Making any elastic modelling code into a local elastic solver

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Summary

Finite-difference (FD) is the simulation method that is most commonly used to model seismic wave propagation. Indeed, this type of modelling is of key importance in many geophysical applications. However, in many of these schemes the region of most interest is small, whereas the entire model is quite large. There are methods to manipulate the source wavefield efficiently and reconstruct synthetical wavefields locally. However, there are few implementations in the elastic domain where it is specifically shown how the injection and reconstruction of wavefields should be done. We discuss the implementation of an efficient algorithm capable of reconstructing elastic wavefields, by injecting monopole sources on a surface surrounding the local model using finite difference methods. Our method is designed to add-on to existing codes, allowing any finite-difference solver to be used as a local wave-equation solver.

Method: Injection of wavefields

To reconstruct synthetical wavefields locally, we implement the multiple points sources (MPS) method within a velocity-stress formulation of the discrete wave equation (Wapenaar 2007). Equation 1 shows the monopole point-sources that are injected in terms of velocity-stress system (Virieux, 1986, equation (2)) on the boundary S,

$$f_x(\mathbf{x}_s, t) = \sum_{s=1}^{N_s} \frac{1}{\rho}[\tau_{xx}n_x + \tau_{xz}n_z]\delta_s$$

$$f_z(\mathbf{x}_s, t) = \sum_{s=1}^{N_s} \frac{1}{\rho}[\tau_{xz}n_x + \tau_{zz}n_z]\delta_s$$

$$h_{xx}(\mathbf{x}_s, t) = \sum_{s=1}^{N_s} [(\lambda + 2\mu)v_x n_x + \lambda v_z n_z]\delta_s$$

$$h_{zz}(\mathbf{x}_s, t) = \sum_{s=1}^{N_s} [\lambda v_x n_x + (\lambda + 2\mu)v_z n_z]\delta_s,, \quad (1)$$

$$h_{xz}(\mathbf{x}_s, t) = \sum_{s=1}^{N_s} \mu[v_x n_z + v_z n_x]\delta_s$$

where $f_x(\mathbf{x}_s, t)$ and $f_z(\mathbf{x}_s, t)$ represent the point-sources injected as velocities in the first two equations. Notice that $f_x(\mathbf{x}_s, t)$ and $f_z(\mathbf{x}_s, t)$ depend on stresses recored on S. The parameters $h_{xx}(\mathbf{x}_s, t), h_{zz}(\mathbf{x}_s, t)$ and $h_{xz}(\mathbf{x}_s, t)$ represent the point-sources injected as stresses in the last three equations, and they are combinations of velocity components recorded   on S. δ_s denotes the delta-function centered at $\mathbf{x}_s \in S$ scaled by the discretization interval to ensure correct amplitude, and N_s represents the number of sources equal to the number of grid points on S.

Results: Numerical Examples

We show the implementation of equations 1, where S is a transparent boundary. For this elastic scenario, we save five different wavefields $v_x(\mathbf{x}_s, t)$, $v_z(\mathbf{x}_s, t)$, $\tau_{xx}(\mathbf{x}_s, t)$, $\tau_{zz}(\mathbf{x}_s, t)$, and $\tau_{xz}(\mathbf{x}_s, t)$.
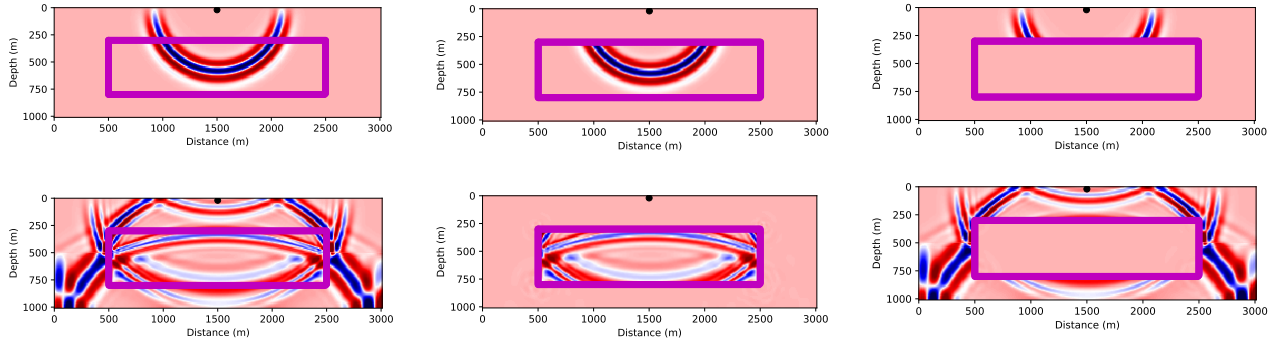

Figure 1: Stress τ_{xx} snapshots of left column: the original wavefield, middle column: injection of the wavefield and right column: difference between left and middle snapshots. Top row: constant velocity model, bottom row: two layer velocity model.

For demonstration purposes, we use a second-order FD scheme to model the response for a source located at $x = 1.5$ km and $z = 0.2$ km in an elastic constant velocity model with $V_p = 2.0\,(\text{m/s})$, $\rho = 2.0\,(\text{kg/m}^3)$ and $V_s = 0.88\,(\text{m/s})$ and a two layer velocity model with $V_{p1} = 2.0\,(\text{m/s})$, $V_{p2} = 4.0\,(\text{m/s})$ $\rho_1 = 2.0\,(\text{kg/m}^3)$, $\rho_2 = 2.3\,(\text{kg/m}^3)$ and $V_{s1} = 0.88\,(\text{m/s})$, $V_{s2} = 1.54\,(\text{m/s})$. We use indices 1 and 2 in the top and bottom layer respectively. The five wavefields for the injection are recovered at the magenta points shown in figure (1), then the recorded data are injected using equation (1). Figure (1) illustrates the results of modeling with the local solver. After doing the difference between the original wavefield and the reconstructed wavefield (Figure 1, right column), we calculate an average error, which is $E_c = 5.6 \times 10^{-5}\,\%$ for the constant model and $E_T = 4.3 \times 10^{-4}\,\%$ for the two layer model. The implementation in any seismic modeling tool is easy. We use Devito to implement the method (Louboutin et al. 2017).

Conclusions

We have described the implementation of the MPS method in a centred staggered FD scheme for the elastic isotropic wave equation in the velocity-stress formulation, and its applicability in an elastic to elastic local solver. We show that using only one surface and five wavefields we are able to accurately retrieve the wavefield within a subdomain. The MPS method thus requires significantly less storage than alternative reconstruction methods such as the method of

Robertsson and Chapman (2000). Because the injection of the wavefield takes place by specifying point sources rather than by changing the parameters of the finite difference stencil, the implementation with any existing seismic modeling tool is straightforward.

Acknowledgment

References

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