Edge-Preserving FWI via Regularization by Denoising

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Summary

The primary objective of Full Waveform Inversion (FWI) is to estimate high-resolution physical properties of geological structures by minimizing the data misfit between observed and modeled seismogram. In spite of its success, the application of FWI in areas with high-velocity contrast remains a challenging problem. Often, quadratic regularization methods are usually adopted to stabilize inverse problems. Unfortunately, edges and sharp discontinuities are not adequately preserved by quadratic regularization techniques. For models with high-velocity contrast where edges of the model were to be adequately reconstructed, an L_1 norm regularization is needed. Here, we propose a regularization by denoising technique to extend the Full Waveform Inversion formulation for models with high-velocity contrasts. One advantage of the regularization by denoising algorithm in FWI is that it's easy to implement. The regularization by denoising technique only requires an image denoising engine to handle the structure of the inverse problems. Our image denoising engine is the primal-dual Total Variation regularization method. We present the BP/EAGE velocity model, where large velocity contrasts and complex salt bodies are present, to highlight the efficiency regularization by denoising technique.

Introduction

Full Waveform Inversion (FWI) aims to estimate high-resolution subsurface physical properties of geological structures by minimizing the data misfit between observed and modeled seismograms through local iterative optimization techniques (Lailly, 1983; Tarantola, 1987; Stekl and Pratt, 1998). FWI is an ill-posed inverse problem (Virieux and Operto, 2009) and, therefore, regularization and preconditioning strategies are often required to obtain stable estimates of velocity models. Regularization methods also serve to impose desired features on subsurface images (Zhdanov, 2002). Quadratic regularization methods are often adopted to deliver smooth models (Constable et al., 1987). In an attempt to preserve discontinuities in images, non-quadratic regularization techniques have become popular in the field of image processing (Künsch, 1994; Geman and Reynolds, 1992; Charbonnier et al., 1997) and FWI (Akcelik et al., 2002; Anagaw and Sacchi, 2011; Guitton, 2012; Smithyman et al., 2015; Esser et al., 2016; Brandsberg-Dahl et al., 2017; Peters and Herrmann, 2017).

The most common regularization method for model buildings in FWI that simultaneously suppressing noises and preserve edges is the Total Variation (TV) regularization (Anagaw and Sacchi, 2012, 2011). Edge-preserving techniques for inverse problems require the resolution of a non-quadratic regularization problem that often contains more than one trade-off parameter (Anagaw and Sacchi, 2011; Farquharson and Oldenburg, 1998). However, our experience indicates that estimating optimal trade-off parameters controlling goodness-of-fit and edge-enhancement is not an easy task when one tries to solve nonlinear inverse problems like FWI. It requires a cumbersome work in finding optimal parameters that control the nonlinearity properties of the model to impose and retrieve desired features on subsurface images. Regularization by denoising technique (RED) proposed by Romano et al. (2016) is an alternative approach to edge preservation via regularization involves a model update using a denoising engine in defining the regularization of the inverse problem. The proposed regularization by denoising method is a more powerful and flexible technique for achieving the same objective as does any image denoising method based on adaptive regularizations.

In this paper, we extend the regularization by denoising scheme originally proposed by Romano et al. (2016) for FWI and propose to use as an edge-preserving FWI via regularization by denoising to preserve edges and discontinuities of velocity models recovered via FWI. One advantage of incorporating the regularization by denoising algorithm in FWI algorithm is that it's easy to implement. The regularization by denoising technique only requires an image denoising engine to handle the structure of the inverse problems. Our image denoising engine is the primal-dual total variation regularization method. The primal-dual total variation algorithm aims to remove some of the singularity caused by the non-differentiability of the L_1 TV norm and is performed by applying a linearization technique.

A 2D acoustic simultaneous-source frequency-domain FWI algorithm (Stekl and Pratt, 1998; Anagaw and Sacchi, 2014) is employed to generate model updates that undergo an edge-preserving FWI via regularization by denoising. We present the BP/EAGE velocity model (Billette and Brandsberg-Dahl, 2005), where large velocity contrasts and complex salt bodies are present, to highlight the efficiency regularization by denoising technique.

Theory

Waveform inversion is a highly nonlinear inverse problem and often uses the least-squares misfit defined as the l_2 norm of the residual between the observed data $\mathbf{d}^{obs}$ and synthetic data $\mathbf{d}^{cal}(\mathbf{m})$:

$$J(\mathbf{m}) = \frac{1}{2} \parallel \mathbf{d}^{obs} - \mathbf{d}^{cal}(\mathbf{m}) \parallel^{\mathbf{2}} . \tag{1}$$

The minimization of the misfit function is a nonlinear problem where from measured wavefields one attempts to estimate the P-wave velocity model $\mathbf{m}$. Our work pertains to acoustic FWI with frequency-domain solvers and source encoding (Stekl and Pratt, 1998; Anagaw and Sacchi, 2014). FWI is an ill-posed inverse problems often requires regularization techniques to stabilize the solution. Regularized full-waveform inversion can be formulated as the following minimization problem:

$$J(\mathbf{m}) = ||\mathbf{d}^{obs} - \mathbf{d}^{cal}(\mathbf{m})||^2 + \lambda R(\mathbf{m}), \tag{2}$$

where $R(\mathbf{m})$ is the regularization term and λ is the weighting regularization parameter that determines the relative balance of the two terms. The minimization of the misfit functions seeks to find the model parameter that best explains the data and the regularization term simultaneously. If a model is blocky and attempts to preserve discontinuities in images, non-quadratic regularization techniques often incorporated regularization schemes.

In this paper, instead of the conventional additive regularization methods, we extend the technique Regularization by Denoising (RED) as an imaging denoising engine to handle the desired structure of inverse problem (Romano et al., 2016). The technique RED is very flexible and is capable of incorporating any image denoising algorithm, only requires a denoising engine. Romano et al. (2016) proposed the following regularization term for an image denoising problem

$$R(\mathbf{m}) = \frac{1}{2}\mathbf{m}^{\mathbf{T}}\left[\mathbf{m} - f(\mathbf{m})\right], \tag{3}$$

where $f(.)$ is any arbitrary denoising engine of choice and $\mathbf{m}$ is model parameter. The idea of RED is to provide an alternative way of achieving the same goal as any image denoising regularization algorithm but with a more flexible and powerful way. To use RED, the denoising engine $f(.)$ need to meet the following two necessary conditions and properties:

• For small positive scaling parameter c, $f(\mathbf{m})$ is local homogeneous

$$f(c\mathbf{m}) = cf(\mathbf{m}) \tag{4}$$

• The Jacobian $\nabla_{\mathbf{m}} f(\mathbf{m})$ of a denoising algorithm is stable and satisfies the condition

$$||f(\mathbf{m})|| \leq ||\mathbf{m}|| \tag{5}$$

With the above conditions, the gradient of the regularization term leads to the following simple equation

$$\nabla_{\mathbf{m}} R(\mathbf{m}) = \mathbf{m} - f(\mathbf{m}) \tag{6}$$

Hence, the gradient of the misift function becomes

$$\nabla_{\mathbf{m}} J(\mathbf{m}) = -\frac{\partial \mathbf{d}^{cal}(\mathbf{m})}{\partial \mathbf{m}}(\mathbf{d}^{obs} - \mathbf{d}^{cal}(\mathbf{m})) + \lambda(\mathbf{m} - f(\mathbf{m})). \tag{7}$$

The primary objective of this work is to estimate high-resolution velocity model in FWI in areas with high-velocity contrast. It aims to preserve edges and sharp discontinuities present in the velocity model while simultaneously perform smoothing and denoising to uniform areas. The most popular regularization technique for edge-preserving and denoising technique is the Total Variation (Osher and Fatemi, 1992). Total Variation is often employed as a regularization for an inverse problem where one attempts to preserve discontinuities models. However, to use the L_1 TV norm as a denoising engine in the RED does not satisfy necessary conditions: 4 and 5. The Primal-Dual Total Variation regularization method is used as an image denoising engine by linearizing the problem to resolve sharp interfaces obtaining solutions where edges and discontinuities are preserved. The edge-preserving FWI algorithm is performed using the simultaneous-source encoding technique. The proposed method of FWI via regularization by denoising is to remove the artifacts created by the simultaneous-source encoding technique in the retrieved model while at the same time preserve edges and sharp discontinuities. We used preconditioning conjugate gradient optimization to estimate the model perturbations (Nocedal and Wright, 2006). The preconditioner is the diagonal of the pseudo-Hessian matrix.

Examples

For our numerical example, we consider a portion of the original BP/EAGE velocity model (Billette and Brandsberg-Dahl, 2005), where large velocity contrasts and complex salt bodies are present, as shown in Figure 1. The BP model is a good representative candidate model to test edge-preserving denoising techniques. Figures 1a & b are the true and smooth velocity models used for FWI, respectively. The actual velocity model contains salt domes of high-velocity contrast, subsalt low-velocity zones under the salt, and several anomalies that offer an ideal velocity model to test the algorithm. Synthetic data were generated via a finite-difference method (Stekl and Pratt, 1998). A total number of 100 sources and 298 receivers were computed. The sources are placed 50 m below the surface, while the receivers on the surface of the earth. To compute the data, we used a source signature modeled by a Ricker wavelet with a 10 Hz central frequency. The source encoding scheme contains four super-shots, and each super-shot includes 25 individual sources.

The edge-preserving algorithm via regularization by denoising technique is to filter and suppresses incoherent cross-talk noise introduced by the source encoding while at the same time to preserve edges and discontinuities present in the model. For inversion, a set of thirteen discrete frequencies between 2. Hz and 12. Hz were selected for the input data to our multiscale FWI algorithm and frequencies were grouped. The multiscale frequency grouping consists of five groups with an overlap of

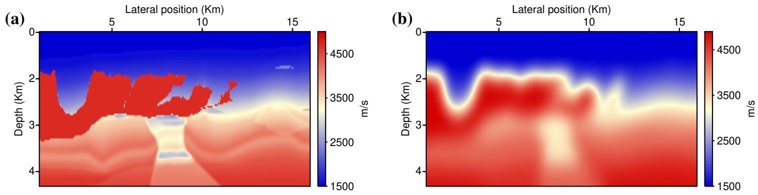

Figure 1: Portion of the original BP/EAGE velocity model. True velocity model (a) and smooth velocity model (b). The smooth velocity model was adopted as the starting model for the inversion.

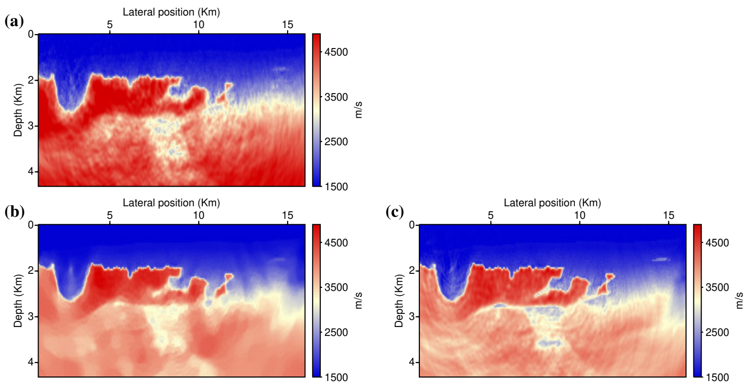

Figure 2: (a) Reconstructed velocity model with classical FWI without employing the RED scheme, via RED $\lambda = 1.5$ (b) and $\lambda = 0.08$ (c).

4 frequencies (Anagaw and Sacchi, 2014). The numerical inversion is then carried out in a sequential approach starting from low to high group frequencies. The final updated model of the lower frequency is used as a starting model for the inversion of the next higher group frequencies data components. To validate the performance and premise of regularization by denoising technique in the framework of edge-preserving FWI, we run the inversion with and without applying the regularization via denoising for a maximum of 70 iterations. For each iteration in the iterative FWI process, the regularization by denoising scheme only needs an image denoising engine to be performed. The image denoising engine deployed for image restoration and denoising in our numerical example is the primal-dual total variation. During the first FWI iteration, we run the conventional FWI model update without the use of image denoising technique. In the preceding iterations, we perform FWI with an image denoising scheme. For primal-dual algorithm, we run the iterative denoising algorithm for up to a maximum of 1000 iteration or when the final TV norm constraint $||\mathbf{m}_k||_{TV} \leq \varepsilon ||\mathbf{m}_{k-1}||_{TV}$ is satisfied. Especially, the stopping criteria has an important influence on the quality of the reconstructed model parameters and on the convergence rate of the algorithm. For this example, we find $0.45 \leq \varepsilon \leq 0.75$ is adequate enough. However, this needs further investigation. The computational cost of computing $f(\mathbf{m}_{k-1})$ is negligible compared to the cost of full waveform inversion algorithm.

Figure 2a is the reconstructed velocity models obtained with the classical FWI algorithm without employing any edge-preserving denoising algorithm. As expected, the inversion didn't delineate the salt edges nor suppresses cross-talk artifacts. Figures 2 b and c depict the reconstructed velocity models obtained with edge-preserving FWI via RED with $\lambda = 1.5$ and $\lambda = 0.08$ weighting regularization parameters, respectively. As we see from the reconstructed models, compared to the result obtained by the classical FWI method, the edge-preserving FWI via RED produces not only provide an image with enhanced resolution, but also salt boundaries were correctly retrieved. The proposed algorithm adequately delineates both shallow and subsalt velocity anomalies. However, the choice of λ plays an important role in achieving the desired results. The weighting regularization parameter λ determines

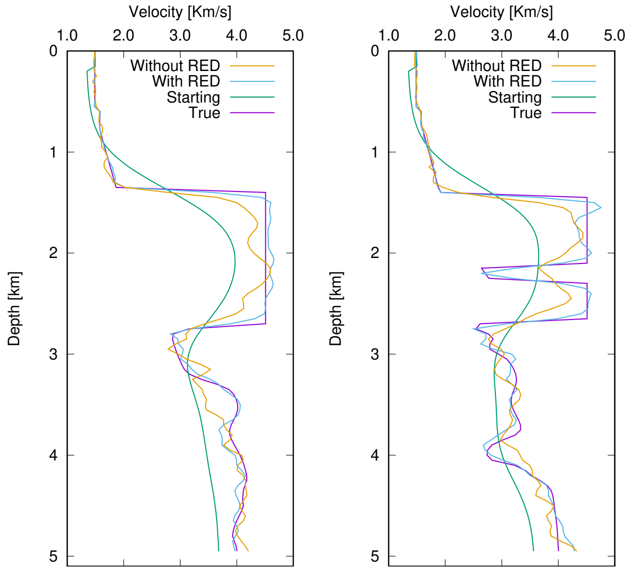

Figure 3: Comparison of vertical velocity profiles of the reconstructed velocity models portrayed in Figure 2a and c. The velocity profiles are extracted at horizontal positions 4.50 Km (a) and 7.20 Km (b).

the relative tradeoff balance of the two terms: the gradient of the data misfit function and the gradient of the regularization term. A higher value of λ relatively produce smooth solutions, Figures 2b and c confirm this. So, it is important to refine the regularization parameter to achieve the desired solutions.

Figure 3 shows vertical velocity profiles extracted at horizontal position 4.50 Km on the left and at horizontal position 7.20 Km on the right, respectively. The results highlight that the proposed algorithm edge-preserving FWI via RED yields images with enhanced velocity contrasts. Results obtained via RED not only gives high resolution but also sharpness of the edges.

Conclusion

We have developed a 2D FWI algorithm with a model update that includes an edge-preserving FWI via a regularization by denoising technique to improve the retrieval of high-contrast images. Unlike other edge-preserving image restoration and denoising regularization algorithms, regularization by denoising scheme is a more powerful and flexible framework of achieving the same goal. The Regularization by Denoising algorithm in FWI is the easiness of implementation. The regularization by denoising technique only requires an image denoising engine to handle the structure of the inverse problems. The primal-dual total variation algorithm is implemented as a denoising engine. The primary objective of the primal-dual total variation denoising technique is to remove some of the singularity caused by the non-differentiability of the L_1 TV norm and is performed by applying a linearization technique. The known edge-preserving, for example, TV-norm constraint method, requires a cumbersome work in finding optimal parameters that control the nonlinearity properties of the model to impose and retrieve desired features on subsurface images. The proposed edge-preserving FWI via a regularization by denoising for high-contrast velocity model proven to provide sharpness of model updates and able to preserve edges while removing unwanted noises.

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