3D Offset dependent Split-Step Fourier (OSSF) migration

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Summary

In this paper, three concepts are covered:
First, the extended split-step Fourier (ESSF) migration is modified with wavefield interpolation, similar to that of phase shift plus interpolation (PSPI), in order to improve the wavefield reconstruction at velocity transitions. Second, a novel offset dependent term is introduced into the split-step Fourier (SSF) correction in the vertical wavenumber of the downward extrapolator of the shot wavefield. This honors the spherical wave expanding nature of the shot while conventional SSF does not. Third, the phase output of OSSF migration is confirmed with the 1-way wave equation migration.

Much research work has been done on the 1-way wave equation migration (WEM) methods. Among them, the phase shift class method is popular and has many desirable characteristics, such as no dispersion, steep 90-degree dip capacity, and relatively low CPU and memory requirement. Unless beyond 90-degree imaging ability is required, WEM can be considered the next migration of choice to the much more resource demanding reverse time migration (RTM). The proposed OSSF method in this paper is not only applicable to conventional structural imaging, it is also applicable to anisotropic migration, converted wave (P-Sv) migration and the current diffraction imaging (DI). OSSF can be used in the recent full wave-field migration (FWM) for imaging multiples back to primaries with the offset dependent term turned off because the excitation of the shot is not an impulse function of expanding spherical wave in nature.

Methods

Concept 1. The basis of OSSF:
SSF (Stoffa et al.,1990) applies a simple translation correction term in the phase shift (PS) extrapolator whenever there is a discrepancy between the actual lateral velocity and the reference velocity. Original SSF uses only one reference velocity per depth step putting a heavy workload on the SSF correction (equation 3), and it can work for small lateral velocity discrepancy or gentle dips. Selection of reference velocities for SSF can be found in Chen (2010). ESSF by Kessinger (1992) extends SSF to use multi reference velocities covering a wide velocity range instead of a single reference velocity PS migration. At each downward extrapolation depth step, ESSF just cuts and pastes the wavefields of different velocities to construct a new composite wavefield without interpolation, while here, the proposed OSSF interpolates the wavefields like that of PSPI in order to give a better wavefield reconstruction at velocity transitions. In a 3D shot, the upward going receiver wavefield U and the downward going shot wavefield D become

$$U(x,y,\omega;z) = \sum_l \left[\Omega_l(x,y) \cdot e^{-ik_l^{SSF}\Delta z} \cdot U(x,y,\omega;z)_{|v_l}\right] \qquad (1),$$

$$D(x,y,\omega;z) = \sum_l \left[\Omega_l(x,y) \cdot e^{+ik_l^{SSF}\Delta z'} \cdot D(x,y,\omega;z)_{|v_l}\right] \qquad (2),$$

$$k_l^{SSF} = \omega\left(\frac{1}{v_l(z)} - \frac{1}{v(x,y;z)}\right) \qquad (3),$$

where Ω_l is the overlapping spatial window l which is identical in shape to that used in the PSPI interpolation of wavefields, k_l^{SSF} the SSF vertical wavenumber, $\Delta z = \Delta z'$ the depth step for now until concept 2, v_l the reference velocity used within the PS spatial window, v the actual velocity. Note that when there is no lateral velocity discrepancy between v_l and v, $k_l^{SSF} = 0$, OSSF will degenerate back to the conventional PSPI, and when $\Omega_l = \delta(l)$, without overlapping windows, the proposed OSSF will become ESSF.

Concept 2. Offset dependency of OSSF:
Here, a novel offset dependent factor is introduced to reduce the SSF limitation and increase its performance. In most migration situations, with the exception of FWM, the downward going shot wavefield $d(x,y,t=0;z=0)$ initial excitation function is an impulse, and it is downward extrapolated with an expanding spherical wave front. However, the conventional SSF correction in equation 3 with $k_l^{SSF} \cdot \Delta z$ is just a simple translation of wavefields without expansion of wave fronts. In the receiver side U, it is acceptable though because the wave front movements are more like a plane wave than a spherical wave. In order to mimic the expanding spherical wave front in the shot wavefield d, a modified depth step $\Delta z'$ is introduced by equation (4), and the ray diagram is shown in figure 1.

$$\Delta z' = \frac{\Delta z}{\cos(\theta)} = \frac{\Delta z}{cos\left(tan^{-1}\left(\frac{h}{z}\right)\right)} \qquad (4),$$

where θ is the emergence angle, h the shot to image offset. Note that when $\theta = 0$, $h = 0$ the image beneath the shot, there is no modification of depth step $\Delta z = \Delta z'$. Also, θ can be limited up to 65 deg in this paper to prevent excessive stretch. This simple idea of mimicking expanding wave fronts without actual migration is similar to the split-step Fourier statics correction for topography (Ng, 2007, 2008).

Figure 2 shows an ideal impulse response of a 3D shot profile prestack depth migration (PSDM) by 1-way WEM using PS. The reference velocity used v_l = 3000 m/s. The result is a half sphere. Figure 3 is the controlled ideal impulse response where the 'geologic' velocity used is $v(x,y,z)$ = 3600 m/s, 120% of the reference velocity v_l used in figure 2. Refer to figure captions for details.
Figure 4 illustrates the impulse response by the conventional SSF in equation (3) trying to correct a migration result of using a reference velocity v_l = 3000 m/s to mimic a migration result of a much faster 'geologic' velocity of 3600 m/s. The result falls short of fitting the blue semi-circle.
Figure 5, similar experiment to figure 4, illustrates the impulse response by the proposed OSSF with the offset consideration described in equation (3) and (4). OSSF shows that it can mimic better than that of the conventional SSF shown in figure 4. Refer to figure captions for details.

Concept 3. The output phases of 1-way OSSF WEM:
It is well known that the correct output phase at the bottom of the impulse response of 2D, both poststack migration and prestack shot profile migration, is -45 deg, and in 3D cases, both

poststack migration and prestack shot profile migration, -90 deg. It is true that the 1-way extrapolator has a correct native output phase in both poststack 2D and 3D cases due to an exploding reflector imaging condition. However, in prestack shot profile 2D and 3D cases, the native output phases are 'incorrectly' doubled to -90 deg and -180 deg respectively. Liu et al. (2006) mathematically proved the 2D cases for 1-way WEM. Alternatively, my intuitive explanation to that phenomenon is simple: the cross-correlation part UD^* of the imaging condition has doubled the phase rotation because one part is contributed by the propagator in U and another equal part by the propagator in D^*. This excessive phase rotation can be easily compensated. As shown in figures 2, 3, and 4, the bottom of the impulse responses (on the red center traces) of prestack 3D shot profile migration are indeed -90 deg. This is also correctly validated for OSSF with offset consideration shown in figure 5.

Conclusions

OSSF uses wavefield interpolation like PSPI to preserve wavefield traveling through velocity transitions better than ESSF. By adding an offset dependency to the SSF correction term in the shot wavefield, OSSF tries to fit the spherical expanding wave front nature while the conventional SSF does not.

References

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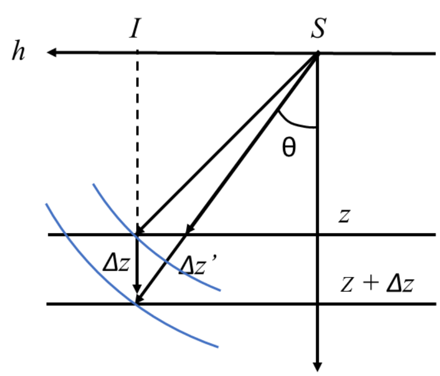

Fig. 1. Expanding wave fronts of the shot S to image points at depth z and $z+\Delta z$

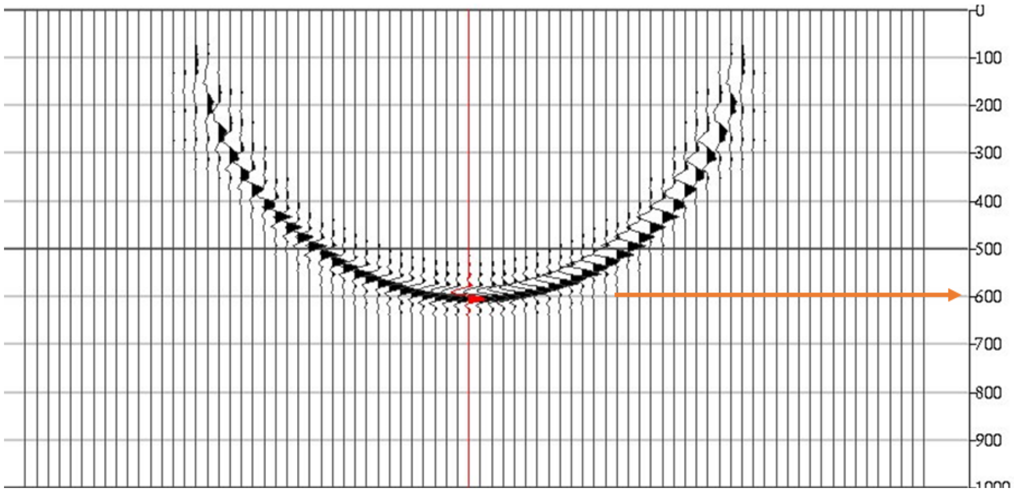

Fig. 2. An ideal impulse response of a 3D shot profile prestack depth 1-way WEM using PS with one single reference velocity used v_l = 3000 m/s. The bottom of the impulse response is at 600 m (brown pointer). The wavelet in the bottom of the impulse response is -90 deg (red wavelet).

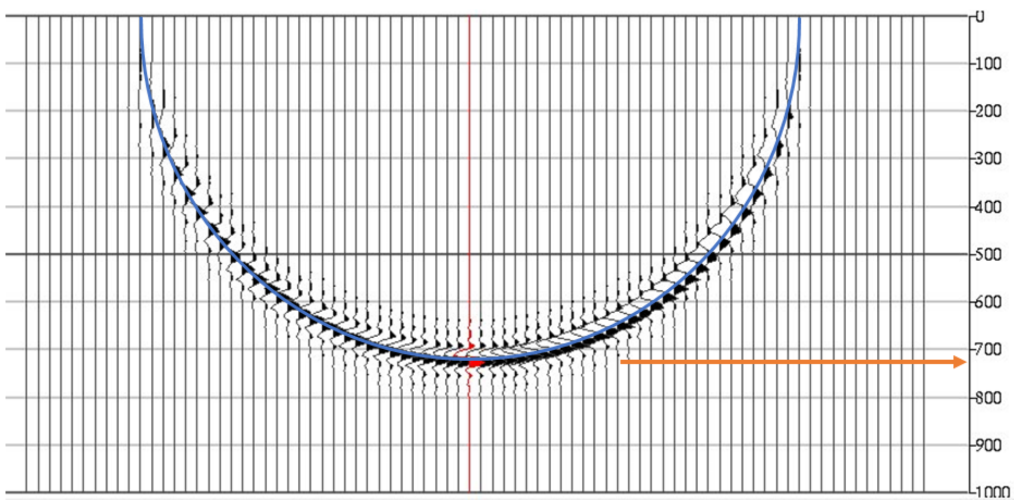

Fig. 3. The 'control' ideal impulse response where the velocity used is $v(x,y,z)$ = 3600 m/s which is 120% of the reference velocity v_l used in figure 2. The bottom of the impulse response is at 720 m (brown pointer). The impulse response is highlighted by the perfect blue semi-circle.

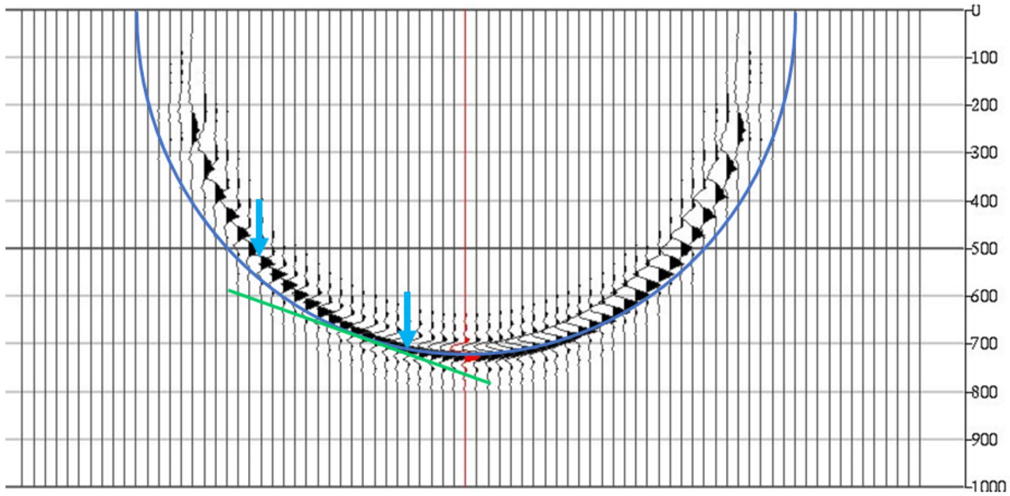

Fig. 4. The impulse response of the conventional SSF (and ESSF) using equation (3) to correct a migration result of using a reference velocity v_l = 3000 m/s to mimic a migration result of a faster velocity $v(x,y,z)$ = 3600 m/s which is 20% higher. The result is simply a vertical translation of the result in figure 1 down to depth 720 m (blue arrows) without expansion of wavefront to fit the blue semi-circle. The dip accuracy is limited to the 20 deg green line. The 'smile' deviates from the ideal control result in figure 3. The wavelet in the bottom of the impulse response is -90 deg (red wavelet).

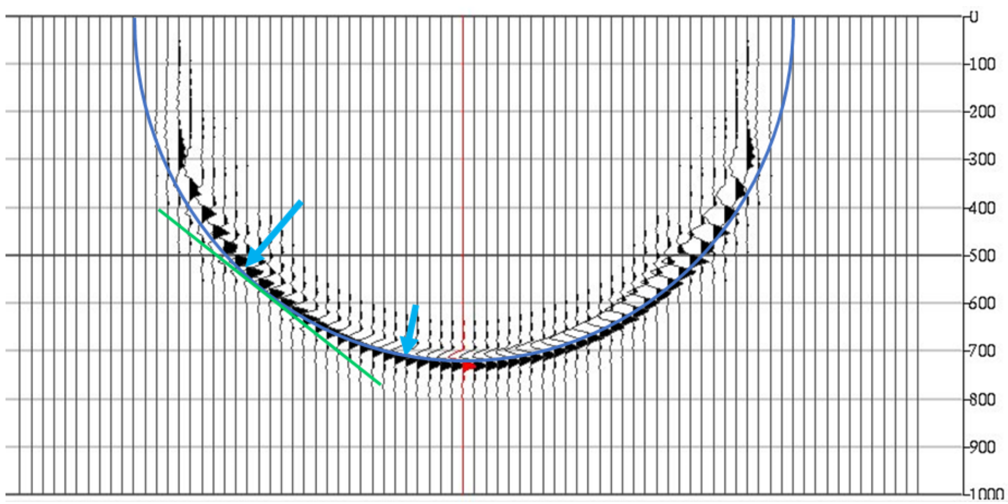

Fig. 5. The impulse response of the proposed OSSF using equations (3) and (4) with offset consideration to correct a migration result of using a reference velocity v_l = 3000 m/s to mimic a migration result of a faster velocity $v(x,y,z)$ = 3600 m/s which is 20% higher. The result shows that it is not a simple vertical translation of wavefronts as done by the conventional SSF in figure 4, but there is an expansion of wavefronts (blue arrows) to fit the blue semi-circle. The dip accuracy is much increased as shown by the 45 deg green line when compared to figure 4 SSF result. The wavelet in the bottom of the impulse response is -90 deg (red wavelet). The overall result is approaching the ideal control result in figure 3.