

## Internal multiple prediction and the downward generator space

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### Summary

Internal multiples continue to degrade seismic images which can lead to errors in interpretation. The signal level of the data relative to noise must also increase as the information demands from seismic data such as inversion and azimuthal analysis become increasingly standard practice. One method to predict and attenuate internal multiples uses the inverse scattering series (Weglein et al. 1997). The method has minimal assumptions and is referred to as data driven as the only inputs required are the data and a search limiting parameter. Though the method has displayed great potential, amplitude errors can arise in the form of mismatches between the actual and predicted multiples. This can be due to the downward generator, the truncation of the series or algorithm assumptions not being met. A variant of the method referred to as internal multiple elimination has improved on the amplitude issue (Ma and Weglein, 2015; Zou et al., 2016). It has also been shown that using higher order terms from the inverse scattering series can assist with certain amplitude errors and erroneous predictions (Liang et al., 2013). When these errors remain, they are often corrected through adaptive subtraction. If the mismatches between prediction and input data varies rapidly or the multiples are coincident with primaries it can be difficult to build a filter that will attenuate the internal multiples and not harm primaries. To assist with this issue a new domain to carry out the adaptive subtraction is introduced. This domain is a more natural space to carry out the adaptive subtraction as systematic amplitude errors due to the prediction algorithm can be corrected in this space. It is displayed how applying 2D adaptive subtraction in this space improves the accuracy of the prediction.

### Theory

The inverse scattering series is infinite but in practice often only the first term ( $b_3$ ) is used. It has been shown that the original algorithm can be simplified to 1D, this formulation is given in Equation 1 (e.g. Eaid et al., 2016).

$$b_3(\omega) = \int_{-\infty}^{\infty} dz_1 e^{-i2\frac{\omega}{c_0}z_1} b_1(z_1) \left[ \int_{z_1+\varepsilon}^{\infty} dz_2 e^{i2\frac{\omega}{c_0}z_2} b_1(z_2) \right]^2 \quad (1)$$

Where  $b_3$  is the internal multiple prediction,  $b_1$  is the input data,  $z_1$  and  $z_2$  are the pseudo-depths that satisfy lower-higher-lower relationship and  $\varepsilon$  is the search limiting parameter. Equation 1 is predicting internal multiples by combining all events from the input data which obey the lower-higher-lower criteria generated by the higher term from the criteria or downward generating horizon  $z_1$ . By storing every downward generator solution and solving for  $b_3(z_1, \omega)$  this creates a 2D matrix of the internal multiple

predictions referred to here as the downward generator space. This result is then inverse Fourier transformed over the frequency dimension to give the prediction in time (Iverson and Innanen, 2017).

An adaptive subtraction algorithm has been developed which has been previously tested on internal multiple attenuation (Keating et al. 2015). This displayed results for cases where either the L1, L2 or hybrid L1/L2 norms were minimized, and It was shown the hybrid norm allowed for the subtraction of unwanted multiples from a trace (Keating et al. 2015). Here the adaptive subtraction is performed by applying a filter designed in the downward generator - time space to the predictions and subtracting the result from the data in time. having too many filter coefficients will result in overfitting of the data, potentially removing multiples (e.g. Guitton and Verschuur, 2004). To prevent this from happening, we minimize

$$\phi(f) = ||d - Mf||_p^p + \phi_R, \tag{2}$$

where d is the time series for the data, M is a matrix representing convolution with the predicted multiples, f is the filter and p is a chosen parameter,  $\phi_R$  is a term penalizing variation of the filter in downward generator space. If the penalty term is chosen to be large, the large number of filter coefficients will change only gradually in the downward generator space direction, preventing overfitting.

### Workflow

To display the benefit of 2D adaptive subtraction in the downward generator space a three-layer model plus half space is used. This contains four first order and several second order internal multiples (Figure 1).

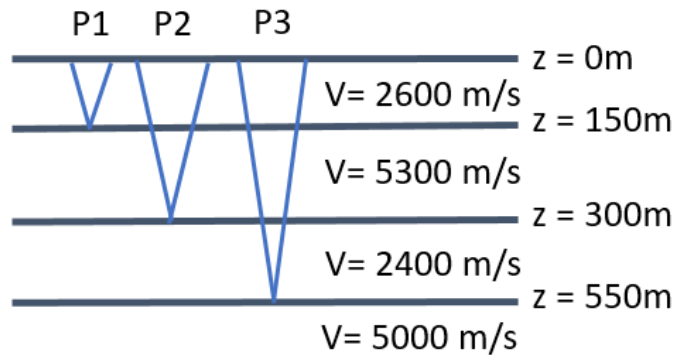


Figure 1. Velocity and depth model used for the 1D prediction.

The prediction using the internal multiple attenuation algorithm is displayed below with no adaptive subtraction (Figure 2). It has predicted the kinematics of the event but with amplitude mismatches compared to the input trace.

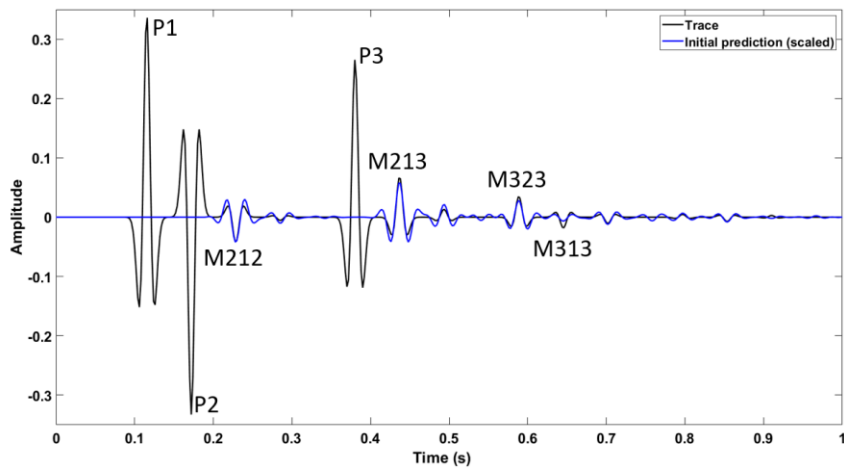


Figure 2. Input trace and 1D internal multiple prediction.

The downward generator space for this model is displayed (Figure 3). The 2D wavelet like events in the space represent an internal multiple. The standard 1D prediction can be calculated from this space by stacking over all the rows.

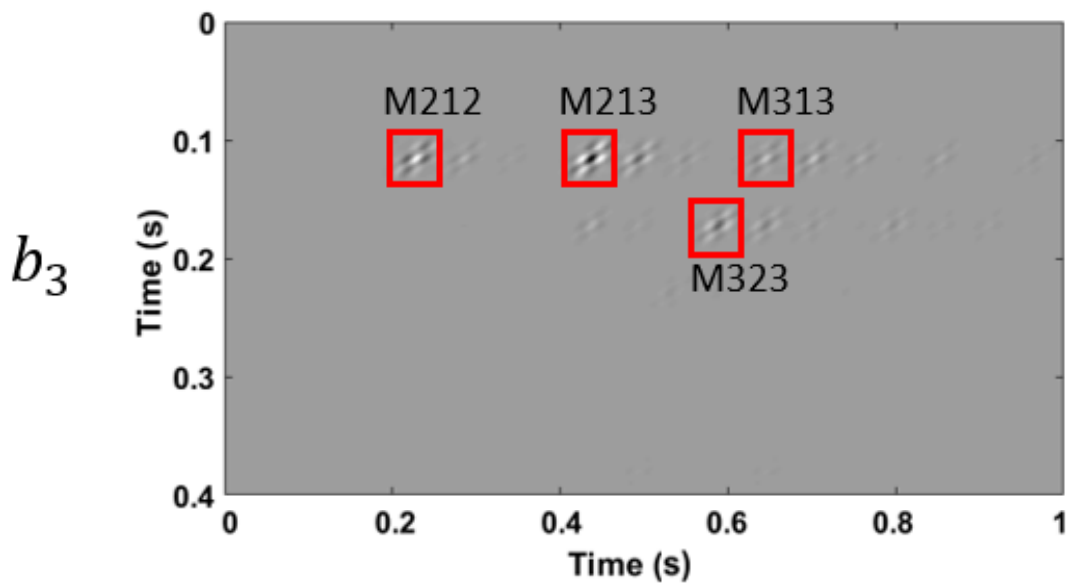


Figure 3. 2D downward generator space displaying internal multiples with first order multiples outlined in red. Prediction time is on the x axis and generator time on the y axis.

The results using both the standard 1D and new 2D adaptive subtraction are displayed (Figure 4). The 2D space is partially stacked along the row direction in practice. This is done to both reduce the computational expense of the subtraction by decreasing the dimensionality of the filter design problem, and at a minimum stack over the width of the wavelet to reduce the potential for overfitting.

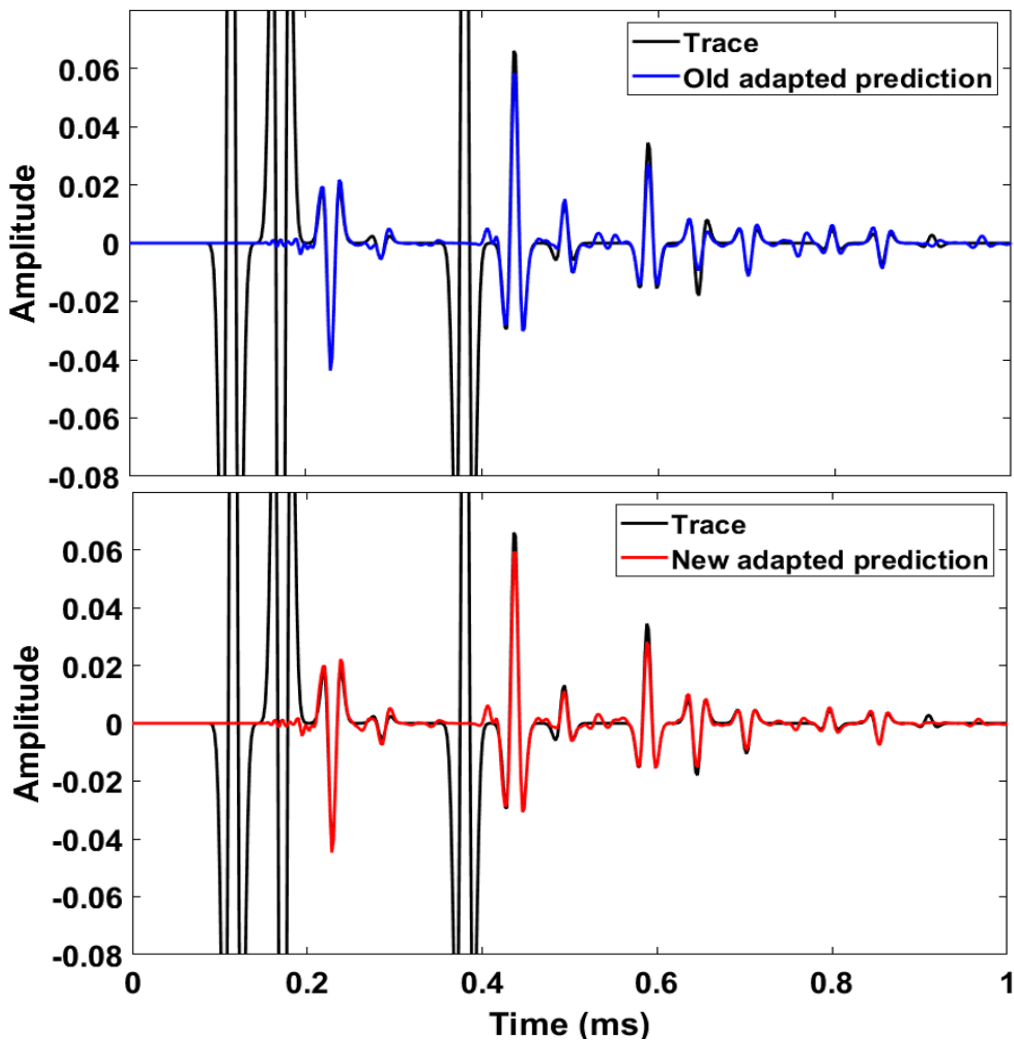


Figure 4. (Top) Internal multiple prediction with 1D adaptive subtraction (Bottom) Internal multiple prediction with 2D adaptive subtraction

## Conclusions

This demonstrates that utilizing the 2D downward generator space to carry out the adaptive subtraction can improve the prediction by attempting to account for the losses due to the downward generator. The method also remains data driven as there are still no subsurface information requirements. This space is also useful as a tool to identify individual internal multiples which can be located in this space.

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