase since Dec. 2006, particularly between 2010 and 2011 in line with increasing injection rates (BC. Oil and Gas commission, 2012). We use the earthquake catalog from Farahbod et al., (2015b) which contains induced earthquakes in the Horn River Basin. We also require catalogs of injected volumes for both the Seismogenic Index and the Hydromechanical nucleation model. For the injected volumes, we use the catalog from Farahbod et al., (2015a), which contains the volumes injected per month at the Horn River basin between Dec. 2006 and Dec. 2011.

To estimate the Seismogenic Index, we simply calculate the total volume injected $Q_c(t)$ up to Dec. 2010, and by knowing the corresponding cumulative a_c -value, we use equation (4) to obtain the Seismogenic Index value. The a_c -value is given by the catalog between Dec-2006 and Dec-2010 with magnitude of completeness $M_c = 2.4$. To estimate the b -value, we apply a modified version of the maximum likelihood method (MLM, Aki, 1965; Wiemer and Wyss, 1997) to the recorded catalog between Dec-2006 and Dec-

2010. Assuming that the estimated Seismogenic Index and b -value from Dec. 2010 are constant, we can predict the expected number of earthquakes per month (Eq.1), given the monthly volume injected. Figure 1 shows the number of earthquakes larger than $M>2.5$ per month, given by the predictions of the Seismogenic Index and observed seismicity. On the other hand, the cumulative $a'_c(t)$ - value for 2011 will be given by adding the Log of the total volume to inject in 2011 $Q_c(t)$ and the Seismogenic Index (Eq.4). Notice that we are using the Seismogenic Index and b -value from Dec. 2010 to secure stable parameters.

The Hydromechanical nucleation model requires multiple parameters that are unknown for the Horn River Basin. As a first approach, we use similar parameters to those described by Norbeck and Rubinstein (2018) in Oklahoma, with modifications to get a better fit between predictions and observed seismicity. We first calculate the pressurization rate \dot{p} (Eq.7) using the catalog of injection rate per time unit $Q(t)$ and the required reservoir parameters. Then, assuming that the Coulomb stressing rate is approximate to the pressurization rate, $s' \approx \dot{p}$, we solve equation (6) to obtain $R(t)$. Finally, by using Eq. (8) we obtain the $a'(t)$ -value resulting from the injection rates. For the b -value, we assume a background b -value=0.86, as given by the 2015 National seismic-hazard model of Canada (Halchuk et al., 2014) for the area. We also compare the monthly number of earthquakes larger than $M>2.5$, given by the predictions of the Hydromechanical Nucleation approach and observed seismicity (Fig.1).

Both models fail to predict the large number of earthquakes for Dec. 2011. This seismicity may be related to an earthquake swarm, a sequence of many earthquakes triggered in a relatively short period of time. Neither model is designed to predict these anomalies, but just the induced main-shock events. We compare the GR parameter given by these predictions with the observed seismicity (earthquake catalog from Farahbod et al., (2015b)) for the year 2011. To calculate the GR parameters of the observed seismicity, we use a $M_c = 2.4$, and we apply the MLM. Figure 2 (Left) shows a comparison between the GR parameters given by the Seismogenic Index, the Hydromechanical model and the GR parameters given by the observed seismicity for the year 2011. Figure 2 (Right) shows the expected number of earthquakes larger than $M=4$ for that year (Non-stationary Poisson distribution, Eq. 2). We evidence a mismatch between the GR parameters from the prediction and the actual seismicity. This difference could be explained by the short duration of the observed catalog (1 year), which give us a short observation period to obtain the parameters from a stochastic process. Another reason is the possible bias in the forecast of GR parameters and the inability to predict large aftershock sequences. For extreme case scenario, we obtained that the most likely number of events with $M>4$ was 0, 5 and 1 for the Seismogenic Index, the hydromechanical model and the GR parameters given by the observed seismicity, respectively. No earthquakes larger than $M>4$ were recorded in the Horn River Basin between Dec 2006 – Dec 2011.

Discussion and conclusions

In this report, we apply two methodologies to predict the Gutenberg-Richter parameters related to injection-induced seismicity, using the Horn River Basin as a case example. Our preliminary results show that there is a relatively good agreement between the predicted number of earthquakes given by both models and the observed seismicity per month. Both models describe the increase and decrease of induced seismicity related to the injection rates, but they fail to capture anomalies like the earthquake swarm in Dec. 2011.

The number of earthquakes for a magnitude range is not enough information for a complete PSHA analysis: it is important to study the magnitude-frequency distribution (GR parameters) in detail. On the other hand, the predictions of the GR parameters from both models tend to either underestimate or overestimate the seismic hazard. However, a question still unsolved is the proper evaluation of the predictions, due to the stochastic nature of earthquakes. Even if we have the appropriate long-term earthquake recurrence parameters, the short-term seismicity will not necessarily follow the long-term seismicity parameters due to the stochastic nature of earthquakes (Aleatory uncertainty). Another reason for this mismatch could be the input data and methods of source parameterization used in the hazard analysis (Stirling, 2014). In many cases, insufficiency or wrongly recorded earthquake catalogs lead to biased a -and b -values. More work to predict accurate GR parameters is required in order to improve the forecasts. For instance, the assumption constant b -value in both models may not be appropriate, and future models may forecast variations in the b -value.

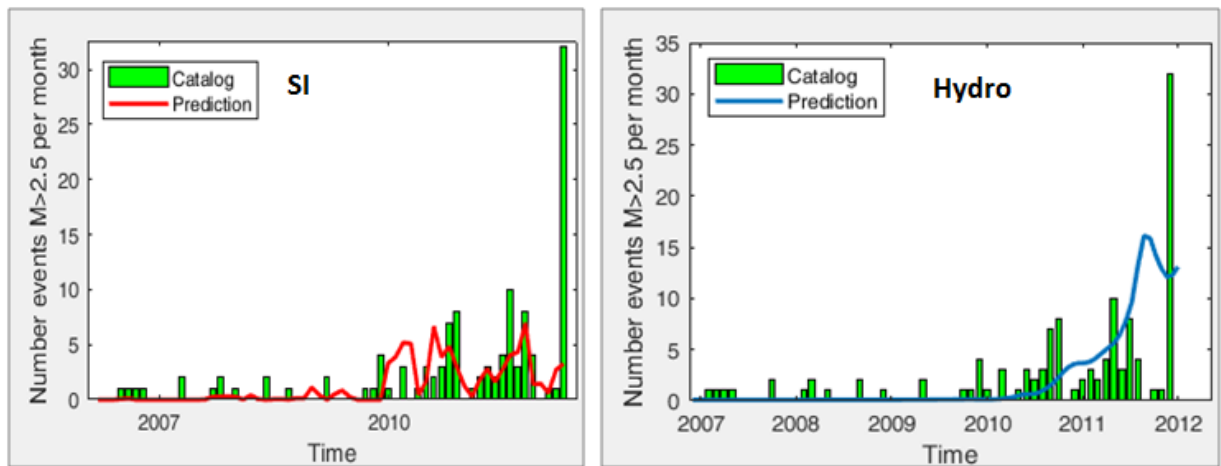


Figure 1. The figure on the left shows a comparison between the monthly number of earthquakes larger than $M > 2.5$, given by the observed seismicity and the Seismogenic Index model. The figure on the right shows the monthly number of earthquakes larger than $M > 2.5$, given by the observed seismicity and the Hydromechanical Nucleation approach.

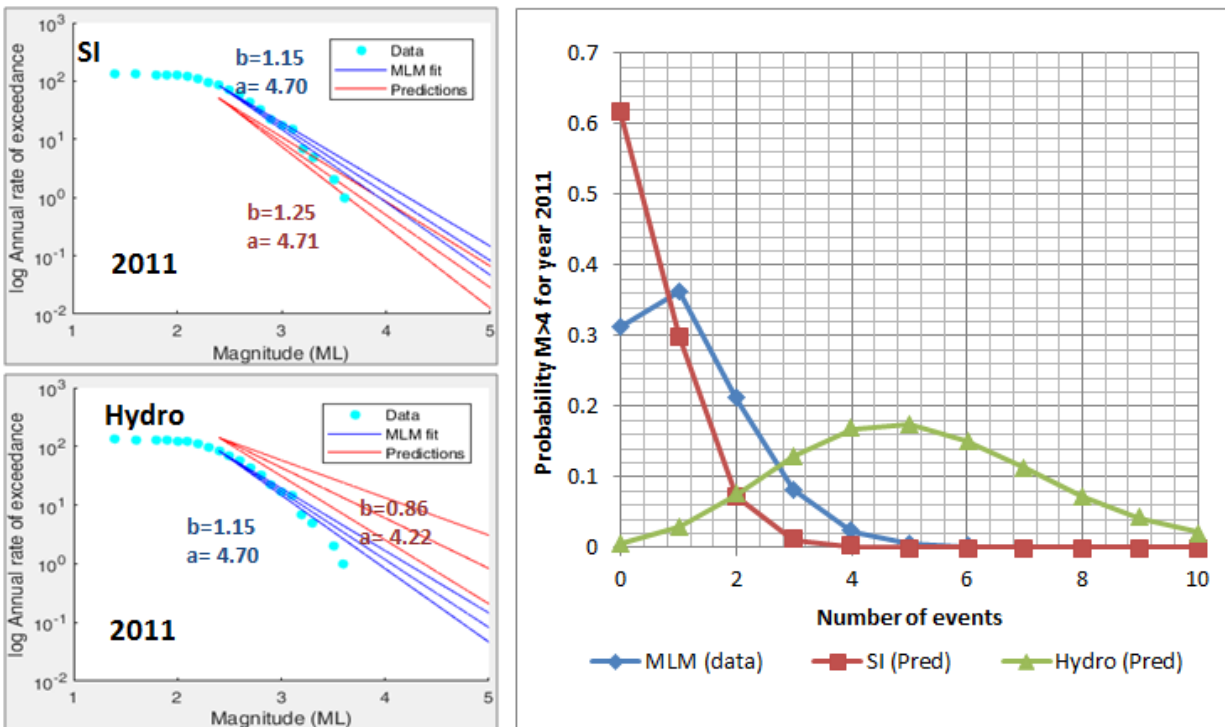


Figure 2. The column on the left shows a comparison between the GR parameters predicted by the Seismogenic Index (SI, top), the Hydromechanical nucleation approach (Hydro, bottom) and the GR parameters of the observed seismicity (MLM fitting), for the year 2011. The figure on the right shows the equivalent probable number of earthquakes in the range $M=[4,5]$ for the year 2011.

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