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Deblending via ADMM and IRLS: A comparative study

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Summary

We compare two methods for solving the deblending problem: Iteratively Reweighed Least-Squares (IRLS) and Alternating Direction Method of Multipliers (ADMM). In both cases, we use IRLS and ADMM to solve for a robust and sparse Radon transform that is adopted to attenuate blending noise and high-amplitude erratic ambient noise. Numerical examples with synthetic and real data show that the robust sparse Radon transform can suppress source interferences and erratic ambient noise effectively. Our findings also indicate that the ADMM method outperforms the IRLS method in computation time and accuracy.

Introduction

Simultaneous source acquisition has drawn attention from both academic and industrial geophysicists because it permits reducing acquisition costs and increases trace density (Beasley et al., 1998; Howe et al., 2008; Kim et al., 2009; Beasley, 2008). Simultaneous source acquisition adopts two or more sources that are fired simultaneously with random time delays. The responses are then recorded and numerically separated by signal processing techniques. In a common receiver gather, simultaneous source data contains coherent events that correspond to the signals one would have expected to acquire via a conventional survey. The common receiver gather is also contaminated by incoherent source interferences (Berkhout, 2008). Source interferences can be modeled as non-Gaussian noise, and therefore, the process of deblending can be posed as robust filtering.

One way to separate simultaneous sources is by stating deblending as an inverse problem in which one minimizes a cost function that includes a data misfit term and a regularization term (Akerberg et al., 2008; Moore et al., 2008; Cheng and Sacchi, 2015, 2016; Lin and Sacchi, 2020). Another way for separating simultaneous source data is by applying robust denoising to pseudo-deblended records (Kim et al., 2009; Beasley, 2008). The latter is often referred to as deblending by denoising. This work falls in the category of deblending by denoising and uses a sparse and robust Radon transform to model data. In essence, the estimated Radon coefficients are used to synthesize data free of source interferences. Recent work on deblending via denoising adopted robust Radon transforms implemented via the IRLS algorithm (Ibrahim and Sacchi, 2013). Our contribution can be summarized as follows: we introduce the ADMM algorithm to solve for the robust and sparse Radon transform. Also, we compare the ADMM algorithm to the classical IRLS method often used for high-resolution frequency and time domain sparse Radon transforms (Sacchi and Ulrych, 1995; Trad et al., 2003).

Robust and Sparse Radon Transforms

IRLS method

One can use the IRLS method to solve for the robust and sparse Radon transform that we will be adopted for erratic noise attenuation. In this case, we need to estimate the Radon coefficients that model the data via the minimization of the following cost function

$$J = \|\mathbf{d} - \mathscr{R}\mathbf{m}\|_1^1 + \mu\|\mathbf{m}\|_1^1. \tag{1}$$

where $\mathbf{m}$ are the Radon coefficients, and $\mathbf{d}$ symbolizes the pseudo-deblended common-receiver gather. The symbol $\mathscr{R}$ indicates the forward Radon operator. The scalar μ is the trade-off parameter. The expression given by equation 1 is the $\ell_1 - \ell_1$ cost function also adopted by Ibrahim and Sacchi (2013) which can be replaced by an $\ell_2 - \ell_2$ cost function of the form

$$J = \|\mathbf{W}_r(\mathbf{d} - \mathscr{R}\mathbf{m})\|_2^2 + \mu\|\mathbf{W}_m\mathbf{m}\|_2^2. \tag{2}$$

where the operator $\mathbf{W}_r$ is a diagonal matrix with elements given by $[\mathbf{W}_r]_{ii} = |r_i|^{-1/2}$ with residuals $\mathbf{r} = (\mathbf{d} - \mathscr{R}\mathbf{m})$ and $[\mathbf{W}_m]_{ii} = |m_i|^{-1/2}$. The weights depend on residuals and model parameters. Thus, the cost function must be solved iteratively. Details are provided in Trad et al. (2003) and Ibrahim and Sacchi (2013).

ADMM method

ADMM is a simple but powerful algorithm (Boyd et al., 2011) which is flexible for solving high-dimensional optimization problems. For obtaining the robust and sparse Radon transform, we solve the following constrained minimization problem

$$\begin{array}{ll} \text{minimize} & \|\mathbf{r}\|_1^1 + \mu\|\mathbf{m}\|_1^1 \\ \text{subject to} & \mathbf{r} = \mathscr{R}\mathbf{m} - \mathbf{d} \end{array} \tag{3}$$

Note that solving the problem given by expression 3 is equivalent to minimizing equation 1. The ADMM method solves problem 3 via the following three steps

$$\mathbf{r}^{k+1} = \arg\min_{\mathbf{r}} \left\{ \|\mathbf{r}\|_1^1 + \frac{\rho}{2}\left\|\mathbf{r} - \mathscr{R}\mathbf{m}^k + \mathbf{d} + \mathbf{u}_1^k\right\|_2^2 \right\} \tag{4}$$

$$\mathbf{m}^{k+1} = \arg\min_{\mathbf{m}} \left\{ \mu\|\mathbf{m}\|_1^1 + \frac{\rho}{2}\left\|\mathbf{r}^{k+1} - \mathscr{R}\mathbf{m} + \mathbf{d} + \mathbf{u}_1^k\right\|_2^2 \right\} \tag{5}$$

$$\mathbf{u}_1^{k+1} = \mathbf{u}_1^k + \left[\mathbf{r}^{k+1} - \mathscr{R}\mathbf{m}^{k+1} + \mathbf{d}\right] \tag{6}$$

where $\mathbf{u}$ is the Lagrangian multiplier, $\rho > 0$ is a penalty parameter. Equation 4 can be solved by the proximity operator ($\mathbf{r}$-update step)

$$\mathbf{r}^{k+1} = \mathrm{prox}_{1/\rho}\left\{\mathbf{d} - \mathscr{R}\mathbf{m}^k + \mathbf{u}_1^k\right\}, \tag{7}$$

which in this case is given by the soft-thresholding operator (Blumensath and Davies, 2008)

$$\mathrm{prox}_\tau\{y\} = \mathrm{sign}(y)\max(|y| - \tau, 0) \tag{8}$$

The $\mathbf{m}$-update step (equation 5) can be rewritten as follows

$$\mathbf{m}^{k+1} = \arg\min_{\mathbf{m}} \left\{ \left\|\mathbf{b}^k - \mathscr{R}\mathbf{m}\right\|_2^2 + \frac{2\mu}{\rho}\|\mathbf{m}\|_1^1 \right\}, \tag{9}$$

where $\mathbf{b}^k = \mathbf{r}^{k+1} + \mathbf{d} + \mathbf{u}_1^k$. Equation 9 is the classical $\ell_2 - \ell_1$ problem, which can be easily solved, for instance, by the Least Absolute Shrinkage and Selection Operator (LASSO) (Tibshirani, 1996) or the Fast Iterative Soft Thresholding Algorithm (FISTA) (Beck and Teboulle, 2009).

Synthetic data examples

To compare the IRLS method and the ADMM method, we first synthesize an example containing five parabolic events with different curvature to mimic a common receiver gather acquired by a conventional survey. The numerical model consists of 60 receivers and 80 shots. The source wavelet was synthesized with a Ricker wavelet of 30 Hz. We apply the robust and sparse Radon transform in the common receiver gather to estimate the Radon coefficients and then to synthesize denoised data. Figure 1a shows clean data. The data contaminated by noise can be found in Figure 1b. Figures 1c and 1d show the denoising and deblending via ADMM and IRLS, respectively. We can observe that the ADMM method yields the best results. The error estimation sections can be found in Figures 1e-1g. In these figures, we can find that the ADMM method outperforms the IRLS method. In Table 1, we provide SNR values and computational times for both technquies.

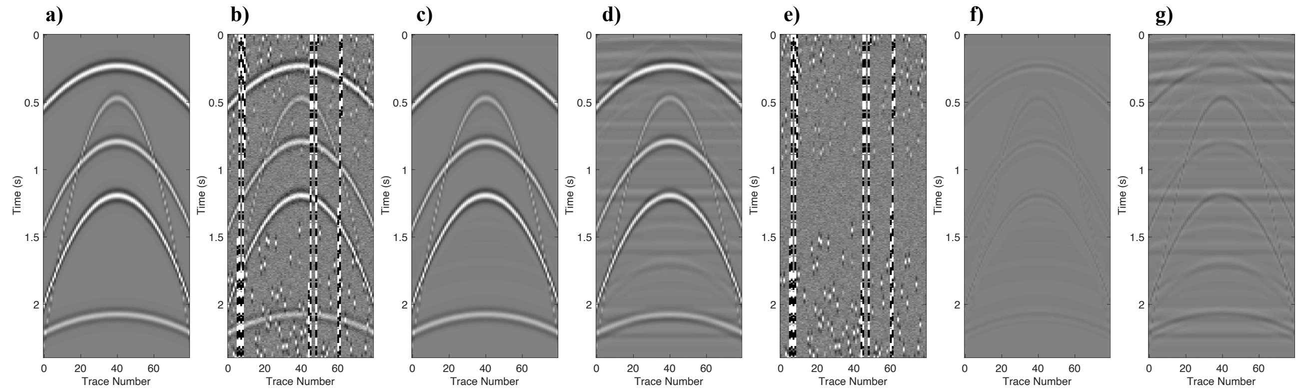

Figure 1: (a) Clean data. (b) Noisy data. (c) Denoising with the ADMM method. (d) Denoising with IRLS method. (e) The difference between (b) and (a). (f) The difference between (c) and (a). (g) The difference between (d) and (a).

Real data example

We also apply our algorithms to a dataset from the Gulf of Mexico (Figure 2). Clean and noisy data can be found in Figures 2a and 2b. The deblending and denoising results can be observed in Figures 2c and 2d, respectively. If we compare Figures 2f and 2g, we observe that the ADMM method has a minimal amount of signal leakage, while IRLS has a amount of noise left. In Table 1, we notice that the SNR values of the ADMM method is similar to the SNR for the IRLS solution.

Case	Algorithm	Time (sec)	SNR Value (dB)
Blending+**Erratic**+**Random** Noise (Synthetic data)	**ADMM**	15.12	22.23
	IRLS	27.44	10.78
Blending+**Erratic**+**Random** Noise (Real data)	**ADMM**	329.29	4.83
	IRLS	1254.84	4.56

Table 1: Comparison of the sparse and robust Radon transform via IRLS and ADMM solutions for a real dataset.

Conclusion

This paper compares the IRLS and the ADMM method for the computation of the robust and sparse Radon transform. For synthetic data, our examples show that the ADMM outperforms IRLS in both the quality of the denoising and computational cost. For real data, the quality of the reconstruction is

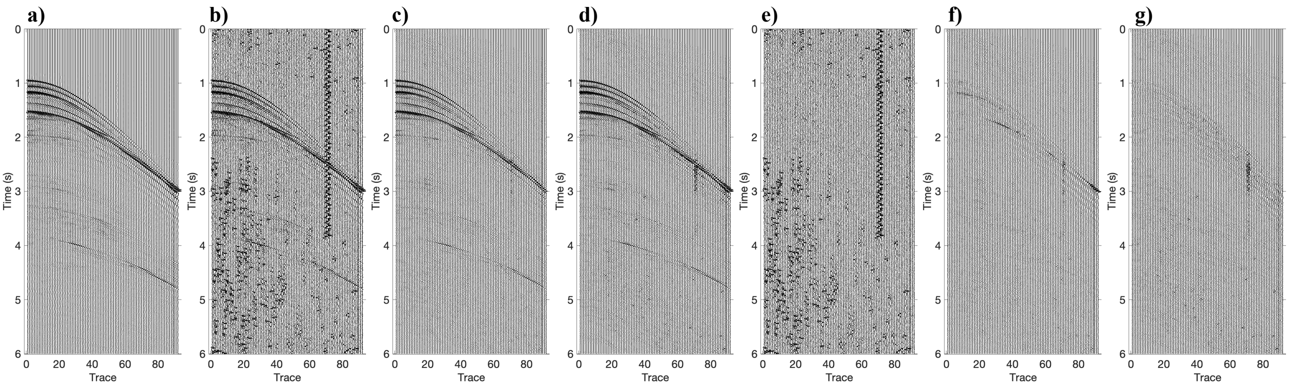

Figure 2: (a) Clean data. (b) Noisy data. (c) Denoising with the ADMM method. (d) Denoising with the IRLS method. (e) Difference between (b) and (a). (f) The difference between (c) and (a). (g) The difference between (d) and (a).

similar for both methods. We believe that the appropriateness of the parabolic Radon transform might be the cause of the differences between synthetic and field data tests. The IRLS method has been profusely used for computing the so-called high-resolution (sparse) Radon transform. We believe that ADMM can replace the IRLS algorithm in a situation where one wants to estimate sparse and robust Radon transforms.

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