Deblending and ambient erratic noise removal via iterative robust singular spectrum analysis

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Summary

We developed an inversion scheme based on robust singular spectrum analysis (SSA) that is capable of simultaneously removing blending and erratic ambient noise. We propose an iterative scheme that adopts the projected gradient method to solve the source separation and denoising problem. The robust SSA acts as the projection operator to suppress blending noise and erratic noise in the frequency-space ($f-x$) domain. We reformulate the singular spectrum analysis filter as a convex optimization problem constraining the low-rank Hankel matrix, which is written as the product of two matrices of lower dimension obtained by bifactored gradient descent (BFGD) method. We evaluate the robust SSA projection operator in deblending and erratic noise removal of a synthetic data example and compare it with the classic (non-robust) SSA method. The results support that the robust one does have a competitive performance for seismic data processing applications.

Introduction

Due to its economic advantages, simultaneous source acquisition has drawn the attention of many practitioners and researchers (Beasley et al., 1998; Howe et al., 2008; Beasley, 2008; Kim et al., 2009). Compared with the conventional seismic acquisition, which adopts a single source, simultaneous source acquisition uses more than two sources and fires them simultaneously with a random time delay (Beasley et al., 1998). One of the major problems associated with this acquisition is that the same receiver records the interferences generated by the other sources. Due to the random time delays, the interferences are incoherent in some domains, such as common receiver gathers (Berkhout, 2008). This characteristic further leads to a variety of separation algorithms.

One way for separating simultaneous sources data is to adopt a denoising method to suppress blending noise in common receiver gathers directly (Beasley, 2008; Kim et al., 2009). The denoising methods mainly rely on the incoherency of the blending noise and aim at separating the coherent unblended data by filtering out the incoherent blending noise in such a domain (Berkhout, 2008).

Another way to separate simultaneous sources is by posing the deblending as an inverse problem in which one minimizes a cost function that includes a data misfit term and a regularization term (Akerberg et al., 2008; Moore et al., 2008). Usually, for the inversion method, the data are transformed into an auxiliary domain, and one can minimize the cost function to make sure that the separated data can reproduce the blended data with a small acceptable error (Abma et al., 2010; Mahdad et al., 2011; Li et al., 2013; Cheng and Sacchi, 2015, 2016). Compared with denoising methods, deblending via inversion methods usually leads to better separation results (Moore, 2010; van Borselen et al., 2012).

In this paper, we borrowed ideas from Cheng and Sacchi (2013) and expanded them into simultaneous deblending and erratic ambient removal by adopting robust SSA instead of traditional SSA as a projection operator. In each iteration, we selected robust SSA to suppress erratic and blending noise simultaneously. The synthetic results support that adopting robust SSA as a projection operator does have a competitive performance compared with adopting the non-robust one.

Theory

Deblending and erratic noise removal via projection operators

In order to suppress blending noise and erratic noise, an iterative inversion scheme can be proposed to preserve data fidelity in order to honor the blending acquisition. In this case, the deblending problem can be summarized by the following cost function

$$\text{minimize} \quad J_1 = \|\mathbf{b} - \mathcal{B}\mathcal{D}\|_2^2 + \mu \parallel \mathcal{D} \parallel_2^2 \tag{1}$$

where $\mathbf{b}$ represents the blended data, $\mathcal{B}$ the blending operator, $\mathcal{D}$ is the desired data cube that one would have acquired via a conventional seismic survey, and μ is the trade-off parameter. The matrix $\mathcal{D}_i$ represents the i-th receiver slice of data cube $\mathcal{D}$ (i.e., a common receiver gather).

A common choice to minimize equation 1 in deblending applications is the projected gradient method (Bertsekas, 1999; Cheng and Sacchi, 2015; Lin and Sacchi, 2020). At each iteration, the solutions are updated in the gradient descent direction and projected to a given set that has desired features. In summary, the projected gradient iterations are given as

$$\mathcal{Z} = \mathcal{D}^k - \lambda \left[\mathcal{B}^* \left(\mathcal{B}\mathcal{D}^k - \mathbf{b}\right) + \mu \mathcal{D}^k\right], \tag{2}$$

$$\mathcal{D}_i^{k+1} = \mathcal{P}_s[\mathcal{Z}_i]. \tag{3}$$

where $\mathcal{P}_s$ represents the projection operator.

In this paper, the robust SSA (Bahia and Sacchi, 2019) is used as a projection operator $\mathcal{P}_s$ due to its capabilities of eliminating erratic and blending noise while preserving the signal for each common receiver gather. Without losing generality, in equation 3, $\mathcal{Z}_i$ denotes the i-th receiver slice of data cube $\mathcal{Z}$. After computing $\mathcal{D}_i,\ i = 1 \dots nr$ for all receiver gathers, we synthesize all $\mathcal{D}_i$ to form a new data cube $\mathcal{D}$.

Robust singular spectrum analysis

We start by representing the classic (non-robust) SSA (Oropeza and Sacchi, 2011) operator by

$$\mathscr{S}\{\cdot\} = \mathscr{A}\{\mathscr{R}\{\mathscr{A}^*\{\cdot\}, k\}\}, \tag{4}$$

where $\mathscr{A}^*\{\cdot\}$ denotes Hankelization operator, $\mathscr{R}\{\cdot, k\}$ represents rank-reduction operator usually done via the Singular Value Decomposition (SVD), k is the desired rank, and $\mathscr{A}\{\cdot\}$ is the anti-diagonal averaging operator. One should notice that the input data for equation 4 are the frequency slices of a given seismic dataset. Further, since the SVD is a least-squares solution (Eckart and Young, 1936), the SSA filter is not resistant to outliers and its performance severely deteriorates in the presence of erratic observations (Chen and Sacchi, 2014; Bahia and Sacchi, 2019).

Alternatively, one can reformulate the SSA filter as a convex optimization problem such as

$$\underset{\hat{\mathbf{H}}}{\text{minimize}} \quad f(\hat{\mathbf{H}}) = \left\|\mathbf{S} - \mathscr{A}\left\{\hat{\mathbf{H}}\right\}\right\|_p, \ s.t. \ \left\{\text{rank}\left(\hat{\mathbf{H}}\right) \le k\right\}, \tag{5}$$

where $\mathbf{S}$ represents the frequency slice of the observed seismic data, $\hat{\mathbf{H}}$ is the desired low-rank Hankel matrix, and $\|\cdot\|_p$ denotes a robust norm of choice.

Again, minimization of equation 5 can be achieved via first-order methods, where the minimizing direction is obtained through the gradient of the cost function

$$\nabla f(\hat{\mathbf{H}}) = -\mathscr{A}^* \left\{ \mathbf{W}(\mathbf{S} - \mathscr{A}\{\hat{\mathbf{H}}\}) \right\} \tag{6}$$

where $\mathbf{W}$ is a diagonal matrix of weights. Adopting Tukey's bisquare function (Trickett et al., 2012; Chen and Sacchi, 2014; Bahia and Sacchi, 2019) the elements of $\mathbf{W}$ for the robust SSA filter can be calculated by

$$\mathbf{W}_{ij} = \begin{cases} \left[1 - \left(\frac{|\mathbf{r}_{ij}|}{\beta}\right)^2\right]^2, & |\mathbf{r}_{ij}| \le \beta \\ 0, & |\mathbf{r}_{ij}| > \beta \end{cases} \tag{7}$$

and $\mathbf{r} = \frac{\left(\mathbf{S} - \mathscr{A}\left\{\hat{\mathbf{H}}^{(n)}\right\}\right)}{\sigma}$ is the normalized residuals by a scale parameter σ, and β is user-defined tunable parameter. Together with σ, β define a threshold to down-weight the large residuals of the outliers, thus reducing its influence in the final output. For non-robust SSA filter, $\mathbf{W}$ can be set as an identity matrix.

The scale parameter σ can be obtained by the normalized median absolute deviation (MAD) (Chen et al., 2014; Bahia and Sacchi, 2019)

$$\sigma = 1.4826\text{MAD}$$

with $\text{MAD} = \text{median}(|\mathbf{r} - \text{median}(|\mathbf{r}|)|)$.

The low-rank constraint in equation 5 can be achieved by factoring the Hankel matrix $\hat{\mathbf{H}}$ as the product of two matrices of lower dimensions

$$\underset{\hat{\mathbf{U}},\hat{\mathbf{V}}}{\text{minimize}} \quad f\left(\hat{\mathbf{U}}\hat{\mathbf{V}}^H\right) = \left\| \mathbf{S} - \mathscr{A}\left\{ \left(\hat{\mathbf{U}}\hat{\mathbf{V}}^H\right)^{(n)} \right\} \right\|_p . \tag{8}$$

Because of the bilinearity of this problem, such factorization leads to nonconvexity in the cost function, and an alternating strategy has to be used in its minimization. Having the gradient of equation 8 with respect to $\mathbf{U}$ and $\mathbf{V}$ as given by (Park et al., 2018)

$$\nabla_{\mathbf{U}} f\left(\mathbf{U}\mathbf{V}^H\right) = \nabla f(\hat{\mathbf{H}})\mathbf{V}, \quad \nabla_{\mathbf{V}} f\left(\mathbf{U}\mathbf{V}^H\right) = \nabla f(\hat{\mathbf{H}})^H \mathbf{U}, \tag{9}$$

one can use the bifactored gradient descent (BFGD) algorithm (Park et al., 2018) to iteratively update the factors $(\mathbf{U}, \mathbf{V})$ following a conventional gradient descent strategy

$$\mathbf{U}_{i+1} = \mathbf{U}_i - \eta \nabla_{\mathbf{U}} f\left(\mathbf{U}_i \mathbf{V}_i^H\right), \quad \mathbf{V}_{i+1} = \mathbf{V}_i - \eta \nabla_{\mathbf{V}} f\left(\mathbf{U}_i \mathbf{V}_i^H\right). \tag{10}$$

Combining equations 6, 9 and 10, we can achieve the robust SSA filter.

Examples

We synthesize an example containing four linear plane-wave events to mimic a common receiver gather in the conventional seismic acquisition. The numerical example consists of 40 receivers and 40 shots. The source wavelet was synthesized with a Ricker wavelet of central frequency 30 Hz. The erratic ambient and random noise are added into each common receiver gather before numerically blended the seismic data.

We first synthesize the data only containing the random noise and blending noise (Figure 1) to compare the deblending and denoising results by robust SSA projection operator and non-robust one. Figure 1a shows the noisy data. Figures 1b and 1d show the denoising and deblending results with robust SSA projection operator and non-robust SSA projection operator, respectively, and Figures 1c and 1e are corresponding error sections.

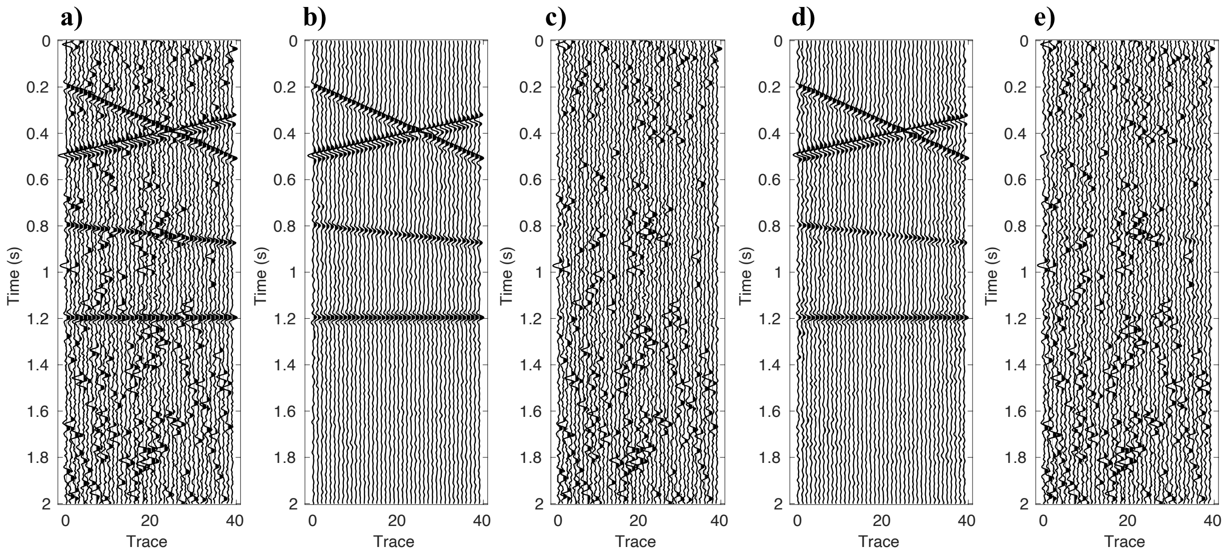

Figure 1: Comparison of denoising and deblending by robust SSA and non-robust SSA projection operator methods. (a) Noisy data, (b) result by robust SSA, (c) difference between (b) and (a), (d) result by non-robust SSA, (e) difference between (d) and (a).

We can observe that the deblending results are quite similar and both methods work well for blending noise and random noise attenuation, although the robust SSA could preserve more signal. Figure 3a shows the SNR variation versus iteration number of the projected gradient method. In Figure 3a, we can observe that adopting robust SSA as a projection operator can achieve higher SNR values than the classic SSA.

The next example is to test the denoising and deblending effect when adding erratic noise (Figure 2). Figure 2a is the noisy data containing random noise, blending noise and erratic noise. The deblending and denoising results can be observed in Figures 2b and 2d. Figures 2c and 2e are corresponding error estimation section.

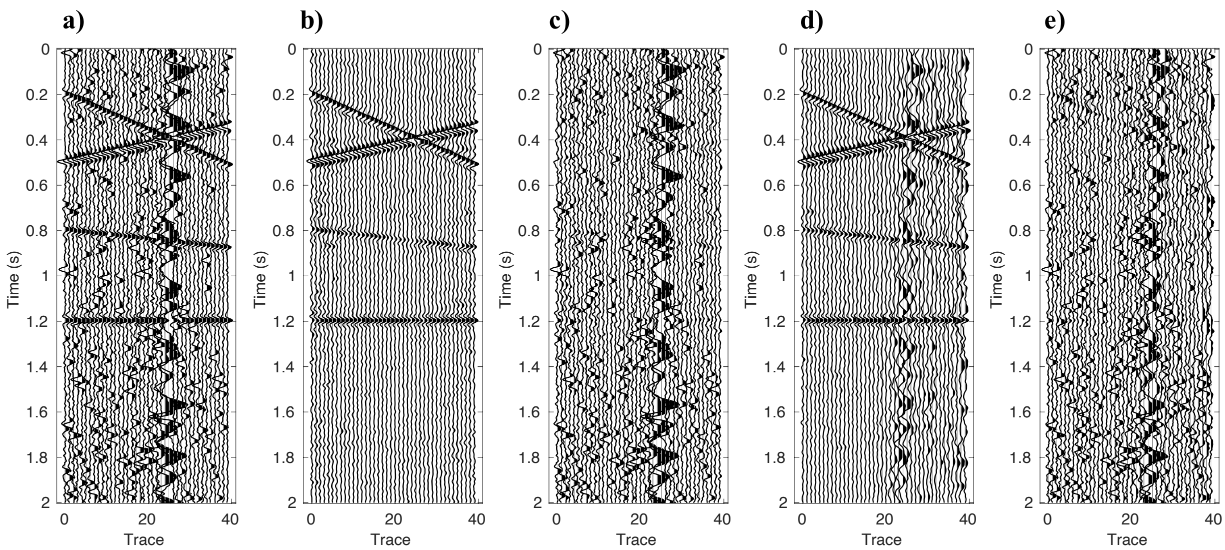

Figure 2: Comparison of denoising and deblending by robust SSA and non-robust SSA projection operator methods by adding erratic ambient noise. a) Noisy data, (b) result by robust SSA, (c) difference between (b) and (a), (d) result by non-robust SSA, (e) difference between (d) and (a).

Comparing Figures 2b and 2d, we notice that the robust SSA projection operator could obtain better deblending and denoising results. The erratic ambient noise are well attenuated, while for the non-robust one we still can observe erratic noise left in the error estimation section (Figure 2e). The SNR variation versus iteration number (Figure 3b) further verifies that adopting the robust SSA as the projection operator could outperform its non-robust counterpart. For the non-robust SSA projection operator, because of characteristic of SSA method, the large singular values of erratic noise are kept, which cause erratic noise left in the deblending and denoising result profile (Figure 2d). This is also the reason why the SNR value decreases according to iteration number (Figure 3b).

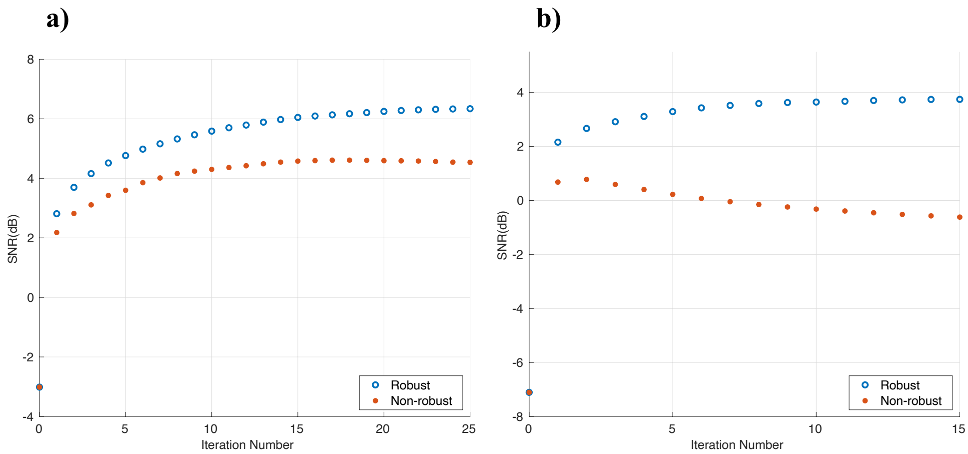

Figure 3: a) SNR vs. iteration number when containing random noise and blending noise. b) SNR vs. iteration number when containing random noise, blending noise and erratic ambient noise.

Conclusion

This paper illustrates an iterative rank reduction algorithm based on robust singular spectrum analysis for simultaneous deblending and erratic noise removal. When existing erratic noise, the classic SSA filter method is not suitable for denoising. Adopting the robust SSA as a projection operator can successfully suppress blending noise an erratic ambient noise. Synthetic examples show that the classic SSA projection operator will keep the largest singular values of erratic noise, which causes erratic noise cannot be attenuated well. The robust SSA projection operator can overcome this problem.

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