Simultaneous signal and ground roll modeling with L2 and L1 constraints

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Summary

We describe an algorithm to concurrently model seismic reflections (hyperbolic events) and ground roll (dispersive linear noise). We first estimate the coefficients that model hyperbolic and coherent noise. These coefficients are determined by solving an inverse problem. Then, the estimated coefficients synthesize reflections and ground roll. We investigate two algorithms. In the first algorithm, we use least-squares data fitting in the $f-x$ domain and a quadratic regularization term to guarantee the stability of the solution. In the second algorithm, we adopt a l_1 regularization term to impose sparsity on the coefficients that model reflections and ground roll. The proposed algorithm operates in the $f-x$ domain. Therefore, it can capture the dispersive nature of the ground roll by estimating its coefficients as a function of frequency and ray-parameter ($f-p$). Reflections are modeled by hyperbolic basis functions of unknown amplitude that are described by unknown τ, v pairs. Finally, we apply our algorithms to field data.

Introduction

Coherent noise, such as ground roll, are Rayleigh waves recorded by geophones. Rayleigh waves arise when P and SV waves interact with the free surface. They display lower velocity, lower frequency, and higher amplitude than body waves. The attenuation of GR is one of the first steps of seismic data processing in land data. In general, ground roll removal is a difficult task because it is dispersive and often aliased. Different methods have been proposed for ground roll attenuation. For instance, transform-based techniques that exploit the separability of noise and signal in an ancillary domain lead to interesting ground roll suppression methods (Askari and Siahkoohi, 2008; Porsani et al., 2009; Naghizadeh and Sacchi, 2018). In this abstract, we follow the work of Perkins and Zwaan (2000) and Le Meur et al. (2008) who proposed to use least-squares inversion to simultaneously fit $f-x$ domain coefficients that model reflections and ground roll. First, we explore the applicability of the proposed inversion algorithm to cases where we do not have a precise knowledge of the τ, v pairs (intercept-time and RMS velocities) of the reflections. Secondly, we investigate l_2 and l_1 regularization techniques to stabilize the problem. Synthetic examples explore the applicability of the proposed method and provide new insights on the parameterization of the operators adopted to model reflections and dispersive noise.

Theory

We assume a 3D shot gather composed of two signals, hyperbolic events, and coherent noise. We begin by creating a frequency domain model for the signal by considering a non-dipping layered model that consists of hyperbolas as the signal of interest.

A superposition of hyperbolas can be represented in the $f-x$ domain via the following expression

$$d_s(f,h) = \sum_{k=1}^{N_s} m_s(f,k)\, e^{-i2\pi f \sqrt{\tau_k^2 + \frac{h^2}{v_k^2}}}\,, \tag{1}$$

where τ is the zero offset travel time, h is the true shot-receiver distance, v is the root mean square velocity and $m_s(f,k)$ contains the unknown coefficients (amplitude $a(k)$ and wavelet $w_s(\omega)$). We assume ground roll modeled via the superposition of linear events via the following expression

$$d_c(f,h) = \sum_{k=1}^{N_c} m_c(f,k)\, e^{-i2\pi f p_k h} \tag{2}$$

where p is the ray parameter.
Our first algorithm utilizes an l_2 norm to measure the fidelity of the modelled data and observed data. The stability of the solution is guaranteed by adding the l_2 model norm as a penalty constraint (Tikhonov and Arsenin, 1977; Menke, 1989)

$$J = \|\mathbf{d} - \mathbf{A}\mathbf{m}\|_2^2 + \mu\,\|\mathbf{m}\|_2^2 . \tag{3}$$

where the matrix $\mathbf{A}$ contains the dispersive linear and hyperbolic events and the vector $\mathbf{m}$ contains the unknown coefficients of hyperbolas and linear events and they are defined as follows

$$\mathbf{A} = \begin{pmatrix} \mathbf{A_s} & \mathbf{A_c} \end{pmatrix}, \mathbf{m} = \begin{pmatrix} \mathbf{m_s} \\ \mathbf{m_c} \end{pmatrix} \tag{4}$$

In our second algorithm, we propose to minimize the following cost function

$$J = \|\mathbf{d} - \mathbf{A}\mathbf{m}\|_2^2 + \mu\,\|\mathbf{m}\|_1 . \tag{5}$$

The solution of the least-squares fitting problem with an l_1 regularization leads to sparse solutions. This technique has been applied for the computation of the so called Sparse (High-Resolution) Radon Transform (Sacchi and Ulrych, 1995; Wang and Nimsaila, 2014), Fourier reconstruction methods with sparsity constraints (Sacchi and Ulrych, 1996; Zwartjes and Gisolf, 2007) and regional-residual separation of potential field data (Guspi and Introcaso, 2000). Different methods were proposed to minimize J. We will adopt the traditional approach of Iterative Reweighed Least-squares (IRLS) (Burrus et al., 1994; Daubechies et al., 2008; Chartrand and Yin, 2008) but bear in mind that other methods exist to minimize J (Tropp and Gilbert, 2007; Beck and Teboulle, 2009; Van den Berg and Friedlander, 2011).
The solution $\hat{\mathbf{m}}$ from both algorithms can be separated into two parts $\hat{\mathbf{m}} = (\hat{\mathbf{m}}_s, \quad \hat{\mathbf{m}}_c)^T$. The recovered amplitudes are finally used to synthesize signal and coherent noise

$$\hat{\mathbf{d}}_s = \quad \mathbf{A}_s\,\hat{\mathbf{m}}_s \tag{6}$$

$$\hat{\mathbf{d}}_c = \quad \mathbf{A}_c\,\hat{\mathbf{m}}_c . \tag{7}$$

In general, we will subtract the modeled coherent noise $\hat{\mathbf{d}}_c$ from the original data.

Examples

Our algorithm is now applied to two data sets to test their effectiveness. The first example is a 3D synthetic shot gather, in which we modeled our data via a superposition of hyperbolic events and two dispersive modes with phase velocity given by

$$v(f) = v_{min} + \frac{(v_{max} - v_{min})}{\sqrt{1 + (\frac{f}{f_0})^4}} . \tag{8}$$

Data consist of $N_x = 50$ receivers per line and $N_y = 5$ receivers lines, the length of each record in 400 samples and the sampling interval is $\Delta t = 4ms$. We assume a realistic situation in which we do not know the exact intercept time and velocity of the reflections, and we adopt a coarse discretization of τ and v. For instance, the intercept axis τ varies from $0.2s$ to $1.4s$ in 40 intervals sampled every $0.0308s$ which is about seven times the sampling interval of the data Δt. Similarly, the velocity axis varies from $1500m/s$ to $5000m/s$ in 20 intervals. We have discretized the p axis by defining 240 parameters in the interval $p \in [0.001, 0.0033]s/m$. Figures 1a-e illustrate the least-squares solution with damping. Similarly, Figures 2a-e shows the least-squares solution with sparse (l_1) regularization.

This experiment shows that one does not need to have precise knowledge of velocities and intercept times to separate ground roll from reflections properly. The analysis also indicates an improvement in the separability of coherent noise from reflections when the inverse problem is regularized via the l_1 (sparse) penalty term.

The second example is a 3D field data shot gather where we use the semblance map to estimate a coarse grid of τ, v. The length of each record is 2000 samples, the sampling interval is $\Delta t = 2ms$ and the time length of the records is $4s$. We have discretized the p axis by defining 2000 parameters in the interval $p \in [0.001 - 0.025]s/m$. From velocity analysis we have estimated $\tau = [1.84, 1.862.22, 3.33]s$ and $v = [2242, 2420, 3126, 4030]m/s$.

Figures 3a-c illustrate the least-squares solution with damping. Similarly, Figures 4a-c shows the least-squares solution with sparse (l_1) regularization. The application of the two algorithms in field data indicates an improved separation of coherent noise from reflections when the inverse problem is regularized through the sparse (l_1) penalty term rather than through quadratic regularization.

Conclusion

We have explored an algorithm for simultaneously modeling reflections and dispersive ground roll in the $f - x$ domain. Two operators are used to model coefficients into hyperbolas and dispersive noise. The coefficients are estimated via least-squares inversion with quadratic and sparse regularization. We show via examples that one can separate the ground roll from reflections when an approximate model of the τ, v pairs is provided.

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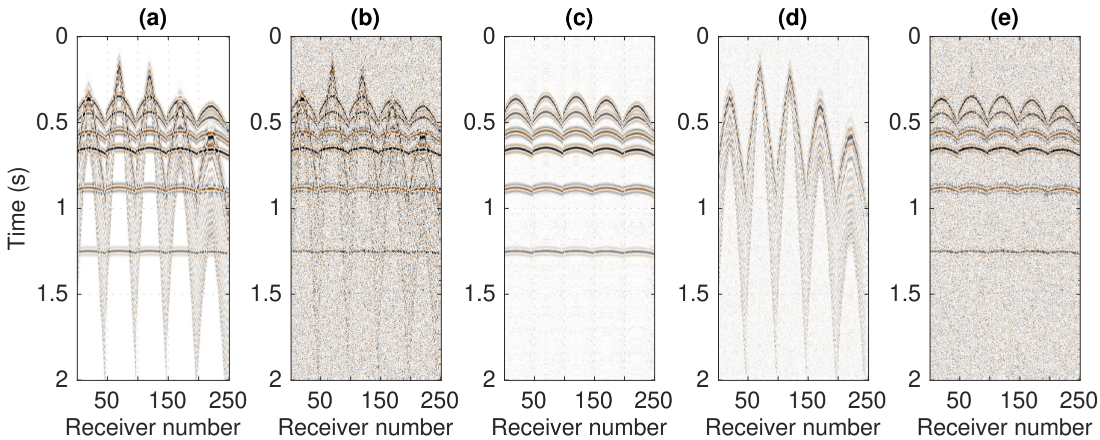

Figure 1: Separation of reflections and ground roll using least-squares inversion with quadratic regularization. The τ, v pairs describing hyperbolic events are unknown. Therefore, we assume a corse discretization of τ, v. a) 3D synthetic shot gather. b) 3D synthetic shot gather contaminated with random noise ($SNR = 1$). c) Estimated signal. d) Estimate coherent noise (ground roll). e) Synthetic observations (b) minus estimated coherent noise (d).

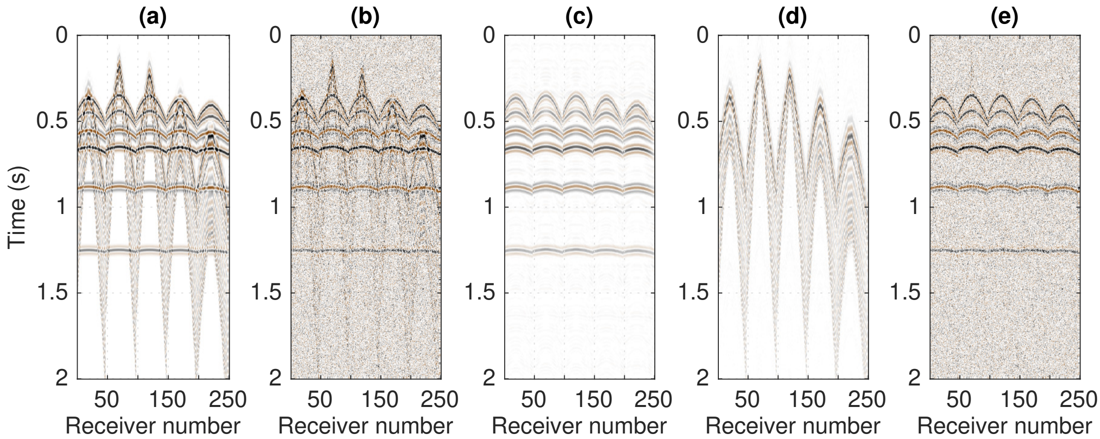

Figure 2: Separation of reflections and ground roll using least-squares inversion with sparse (l_1) inversion. The τ, v pairs describing hyperbolic events are unknown for the inversion process. Coarse discretization of the pairs τ, v is adopted. a) 3D synthetic shot gather. b) 3D synthetic shot gather contaminated with random noise ($SNR = 1$). c) Estimated signal. d) Estimate coherent noise (ground roll). e) Synthetic observations (b) minus estimated coherent noise (d).

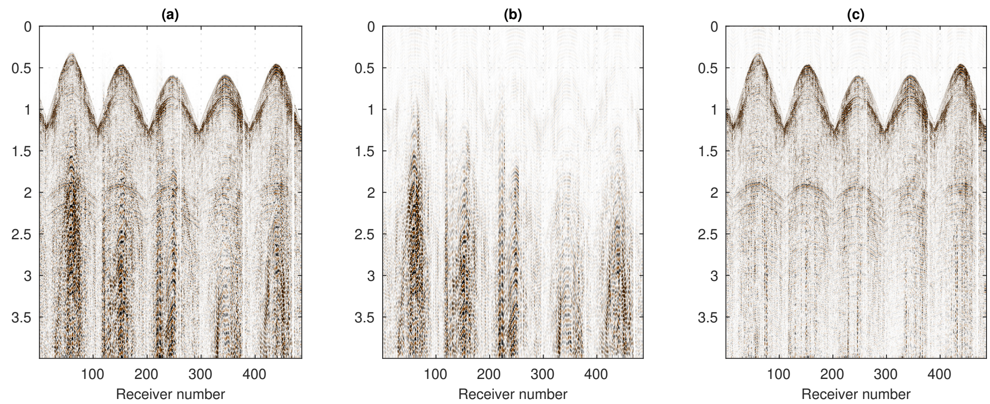

Figure 3: Separation of reflections and ground roll using least-squares inversion with quadratic regularization. a) 3D real shot gather. b) Estimated coherent noise (ground roll). c) Real observations (a) minus estimated coherent noise (b).

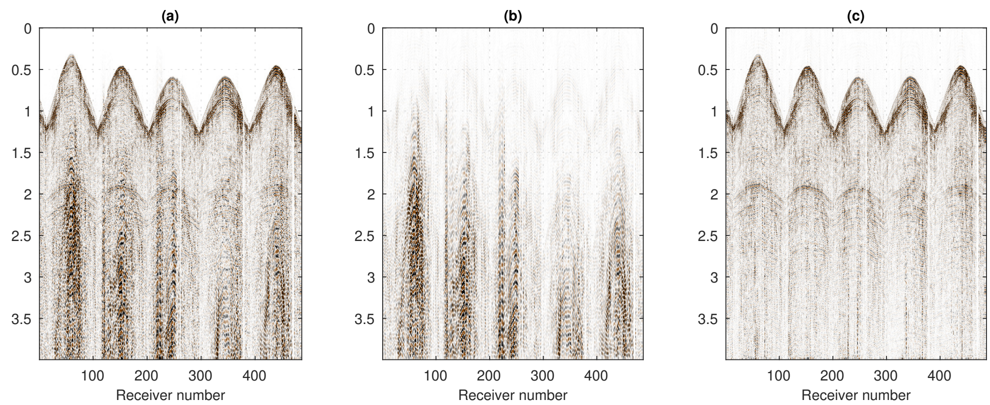

Figure 4: Separation of reflections and ground roll using least-squares inversion with sparse (l_1) regularization. a) 3D real shot gather. b) Estimated coherent noise (ground roll). c) Real observations (a) minus estimated coherent noise (b).

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