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Using targeted full waveform inversion uncertainty quantification to understand acquisition requirements

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Summary

Full waveform inversion (FWI) is a powerful tool for recovering the physical properties of the subsurface, but uncertainty exists in the recovered properties. The degree of confidence we can place in an inversion output has a major impact on how we are able to use that inversion. The data used in the inversion, especially the acquisition geometry and available frequencies, play a major role in determining the confidence in the inverted model. Unfortunately, general quantification of uncertainty in an inversion result is very challenging in FWI, but the model features which have the largest impact on the interpretation of results are typically very specific. Here, we explore a method for targeted uncertainty quantification, and use this approach to identify the acquisition geometries required to recover key model features in a set of numerical examples.

Introduction

Typically, FWI is framed as a numerical optimization procedure, seeking to minimize an objective function quantifying the discrepancy between measured data and the data simulated from the current estimate of the subsurface properties (Tarantola, 1984). The subsurface model output by this procedure is generally a much better estimate of the subsurface than the starting model but is uncertain for a number of reasons. Specifically, an inversion result may be imperfect because 1) it erroneously attempts to match noise in the data, 2) it neglects complexities of seismic wave propagation that have a major impact on the data, 3) computational cost considerations prevent the FWI result from being truly optimal, and 4) the data simply do not constrain the subsurface adequately: different subsurface models agree with the measured data equally well. The first two of these causes of uncertainty involve questions of which data should or shouldn't be matched in the inversion, which we do not discuss here. The third and fourth sources of uncertainty relate directly to the FWI objective function; they mean that in general there are models other than the FWI output that are equivalent or better according to the objective function. Here, we explore a strategy for quantifying this type of uncertainty and investigate how it can be used to better design seismic acquisition.

Theory

Consider a model output by an FWI procedure, *m*. We expect in general that this model lies near, but not at, a minimum of the objective function. This means that the objective function near *m* can be approximated as

$$\Phi(m+\alpha\delta m) \approx \phi_0 + \alpha g_0 \delta m + \frac{\alpha^2}{2}\delta m^T H_{GN} \delta m,$$

where ϕ_0, g_0, and H_{GN} are the objective, gradient, and Gauss-Newton approximation of the Hessian at *m*, α is a step length, and δm is a model-space step. The longest step $\alpha\delta m$ that can be taken in a given model-space direction without increasing the objective function higher than ϕ_0 is the step which gives $\Phi = \phi_0$, which is achieved when

$$\alpha = \frac{2 g_0 \delta m}{\delta m^T H_{GN} \delta m} .$$

Given the definition of α above, the step $\alpha\delta m$ is a nullspace shuttle: a model-space step which does not change the objective function (Deal and Nolet, 1996). Nullspace shuttles are useful for gauging uncertainty because they represent the maximum change in a model-space direction that can be taken without worsening the objective function.

While calculating a large number of nullspace shuttles for an FWI output is generally not feasible, they can be useful tools for targeted uncertainty quantification. In a targeted approach, a scalar function ψ can be defined, which characterizes the presence or absence of a chosen feature of the inversion result. Finding the uncertainty in this feature can be treated as an optimization over nullspace shuttles:

$$\Delta m = \min_{\alpha\delta m} \psi(\alpha\delta m)$$

The nullspace shuttle minimizing ψ is an estimate of the model-space step which maximally removes the feature defined by ψ while remaining equivalent to or better than the inversion output in objective function. In this way it is an important measure of the uncertainty in ψ.

Numerical tests allow us to use uncertainty quantification to learn about the data required to gain confidence in different model features. This can be done by comparing the confidence in an important feature of an inversion result given access to different datasets. If the model considered in the numerical tests is representative what is expected in the subsurface, these tests can be informative about which acquisition geometries are needed to constrain important subsurface features.

Example

Here, we consider a numerical example of using targeted uncertainty quantification to identify ideal seismic acquisition. For this example, we consider the model shown in figure 1 to be the inversion result we are interested in characterizing. One of the notable features of this model is a low v_P anomaly, which occurs at the same location as a notable drop in density and Q_P (at about 700 m x-position and 200 m depth). This anomaly is associated with a substantial drop in v_P/v_S ratio. In this example, we try to determine how well constrained this anomaly is, given access to data from different acquisition geometries.

Figure 2 shows the v_P/v_S ratio of the inversion output shown in figure 1 (top left), as well as that of the model after applying the best calculated null-space shuttle for removing the v_P/v_S anomaly given only 400 m of sources and receivers at the surface (top right), given sources and receivers along the entire surface (bottom left), and given sources along the entire surface as well as receivers along the entire surface and bottom of the model (bottom right). None of the acquisition geometries investigated have total confidence in the extent of the v_P/v_S low, but a clear difference in the uncertainties is evident here. The narrow surface acquisition is unable to constrain a v_P/v_S low of any kind; the top right panel of figure 2 shows a model with the same FWI objective function as the inversion output, and this model has little or no v_P/v_S ratio drop at the anomaly. The shuttled result for the broad acquisition geometry (figure 2, bottom left) preserves a v_P/v_S low at the anomaly but shows that the magnitude of this anomaly could be significantly underestimated without changing the objective function. This type of acquisition could allow for high confidence in determining whether such an anomaly is present, but much lower confidence in the recovered magnitude. The shuttled result for the surface-and-bottom type acquisition is shown in the bottom right of figure 2. In this case, the shuttled result is quite similar to the inversion output; there is relatively high confidence in the recovered anomaly in this case.

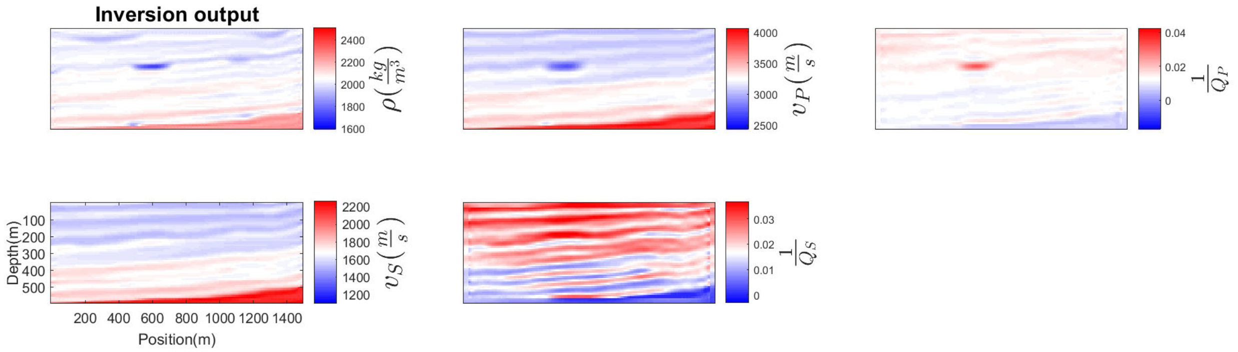

Figure 1. Viscoelastic inversion output considered in the numerical examples. The key feature being investigated here is a region of anomalously low v_P/v_S ratio, at about 700 m x-position and 200 m depth.

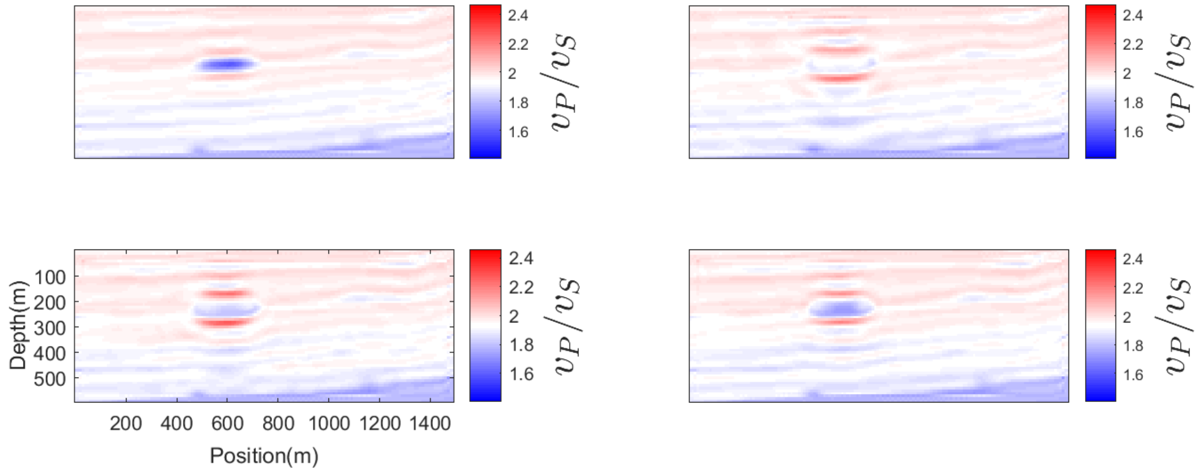

Figure 2. Ratio of P- and S-wave velocities for inversion output (top left), and equivalent-objective models with narrow surface acquisition (top right), wide surface acquisition (bottom left), and combined surface and bottom acquisition (bottom right). The more comprehensive acquisitions are better able to constrain the anomaly in this case.

Conclusions

Uncertainty quantification can be a useful tool for deciding which seismic data to acquire. The very large dimensionality of FWI, however, makes complete uncertainty quantification challenging. Here, we investigate a targeted method for uncertainty quantification of FWI results using nullspace shuttles. In a synthetic test, we demonstrate that this approach is capable of determining the confidence in an inversion feature given access to different sets of data. This type of information could be used to better inform acquisition design.

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