geoconvention
Calgary • Canada • May 13-17 2019

Modeling Seismic Waves in Layered Anelastic Media – beyond the Viscoelastic Approach

Yichuan Wang and Igor B. Morozov
University of Saskatchewan
yichuan.wang@usask.ca, igor.morozov@usask.ca

Summary

Modeling of seismic wavefields in anelastic media is used in a vast number of applications and implemented by numerous methods and software algorithms. Historically, all of these methods are based on the viscoelastic model, which explains wave attenuation by the *Q*-factor, time-dependent "material memory", and frequency-dependent material properties. However, none of such material properties actually exist in rocks or Earth media. It is important to realize that the *Q* commonly used to parameterize attenuation models is an "apparent" property, i.e. an ambiguous quantity inserted empirically in order to simulate the observed attenuation effects by using (generally) inaccurate equations. The differences of true attenuation effects from the predictions of *Q*-based models are the strongest for gas- and fluid saturated porous rocks, which are of the most importance for seismic interpretation of oil and gas reservoirs.

It is therefore important to be able to perform seismic modeling by using not an empirical "*Q*" but true physical and geological properties of the subsurface. In contrast to the single *Q*, there exist many physical properties responsible for wave attenuation. Such properties include elasticity and viscosity matrices, pore-fluid content, fluid mobility, and internal structure of the material. In this paper, we show how such properties can be used for accurate modeling of wave propagation in layered anelastic media. Attenuation effects such as the apparent *Q*, velocity dispersion, and tuning and reflection AVO/QVO effects are modeled in arbitrary layered structures consisting of poroelastic, squirt-flow, and the so-called Standard Linear Solid (Zener) rheologies. Interestingly, the modeling shows that combination of many layers traversed by the seismic wave may naturally explain the near-constant *Q* or broad attenuation peaks often observed in field and laboratory seismic studies.

Introduction

Inverse viscoelastic (VE) *Q* factors are broadly used in seismic interpretation and are believed to be related to reservoir parameters such as porosity, saturation, and fluid viscosity (e.g., Chen et al., 2018). However, although firmly established in geophysical curricula, the VE model still contains significant uncertainties and unacceptable physical inaccuracies. These shortcomings are caused by relying on the phenomenological "material memory" instead of the actual physical properties, and by not recognizing the effects of boundaries and contrasts in material properties on anelastic attenuation. These problems do not manifest themselves in cases where the spatial variations in material properties are not considered, such as for waves in uniform media or in laboratory observations with rock samples. Nevertheless, for waves in heterogeneous media including most practical cases in exploration seismology, the physical inaccuracy of the VE model is substantial and critical for interpretation (Morozov and Baharvand Ahmadi, 2015).

In section "Method" below, we briefly describe an alternate to VE, rigorous, continuum-mechanics based approach to modeling seismic wavefields without assuming a *Q* factor but based on the "General Linear Solid" formulation by Morozov and Deng (2016a, 2016b). We apply this approach to numerical modeling of reflected and transmitted seismic-wave responses in 1-D layered media. As a specific example, we examine a single reflective layer and further approximate the elastic and anelastic structure of the Weyburn oil reservoir in southern Saskatchewan. The results are interpreted in terms of several types of *Q* measured from the modeled wavefields.

Because the mechanical rheologies for rock are poorly known (only several estimates were recently made by Deng and Morozov, 2016, Morozov et al., 2018, and Deng and Morozov, 2018a, 2018b), we assume that each layer represents a mixture of the poroelastic (Biot, 1956) and the Standard Linear Solid (SLS) rheologies. Such parameterization of the layers as SLS is consistent with many classic and recent studies modeling and estimating *Q* from seismic data (e.g., Liu et al., 1976; Chen et al., 2018; Qadrouh et al., 2018).

Another major, but unfortunately also unnoticed omission of conventional VE modeling consists in disregarding the boundary conditions for internal variables within an anelastic medium (Morozov and Deng, 2018). Different types of internal boundary conditions cause significant variations in the modeled *Q* values for waves, even though the materials of the layers remain the same. Two types of such boundary conditions are illustrated in section "Results".

Method

In the standard VE theory, propagation of seismic waves in an anelastic medium or deformation of a rock sample in a laboratory experiment is governed by integro-differential ("material memory") equations in time combined with differential-only equations in space (for example, see the book Carcione (2007) and many papers by the same author). Nevertheless, the material memory is still a hypothetical (empirical, effective, or "apparent") material property. In rigorous physics, the dynamics of an anelastic material should be described by only "instantaneous" partial-differential equations. These equations can be obtained from two basic observations:

1) Anelasticity of the medium results not from "memory" but from parts of its internal structure (such as pores, fluids, or grains) moving relative to each other. This means that some *internal variables describing these movements must always be present in an anelastic medium*.
2) All equations of motion must follow from *rigorous continuum mechanics* and utilize only *macroscopic, time- and frequency-independent material properties* (Landau and Lifshitz, 1986).

For linear elastic and frictional interactions within the medium, these principles give the most general governing equations for the field $\vec{\mathbf{u}}\left(\vec{x},t\right)$ (Morozov and Deng, 2016a):

$$\begin{cases} \boldsymbol{\rho}\ddot{\mathbf{u}}_i = -\mathbf{d}\dot{\mathbf{u}}_i + \partial_j \boldsymbol{\sigma}_{ij}, \\ \boldsymbol{\sigma}_{ij} = \mathbf{K}\boldsymbol{\Delta}\delta_{ij} + 2\boldsymbol{\mu}\tilde{\boldsymbol{\varepsilon}}_{ij} + \boldsymbol{\eta}_K \dot{\boldsymbol{\Delta}}\delta_{ij} + 2\boldsymbol{\eta}_\lambda \dot{\tilde{\boldsymbol{\varepsilon}}}_{ij}, \end{cases} \tag{1}$$

where indices $i, j = 1, 2,$ or 3 denote the spatial dimensions, $\boldsymbol{\varepsilon}_{ij} \equiv \left(\partial_i \mathbf{u}_j + \partial_j \mathbf{u}_i\right)/2$ is the strain tensor, $\boldsymbol{\Delta} \equiv \boldsymbol{\varepsilon}_{kk} \equiv \mathrm{tr}\,\boldsymbol{\varepsilon}$ is the volumetric strain, $\tilde{\boldsymbol{\varepsilon}}_{ij} \equiv \boldsymbol{\varepsilon}_{ij} - \boldsymbol{\Delta}\delta_{ij}/3$ is the deviatoric strain, and summations over pairs of repeated spatial indices are implied. Tensor $\boldsymbol{\sigma}_{ij}$ (matrix in model space) is the total stress. Material properties are given by the density matrix $\boldsymbol{\rho}$, the bulk ($\mathbf{K}$) and shear ($\boldsymbol{\mu}$) elastic moduli matrices, the corresponding viscosity matrices $\boldsymbol{\eta}_K$ and $\boldsymbol{\eta}_\mu$, and the drag (inverse mobility) matrix $\mathbf{d}$. Each of these matrices is square, symmetric, and positive definite (Morozov and Deng, 2016a).

The model in eqs. (1) represents the most general case of linear anelastic interactions and is therefore suitable for practically all types of rocks and fluids. In poroelasticity-type models with single or multiple porosities, the principal source of internal friction is the Darcy drag force $-\mathbf{d}\dot{\mathbf{u}}_i$ in the first eq. (1). This body force is also the principal mechanism of interaction between the rock matrix and partial melt in rock compaction models (McKenzie, 1984). By contrast, all conventional VE models are obtained by setting $\mathbf{d} = \mathbf{0}$, and therefore the divergence of stress $\partial_j \boldsymbol{\sigma}_{ij}$ is the only type of frictional force considered in them. This is a significant limitation when using the conventional VE-type seismic modeling (Morozov and Deng, 2016b).

A fundamental difference of rigorous equations of mechanics from their VE counterparts (Blanch et al., 1995; Carcione, 2007) consists in eqs. (1) generally predicting N P-wave and N S-wave modes, whereas the VE model uses only one P and one S wave (we do not consider the anisotropy or inhomogeneous waves here). The multiple wave modes are analogous, for example, to the primary (ordinary, or "fast") and secondary ("slow") P waves well known in Biot's (1956) poroelasticity. Thus, in an anelastic medium, we should generally expect to see one primary and $N-1$ secondary wave modes. The secondary modes are usually diffusive or "evanescent" and therefore not easily observable, but their effects should nevertheless be significant in media with ~10-cm to 1-m scale layering (Morozov and Deng, 2018).

In addition to the differential equations (1), boundary conditions at the source and on layer boundaries are required in order to determine the resulting wavefield. These boundary conditions should also be applied to internal variables u_J (with $J > 1$), similar to the conditions for open and closed pore flows in poroelasticity (e.g., Dunn, 1987). Similarly to poroelasticity, internal boundary conditions affect the amplitudes of secondary wave modes and the overall attenuation and wave dispersion (Morozov and Deng, 2018).

Model

To illustrate the wavefields predicted by eqs. (1), we generated a 1-D column of physical properties based on the available velocity, density, and porosity well logs from Weyburn oilfield. A fragment of this column is schematically shown in Figure 1. The reservoir interval is modeled as poroelastic rock with squirt flows (Deng and Morozov, 2016) or as SLS rheologies, and the layers outside of this interval are modeled as elastic. Numerical wavefield modeling is performed as described by Morozov and Deng (2016a, 2016b) for oblique incidence of harmonic waves with several values of ray parameter p. By using the inverse Fourier transform, time-domain records are obtained (not shown for brevity). Finally, by recording the transmitted and reflected amplitudes above and below the reservoir zone, AVO responses and the

corresponding apparent Q values are obtained. Figure 2 shows an example of reflection amplitudes from a simplified case with a single low-velocity layer simulating gas sandstone within wet sandstone (parameters from Dutta and Odé, 1979). Note that both the reflection amplitudes and AVO patterns strongly depend on the internal-variable conditions on the boundaries of the layer, which are not considered in standard modeling approaches.

Modeling of the apparent $Q(f)$ dependencies shows that if each layer of a medium represents an SLS, the stack of layers behaves as a "generalized SLS" (GSLS) producing a band of slowly-varying attenuation $Q^{-1}(f)$ that can fit many of the existing attenuation datasets (Deng and Morozov, 2018a). Note that the GSLS is routinely used in seismology for explaining observations of near-constant Q attenuation (Liu et al., 1976) and for implementing the Q in finite-difference seismic modeling software (e.g., Blanch et al., 1995). Thus, our results suggest that the observed appearances of broad-band or near-constant $Q(f)$ for seismic waves may be due to the averaging of attenuation effects in layered media. At the same time, the rocks within the layers do not have to possess a near-constant $Q(f)$ or even any definite values of Q at all.

Conclusions

The conventional modeling of seismic wavefields based on empirical Q and used in most current approaches is greatly over-simplified and may be insufficiently physically accurate, particularly for fluid-saturated reservoir rock. We propose an alternate, continuum-mechanics based approach and illustrate it on a realistic 1-D models with fine layering.

Acknowledgements

We thank Wubing Deng for estimating material properties and numerous contributions to the methods of this work.

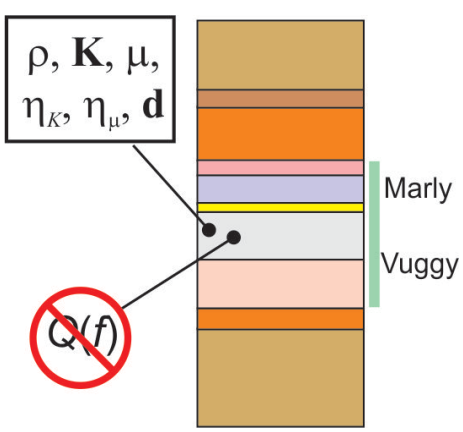

Figure 1. Schematic model of the layering showing the physical properties associated with each layer. Reservoir zone is indicated by green bar. Fine layering is also included in each lithological interval. There is no $Q(f)$ dependence assumed for any rock in the model.

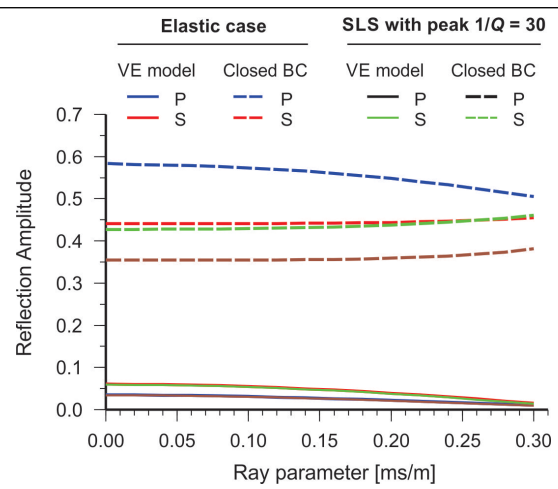

Figure 2. Reflection AVO responses for a 0.5-m thick gas-sandstone layer modeled at frequency 29 Hz assuming elastic, viscoelastic (VE) and exact SLS rheologies with closed internal boundary conditions (BC; legend).

References

Biot, M.A., 1956. Theory of propagation of elastic waves in a fluid-saturated porous solid, II. Higher-frequency range: Journal of the Acoustical Society of America., 28, 168–178.

Blanch, J.O., J.O.A. Robertsson. and W. W. Symes, 1995. Modeling of a constant *Q*: methodology and algorithm for an efficient and optimally inexpensive viscoelastic technique. Geophysics, 60, 176–184.

Carcione, J.M., 2007, Wave fields in real media: Wave propagation in anisotropic anelastic, porous, and electromagnetic media. Second Edition: Elsevier

Chen, H., K. A. Innanen, and T. Chen, 2018, Estimating P- and S-wave inverse quality factors from observed seismic data using an attenuative elastic impedance: Geophysics, 83, No. 2, P. R173–R187, doi: 10.1190/geo2017-0183.1.

Deng, W., and I. B. Morozov, 2016. Solid viscosity of fluid-saturated porous rock with squirt flows at seismic frequencies, Geophysics, 81, No. 4, D381–D390.

Deng, W. and I. B. Morozov. 2018a, Causality relations and mechanical interpretation of band-limited seismic attenuation, Geophysical Journal International., doi: 10.1093/gji/ggy354

Deng, W. and I. B. Morozov, 2018b, Mechanical interpretation and generalization of the Cole-Cole model in viscoelasticity, Geophysics, 83, No. 6, MR345–MR352, doi: 10.1190/GEO2017-0821.1

Dunn, K.-J., 1987. Sample boundary effect in acoustic attenuation of fluid-saturated porous cylinders, Journal of the Acoustical Society of America, 81, 1259–1266.

Dutta, N. & H. Odé, 1979. Attenuation and dispersion of compressional waves in fluid-filled porous rocks with partial gas saturation (White model)-Part II: Results, Geophysics, 44, 1789–1805.

Landau L. D., and E. M. Lifshitz, 1986, Course of theoretical physics, volume 7 (3rd English edition): Theory of elasticity, Butterworth-Heinemann, ISBN 978-0-7506-2633-0.

McKenzie, D., 1984, The generation and compaction of partially molten rock: Journal of Petrology, 25, 713-765.

Morozov, I.B. and A. Baharvand Ahmadi, 2015. Taxonomy of *Q*, Geophysics 80, No. 1, T41–T49.

Morozov, I. B. and W. Deng, 2016a. Macroscopic framework for viscoelasticity, poroelasticity, and wave-induced fluid flows—Part 1: General linear solid, Geophysics, 81, No. 1, L1–L13.

Morozov, I. B. and W. Deng, 2016b. Macroscopic framework for viscoelasticity, poroelasticity, and wave-induced fluid flows—Part 2: Effective media, Geophysics, 81, No. 4, D405–D417.

Morozov, I. B, and W. Deng, 2018. Internal boundary conditions in heterogeneous anelastic media: Geophysical Journal International, ggy387, https://doi.org/10.1093/gji/ggy387

Morozov, I. B., W. Deng, and L. Lines, 2018. Ultrasonic wave propagation and reflections in “heavy oil” (Crisco): Canadian Journal of Exploration Geophysics, 43, 1-12.

Qadrouh, A.N., J. M. Carcione, J. Ba, D. Gei, and A. M. Salim, 2018. Backus and Wyllie averages for seismic attenuation, Pure and Applied Geophysics, 175, 165–170.