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Improving Formation Factor and Permeability Estimation in Clean Sediments for Reservoir Characterization of Oil Sands

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Summary

Clean sand/silt values of permeability and water saturation in the Athabasca oil sands (Alberta, Canada) are important inputs to resource volume calculations, pay and thief zone identification, and geomodels for flow simulation to rank projects and make development decisions. Often a challenge to estimate accurately, these two petrophysical properties are particularly troublesome in oil sands owing to problems in obtaining sufficient, reliable, and inexpensive lab measurements to use as ground truths for log analysis. Typical absolute permeability and formation factor tests often yield questionable and unreliable results due to the unconsolidated nature of the sediments and the occurrence of the oil as bitumen, which causes sample deformation in handling and testing (Brabant and Al-Adani, 2014).

A solution to achieving reasonably accurate and reliable results at no extra cost has been found through the adaptation of established equations developed for unconsolidated material, which have been tested with new, theoretical, pore geometry models and published, lab measured data to ensure integrity as described here. These equations use reliable and accurate data that for oil sands are widely available, bypassing troublesome and expensive techniques and providing extensive spatial coverage to improve reservoir characterization. Application to oil sands has shown good agreement with measured data and explains why significant differences in reservoir performance occur between recovery project areas, as well as validating the possibility of very high permeability values. The new methods provide superior clean sand/silt values for use in mini-models (Etris et al., 2012) and micro-models (Manchuk et al., 2015) to scale-up values for reservoir models.

Theory / Method / Workflow

Substantial research work in the 1940s-1950s on electrical and fluid flow determined important insights and equations that are verified here and shown to be relevant to oil sands formation evaluation. Winsauer et al. (1952) derived that formation factor (F) was the result of only two properties: porosity (ϕ) and sinuosity (τ, which they called tortuosity), also developing the concept of electrically effective cross-sectional porosity (ψ) as being equal to porosity divided by sinuosity, and experimentally proved them. In the work presented here these relationships are validated mathematically using a simple capillary tube model in which the circular tube is straight but dips, and using models of cubic and rhombohedral sphere packs per Graton and Fraser (1935), with the concept of sinuosity angle (θ) from Carman (1956) (Figures 1 and 2).

The sphere pack models produce the same results as lab measurements on actual sphere packs from Wyllie and Gregory (1955), which agree with values that Pirson (1947) derived through geometry analysis of sphere packs of all packing types. From these it can be shown that for any packing of spheres of constant size, the cementation exponent (m) of the Archie formation factor equation (Archie, 1942) is constant at 1.3 (equation 4 shown below). Due to the constant value of m for spheres, a relationship between formation factor involving porosity as the only input can be derived, and equations have been proposed by Maxwell, by Fricke, and by Slawinski (Wyllie and Gregory, 1953). Lab data from Wyllie and Gregory (1955) show Slawinski's equation is excellent and best for perfect spheres, and is best but slightly low for complex grain shapes, therefore a modified Slawinski equation is used here. Typical values of m for oil sands from the equation are approximately 1.5, which matches quite well with the natural sands example in Winsauer et al. of approximately 1.6 (the unconsolidated sample).

In addition to its role in formation factor, sinuosity is involved in the Kozeny-Carman equation for permeability (Carman, 1937) in a factor called tortuosity (T), which is consistently defined as sinuosity squared (τ^2) but different relationships between T, F, and ϕ were proposed by different workers (Wyllie and Gregory, 1955). The capillary tube and sphere pack models prove T is as derived by Winsauer et al. and supported by their lab experiments, suggesting that electrical tortuosity is T and fluid flow tortuosity is T^2. An alternative explanation keeping tortuosity the same for both is proposed here through dividing the Kozeny porosity term by tortuosity, akin to Winsauer et al. dividing porosity by sinuosity for formation factor, and results in an enhanced Kozeny-Carman equation providing consistency with the lab results of Wyllie and Gregory (1955) and Winsauer et al. as well as with oil sands data as shown here. It also yields a Kozeny constant (K_{cf}) for spheres of 5.05, very close to Carman's suggestion of 5.0.

A longstanding concern with the Kozeny-Carman permeability equation is the existence of a shape factor (s_{hf}) whose assignment is arbitrary for irregular pore shapes (Wyllie and Gregory, 1955). A solution for deterministically calculating its value is proposed here through an equation in which porosity is the only input, based on regression between a combination of back-calculated values from the Wyllie and Gregory (1955) data set with values from Pirson, and matching to the Winsauer et al. data set. The result is a new equation (equation 5 shown below) whose values for low porosities approximate the rigorously solved lowest values—square (1.78) and triangle (1.67) shapes—and for high porosities approximate the rigorously solved highest values—rectangular (2.65) and slit (3.0) shapes—(Carman, 1956).

The last parameter required in the Kozeny-Carman permeability equation is the specific surface area (S_0). For spheres, there is a simple equality between S_0 and grain diameter (D) (Carman, 1937, Wyllie and Gregory, 1955). To test the applicability of using an average grain size to calculate permeability, the Winsauer et al. data set was used and, despite the lithified nature of the samples, reasonably good results were obtained, although a maximum average size (geometric mean) of 0.23 mm was required to handle all samples. This same limit was seen in the match to oil sands samples. There is no theoretical reason for this limitation, and given that it only happens in a small number of samples with unusually large grain sizes it is suspected to be due to poor sorting, but more work is needed to investigate this limitation.

Through this method, the final equation for formation factor (equation 2 shown below) requires only porosity as input, and the final equation for permeability (equation 8 shown below) requires only porosity and mean grain size as input, all of which are commonly available for oil sands, and are measurable by highly accurate and relatively inexpensive processes.

Results, Observations, Conclusions

A simple workflow results from the above (all referenced equations shown in next section):

1) Determine effective porosity from core or log analysis.
2) Calculate formation factor from porosity using modified Slawinski equation.
3) Calculate sinuosity from formation factor and porosity using Winsauer et al. equation.
4) Calculate cementation exponent using new equation from this paper (optional).
5) Calculate shape factor from porosity using new equation from this paper.
6) Determine geometric mean grain diameter using particle size analysis.
7) Calculate specific surface area from grain diameter using equation for spheres.
8) Calculate permeability from sinuosity, porosity, shape factor, and specific surface area using enhanced Kozeny-Carman equation.

No parameters that must be arbitrarily changed per data set are involved. This workflow has been tested against experimental data from published literature and from oil sands samples with correlations (R^2) between calculated and measured permeability ranging from 0.75 to 0.99, far superior to porosity versus permeability correlations for the same data sets.

Novel/Additive Information

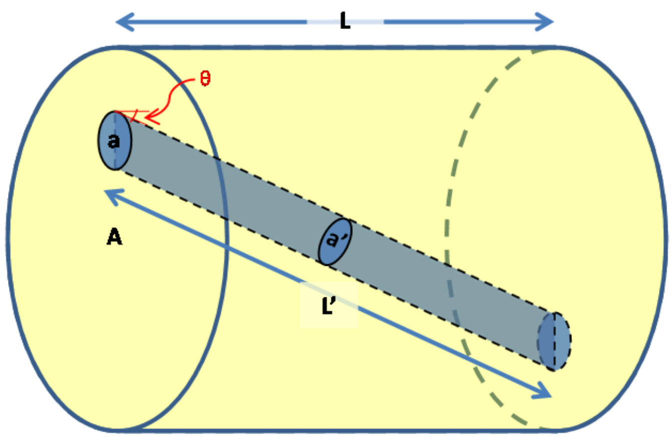
A simple capillary tube model reveals how sinuosity affects porosity to create an electrically effective porosity factor. A new analysis of sphere packs reveals that pore geometry and flow-path sinuosity are essentially the same thing, and explains why m is a constant value of 1.3 regardless of packing. Together, they confirm the correct way to calculate sinuosity from formation factor and porosity, and suggest that the Kozeny-Carman equation for permeability requires an additional tortuosity factor. A new method for calculating shape factor removes subjectivity and is consistent with rigorously derived values. A modified Slawinski equation allows estimation of formation factor for non-spherical grains using only porosity. The use of the geometric mean of grain size is shown to be a valid approximation for specific surface area, at least up to a maximum average grain size 0.23 mm. Oil sands can have an approximate m of 1.5 and permeabilities can reach >25 Darcies, at least for small samples (core plugs).

The following are the equations and methods used:

(1) Porosity (fraction): $\emptyset = \text{Effective Porosity from Core or Log Analysis}$
(2) Modified Slawinski Formation Factor Equation: $F = \frac{(1.45-(0.35\emptyset))^2}{\emptyset}$
(3) Winsauer et al. Sinuosity Equation: $\tau = \sqrt{F\emptyset}$ (Note: use lab F if available)
(4) New Cementation Exponent Equation: $m = \left[\frac{log\left(\frac{\emptyset}{\tau^2}\right)}{log(\emptyset)}\right] = \left[\frac{log(\emptyset \cos^2 \theta)}{log(\emptyset)}\right]$ (Note: where $F = \emptyset^{-m}$)
(5) New Shape Factor Equation: $s_{hf} = 4.2\sqrt{\emptyset}$ (Note: use lower limit of 1.2 (Carman))
(6) Average Grain Diameter (cm): $D = \text{Geometric Mean of Measured Grain Sizes}$
(7) Standard Specific Surface Area Equation (cm^2/cc): $S_0 = \left(\frac{6}{D}\right)$
(8) Enhanced Kozeny-Carman Equation (mD): $k = \frac{1}{S_0{}^2} \cdot \frac{1}{s_{hf}(\tau^2)} \cdot \frac{\emptyset^3}{(1-\emptyset)^2} \cdot \frac{1}{(\tau^2)} \cdot 1.01327\text{x}10^{11}$ where $1.01327\text{x}10^{11}$ converts cm^2 to mD (Amyx, Bass, Whiting, 1960)

Figure 1.— Capillary Tube Model

Derivations of formation factor and related properties for a capillary tube model of a straight, cylindrical pore (blue) dipping across a cylindrical rock volume (yellow). Dimensions and definitions are given to the right of the graphic. Equations 2 and 3 show the complete derivation of electrically effective cross-sectional porosity that Winsauer et al. (1952) stated but did not fully derive, which is volumetric porosity divided by sinuosity (equation 1), and equation 5 shows its significance. Equations 2 and 5 show where Wyllie and Gregory (1953, 1955) and others erred in their use of volumetric porosity in the derivation of formation factor (F=τ/ϕ) and why the Winsauer et al. (1952) derivation using electrically effective cross-sectional porosity (F=τ/ψ) is correct. Equation 6 confirms the true derivation of sinuosity obtained from electrical experiments, which also applies to permeability, and equation 7 confirms the true derivation of Archie's cementation exponent, *m*, as a correction factor to volumetric porosity due to sinuosity.



$\mathrm{L} =$ Bulk Length ; $\mathrm{A} =$ Bulk Area

$\mathrm{L}' =$ Pore Length ; $\mathrm{a}' =$ Pore Area

$\mathrm{Vb} =$ Bulk Volume ; $\mathrm{Vp} =$ Pore Volume

$\mathrm{R} =$ Resistance ; $\rho =$ Resistivity

$$\cos\theta = \frac{L}{L'} \Rightarrow L' = L\left(\frac{1}{\cos\theta}\right)$$

$$a = \pi r^2 \; ; \; a' = \pi r'^2$$

$$\cos\theta = \frac{r'}{r} \Rightarrow r' = r\cos\theta$$

$$\mathit{Carman}\ (1956):\ \tau = \text{Sinuosity} = \frac{L'}{L} = \frac{1}{\cos\theta} \qquad \textbf{(1)}$$

$$\Phi = \text{Volumetric Porosity} = \frac{Vp}{Vb} = \frac{L'a'}{LA} = \left(\frac{L'}{L}\right)\left(\frac{a'}{A}\right) = \tau\,\psi \qquad \textbf{(2)}$$

$$\psi = \text{Cross} - \text{Sectional Porosity} = \frac{a'}{A} = \frac{\Phi}{\tau} = \Phi\cos\theta \qquad \textbf{(3)}$$

$$\Phi = \left(\frac{L'}{L}\right)\left(\frac{a'}{A}\right) = \left(\frac{L'}{L}\right)\left(\frac{\pi r'^2}{A}\right) = \left(\frac{1}{\cos\theta}\right)\left(\frac{\pi r^2\cos^2\theta}{A}\right) = \frac{\pi r^2\cos\theta}{A} = \left(\frac{a}{A}\right)\cos\theta \qquad \textbf{(4)}$$

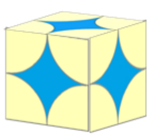
$$\mathit{Winsauer\ et\ al.}\ (1952):\ F = \text{Formation Factor} = \frac{R'}{R} = \frac{\rho\left(\frac{L'}{a'}\right)}{\rho\left(\frac{L}{A}\right)} = \left(\frac{L'}{a'}\right)\left(\frac{A}{L}\right) = \left(\frac{L'}{L}\right)\left(\frac{A}{a'}\right) = \frac{\tau}{\psi} = \frac{\tau}{\left(\frac{\Phi}{\tau}\right)} \qquad \textbf{(5)}$$

$$T = \text{Tortuosity} = \tau^2 = F\Phi \Rightarrow \tau = \sqrt{F\Phi} \qquad \textbf{(6)}$$

$$F = \frac{\tau}{\left(\frac{\Phi}{\tau}\right)} = \frac{\tau^2}{\Phi} = \frac{1}{\Phi^m} \Rightarrow \Phi^m = \frac{\Phi}{\tau^2} \Rightarrow m\log\Phi = \log\left(\frac{\Phi}{\tau^2}\right) \Rightarrow m = \frac{\log\left(\frac{\Phi}{\tau^2}\right)}{\log\Phi} = \frac{\log(\Phi\cos^2\theta)}{\log\Phi} \qquad \textbf{(7)}$$

Figure 2.— Sphere Pack Models

Definitions and derivations for the four sphere-pack model types per Graton and Fraser (1935). Equation 8 assumes overall sinuosity is the average of the maximum sinuosity flow-path (hugging the edges of the spheres) and the minimum flow-path (the least sinuous single plane in a given direction). Equation 9 proves the same relationship between specific surface area and grain diameter in the unique case of perfect spheres as other authors have done (e.g., Carman, 1937). Equations 7a and 7b are from figure 1 of this paper. Results special to each sphere packing case are shown next to the graphic of the case, as input to the equations in the box on the right to calculate the specific values for each case. Using the original Archie equation ($F = Ø^{-m}$), it is demonstrated that cementation exponent is always 1.3, regardless of packing type. Formation factor values are near perfect matches to those derived from Pirson (1947) from a different method not using sinuosity or flow-path geometry and based on Ohm's Law. All these values are confirmed by experimental tests in Wyllie and Gregory (1955).



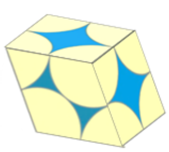
Cubic Packing (Isotropic)

$Vb = 8.00\,r^3$; $\Phi = 47.64\%$

$Sf = D^2$; $Sm = D^2$

$\tau_{max} = 1.571$; $\tau_{min} = 1.000$; $\theta = 26.8°$

$F = 2.64$; $m = 1.307$



Orthorhombic Packing (Anisotropic)

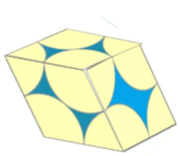
$Vb = 6.928\,r^3$; $\Phi = 39.54\%$

$Sf_1 = D^2$; $Sm_1 = D\frac{2\pi r}{3}$; $Sf_2 = Sm_2 = 2\,r\,D\sin 60°$

$$\theta_{min} = 0.33\cos^{-1}\left(\frac{Sf_1}{Sm_1}\right) + 0.67\cos^{-1}\left(\frac{Sf_2}{Sm_2}\right)$$

$\tau_{max} = 1.571$; $\tau_{min} = 1.007$; $\theta = 29.7°$

$F = 3.35$; $m = 1.304$



Tetragonal-Sphenoidal Packing (Anisotropic)

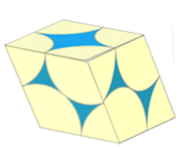
$Vb = 5.6567\,r^3$; $\Phi = 30.19\%$; $Sf = 2rD\sin 60°$

$$Sm_1 = D\sin 60°\left(r + \frac{2\pi r}{6.218}\right); Sm_2 = D\sin 60°\left(\frac{2\pi r}{3}\right)$$

$$\theta_{min} = 0.33\cos^{-1}\left(\frac{Sf_1}{Sm_1}\right) + 0.67\cos^{-1}\left(\frac{Sf_2}{Sm_2}\right)$$

$\tau_{max} = 1.571$; $\tau_{min} = 1.026$; $\theta = 33.6°$

$F = 4.77$; $m = 1.305$



Rhombohedral Packing (Isotropic)

$Vb = 6.00\,r^3$; $\Phi = 25.95\%$

$Sf = D^2$; $Sm = \frac{4\pi r^2}{3}$

$\tau_{max} = 1.571$; $\tau_{min} = 1.047$; $\theta = 35.5°$

$F = 5.81$; $m = 1.304$

D = Grain Diameter ; r = Grain Radius

Vb = Bulk Volume of Unit Cell ; Vg = Grain Volume $= \frac{4}{3}\pi r^3$

$$\Phi = \text{Porosity} = \frac{Vp}{Vb}$$

Sg = Grain Surface Area $= 4\pi r^2$

P = Grain Perimeter $= 2\pi r$

Sf = Surface Area of Plane Parallel to Unit Cell Edges

Sm = Surface Area of Minimum Area Plane Around Spheres

θ_Δ = sinuosity angle due to grain contacts = 1.6°

$\Rightarrow \tau_\Delta = 1.0004$

$$\tau = \text{Sinuosity} = \frac{1}{\cos\theta} \Rightarrow \cos\theta = \frac{1}{\tau} \Rightarrow \theta = \cos^{-1}\left(\frac{1}{\tau}\right)$$

$$\theta = \frac{\theta_{max} + \theta_{min}}{2} = \frac{\cos^{-1}\left(\frac{1}{\tau_{max}}\right) + \cos^{-1}\left(\frac{1}{\tau_{min}}\right)}{2}$$

$$\tau = \frac{1}{\cos\left(\frac{\cos^{-1}\left[\frac{2Sf}{Sg}\right] + \cos^{-1}\left[\frac{Sf}{Sm}\right]}{2} + \theta_\Delta\right)} \qquad \textbf{(8)}$$

$$So = \text{Specific Surface Area} = \frac{\left(\frac{Sg}{Vb}\right)}{(1-\Phi)} = \frac{\left(\frac{Sg}{Vb}\right)}{\left(1-\left(\frac{Vp}{Vb}\right)\right)} = \frac{Sg}{Vb\left(1-\left(\frac{Vp}{Vb}\right)\right)}$$

$$= \frac{Sg}{Vb - Vp} = \frac{Sg}{Vg} = \frac{4\pi r^2}{\frac{4}{3}\pi r^3} = \frac{1}{\frac{1}{3}\pi} = \frac{3}{r} = \frac{6}{D} \qquad \textbf{(9)}$$

$$\textit{Winsauer et al.}\,(1952){:}\ F = \frac{\tau^2}{\Phi} \qquad \textbf{(7a)}$$

$$\textit{This Paper}{:}\ m = \frac{\log(\Phi\cos^2\theta)}{\log\Phi} \qquad \textbf{(7b)}$$

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