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An implicitly weighted least squares algorithm for time domain Radon transform

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Summary

High resolution Radon transform is proven to be an effective tool to enhance signal and suppress unwanted seismic events. The algorithm of weighted least squares has been successfully adopted in high resolution Radon transform performed in frequency domain. However, due to the lack of high resolution in time for frequency domain radon transform, time domain radon transform may be found more attractive. In this paper, we formulate time domain weighted least squares radon transform as a simple linear matrix equation inverse problem and the weighting function is implicitly embedded in the coefficient matrix of the equation. Because the none-zero elements in the coefficient matrix are very sparse, the solver of conjugate gradient can be used for computational efficiency.

Introduction

Radon transform can be expressed by the following operator that is an integration of the data along a given travel–time curve $\emptyset(\tau,\Delta,q)$ which depends on the reduced time τ, offset Δ and ray parameter q:

$$m(\tau,q)=\sum_i d(t=\emptyset(\tau,q,\Delta),\Delta_i) \qquad (1)$$

Equation 1 represents a simple mapping from data space to the transform domain. To obtain more focus in Radon domain, the Radon transform is carried out in an inverse formulation

$$d(t,\Delta)=\sum_q m(\tau=\emptyset^{adj}(t,q,\Delta),q) \qquad (2)$$

where, $\emptyset^{adj}$ is the adjourn operator of ∅. The advantage of this formulation is that in inverse problem, the priori information regarding to model variable m can be added to obtain what is known as high resolution Radon transform, e.g. Sacchi, 2009.

Solution is commonly obtained in frequency domain due to generally accepted computational efficiency and simplicity of the formulation. Application of Fourier transform to both sides of equation (2) yields expression for each frequency, so each component can be processed independently (Sacchi, 2009):

$$D(\omega,\Delta_k)=\sum_j M(\omega,q)\,e^{-i\omega\varphi(\Delta_k,q_j)} \qquad (3)$$

Equation (3) can be written as matrix equation:

$$\begin{bmatrix} D(\omega,\Delta_1 \\ D(\omega,\Delta_2 \\ \vdots \\ D(\omega,\Delta_n \end{bmatrix} = \begin{bmatrix} e^{-i\omega\varphi(\Delta_1 q_1)} & e^{-i\omega\varphi(\Delta_1 q_2)} & \cdots & e^{-i\omega\varphi(\Delta_1 q_m)} \\ e^{-i\omega\varphi(\Delta_2 q_1)} & e^{-i\omega\varphi(\Delta_2 q_2)} & \cdots & e^{-i\omega\varphi(\Delta_2 q_m)} \\ \vdots & \vdots & \vdots & \vdots \\ e^{-i\omega\varphi(\Delta_n q_1)} & e^{-i\omega\varphi(\Delta_n q_2)} & e^{-i\omega\varphi(\Delta_1 q_1)} & e^{-i\omega\varphi(\Delta_n q_m)} \end{bmatrix} \begin{bmatrix} M(\omega,q_1) \\ M(\omega,q_2) \\ \vdots \\ M(\omega,q_m) \end{bmatrix}$$

or simply:

$$D(\omega)=A(\omega)M(\omega) \qquad (4)$$

Based on linear inverse theory, the priori information related to $M(\omega)$ can be added and the solution can be obtained via the normal equation (Menke William 1989)

$$(A^TA+\lambda W^TW)M=A^TD \qquad (5)$$

Where the matrix W is the priori information regarding to model M and A^T is the transpose of A. When the matrix W is defined as diagonal matrix with diagonal element proportional to the desired model M, the equation becomes so called high-resolution Radon and solution is obtained by re-weighted least squares (e.g. Trad, 2003).

Frequency domain solution lacks high resolution for variable τ because this variable is not explicitly present in the equation. Dividing the data into a series of time windows may help increase resolution for τ but this may lead to an artifact due to the seismic events truncation (Cary, 1998). Hence, the time domain Radon transform can be preferable. Time domain Radon transform with matching pursuit technique can be found in recent publications, e.g. Wang, 2018. The problem is solved by iteratively collecting the largest model component $m(\tau_i, q_j)$ from the residual of the data. Essentially Radon transform, i.e. equation (1), can be considered as a moveout correction and stacking and ideally a seismic event in Radon domain should be a wavelet due to frequency band limit. Therefore, the best performance of matching pursuit needs wavelet information that may be difficult to obtain. On the other hand, re-weighted least squares method that uses previous model as a weighting function may implicitly handle wavelet properly.

In this paper, we present an algorithm that based on re-formulated direct matrix equation formulation for weighted least squares Radon Transform.

Formulation of Time domain implicitly weighted least squares Radon Transform

Corresponding to equation (1), the matrix equation for time domain can be formulated as

$$m(\tau_i, q_j) = \sum_k \sum_l A(\tau_i, q_j; t_k, \Delta_l)\, d(t_k, \Delta_l)$$

Mathematically, we can unfold the equation into 2D matrix equation. Unfold is based on the rule

$\begin{bmatrix} a_{11} & a_{12} & b_{11} & b_{12} \\ a_{21} & a_{22} & b_{21} & b_{22} \end{bmatrix} \rightarrow \begin{bmatrix} a_{11} & b_{11} \\ a_{21} & b_{21} \\ a_{12} & b_{12} \\ a_{22} & b_{22} \end{bmatrix}$ then, we can come to a standard matrix equation for linear equation system:

$$M_{nt*nq\times 1} = A_{nt*nq\times nt*nx} D_{nt*nx\times 1} \tag{7}$$

If we define M as an inverse problem that comparable to equation (2), then we may have

$$D_{nt*nx\times 1} = A^T_{nt*nq\times nt*nx} M_{nt*nq\ \times 1} \tag{8}$$

where A^T is the transpose of A. The weighted least squares solution to equation (8) has the same form as equation (5).

In equation (5), assuming the weighting matrix is invertible, we can substitute:

$$\widehat{M} = WM \quad and\ \hat{A} = A^T W^{-1}$$

Then the normal equation can be written as:

$$(\hat{A}^T \hat{A} + \lambda I)\widehat{M} = \hat{A}^T D \tag{9}$$

The advantage of equation (9) is that the weighting function is embedded into transformed operator, it is possible to exploit the weighting even without an explicit regularization term, i.e. $\lambda = 0$, then it becomes solving a simple matrix equation problem, i.e.

$$\hat{A}\widehat{M} = D \tag{10}$$

Note, that $\hat{A}$ is a very large matrix with very few non-zero elements and its dimension depends on the method of interpolation for fractional data time samples. We adopt linear interpolation shown in figure 1, the maximum number of none zeros elements is *2*nt*nx* and the compact version of matrix $\hat{A}$ will have dimension of *nt*nq* by *2*nx*. Therefore, equation (10) can be efficiently solved with Conjugate gradient algorithm and *M* can be obtained by $W^{-1}\widehat{M}$. (please note that the straight line shown in figure 1 is just an example for illustration, the line can be of any shape).

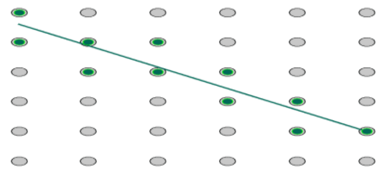

Figure 1. The elements are involved for a line integration

The physical meaning of equation (10) is obvious, i.e. more weighing impact on the components of *M* that have more energy, which is comparable to matching pursuit/greed method where components related to the largest model parameters are selected for inversing data. Essentially when the diagonal elements of weight matrix are sparse, then equation (10) is equivalent to orthogonalized matching pursuit method, e.g. Cai & wang, 2011, wang, 2018. However, instead of adding one component and re-do least squares solving procedure in orthogonal matching pursuit, equation (10) directly apply weighting function to all components.

Example

The example shows resolution comparison of the Radon transform in frequency and time domains with same number of iterations for the solver of conjugate gradient. Figure 2 shows input synthetic data that contain horizontal and parabolic moveout events and the results of frequency domain and time domain transforms. While calculations in both domain can produce high resolution for in q-axis, time domain Radon shows better resolution in the τ axis direction. The next test was to remove second event by applying same mute function to the data in Radon domain (Figure 3). The reconstructed data from muted Radon transform and the removed event are shown in figure 4, which shows the advantage of time domain Radon Transform.

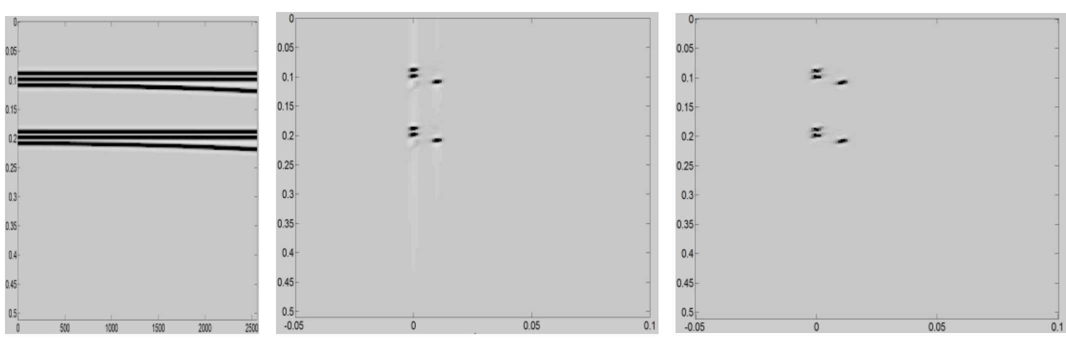

Figure 2: from left to right: input data, Freq. domain Radon transform and time domain Radon Transform

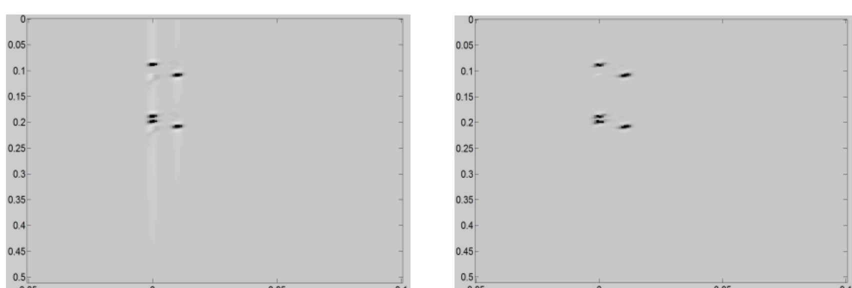

Figure 3. Applied mute to Freq. domain Radon transform (left) and time domain Radon Transform (right)

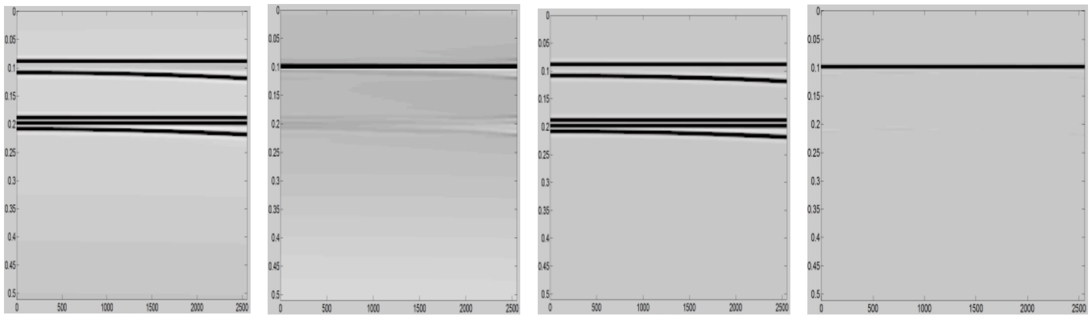

Figure 4. Reconstructed data and multiple: Freq. domain Radon transform (left) and time domain Radon Transform (right)

The second example shows application of the proposed Radon transform to real data multiples attenuation. Figure 5, (a) shows NMO corrected input data contaminated with multiples. The results of primaries and multiples separation are shown in (b) and (c), respectively.

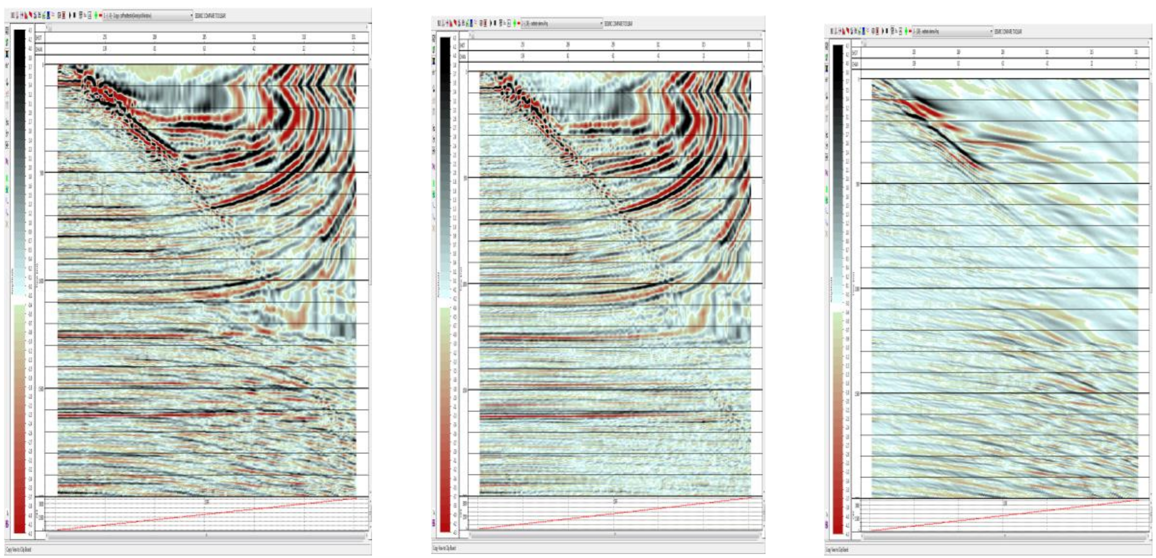

Figure 5: (a) input data; (b) and (c) primary and multiples, respectively.

Conclusions

We presented an implicitly weighted least squares algorithm for time domain Radon transform to achieve better resolution in time dimension over that of in frequency domain. In our formulation, the weighting function directly impact on related components of q in radon domain, which can be an alternative of orthogonal matching pursuit approach. However, instead of adding one component and re-do least squares solving procedure in orthogonal matching pursuit, our algorithm directly apply weighting function to all components and therefore, it is of more computational efficient. The numerical example shows that our algorithm can be applied to remove unwanted event that has the same moveout as others.

Acknowledgements

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