

## Full waveform inversion of multimode surface wave data: numerical insights

Raul Cova,<sup>1,2</sup> and Kris Innanen<sup>2</sup>

<sup>1</sup>Qeye, <sup>2</sup>CREWES/University of Calgary

### Summary

Full waveform inversion has been demonstrated to be a powerful tool for high resolution velocity model building. However, using surface wave data in FWI presents many challenges. In particular, the dispersive nature of surface waves results in the amplification of cycle skipping problems. Here, we propose decomposing surface waves into their fundamental and higher order modes and inverting them following a sequential approach to mitigate this problem. Despite the fundamental mode amplitudes being typically larger than the higher order modes, the latter ones can travel in the deeper parts of the near-surface at higher frequencies. Therefore, we use the fundamental mode to produce an initial approximation to the near-surface S-wave velocities and then perform an additional step of FWI using the higher order modes to produce a more detailed velocity profile, particularly at larger depths. We also argue that although each individual higher mode is less energetic than the fundamental mode, as a group, the combination of all higher modes surpasses the energy and reach of the fundamental mode. We performed the modal separation by designing a mask in the F-K domain that roughly contains the energy of each mode. Results obtained from synthetic data demonstrate the potential of this approach to avoid cycle skipping and to improve the resolution of inverted S-wave velocity models.

### Surface wave theory

In a layered medium surface wave propagation is dispersive. This means that each frequency component will travel with a different velocity. The velocities are controlled by the S-wave velocity profile of the near-surface. In addition, surface wave propagation in a layered medium is a multimode phenomenon. This means that at each frequency different modes of vibration might exist as the result of constructive interference occurring among waves undergoing multiple reflections (Foti et al., 2018).

Full waveform inversion (FWI) of surface wave data attempts to retrieve the elastic properties of the near-surface by using all the wave modes present in the data. In its original formulation, FWI is posed as a local optimization problem, which can get stuck at local minima if a solution close to the true subsurface parameters is not provided as an initial guess. This results in an effect commonly known as cycle skipping. The dispersive character of the surface wave amplifies the cycle skipping problem, making the FWI of surface wave very challenging. Alternative misfit functions have been proposed to mitigate this problem (Masoni et al., 2013; Pérez Solano et al., 2014; Yuan et al., 2015; Borisov et al., 2018). Masoni et al. (2016) follow a layer stripping approach, moving from narrow-offset high-frequency components, which contain information about the shallower parts of the near-surface, towards wide-offset low-frequency components which provide information about the lower part of the near-surface.

Here, we propose using modal decomposition as a proxy for layer stripping. We perform an initial step of FWI using only the fundamental mode data, which should provide detailed information about the shallower part of the near-surface and just the low frequency component of the deeper parts. Then, we perform one additional FWI step using the high order modes to add resolution to the lower part of the model. This approach can be combined with the strategy proposed by Masoni et al. (2016), for further stability. We use the FWI implementation developed by Pan et al. (2018), which exploits the computation of the misfit kernels and forward modelling capabilities of the Specfem2d package using the spectral element method (Komatitsch and Tromp, 1999).

### Multimodal surface wave FWI gradients

To understand the extent of the influence of each surface wave mode in an FWI iteration, we first computed synthetic vertical and horizontal component data using Specfem2d (Figure 1a and 1c, respectively). The dispersion spectra of these data are shown in Figure 1b and 1d. The white dash line separates the region of the spectra that is dominated by the high-energy low-velocity components that represent the fundamental mode, from the less energetic but faster components that are the result of the higher order modes.

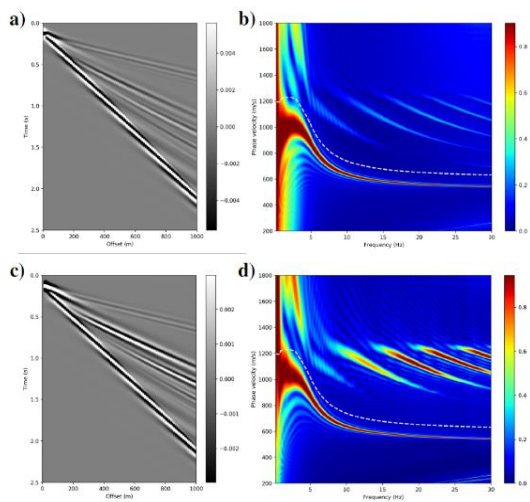


Figure 1. Raw (a) vertical- and (c) horizontal-component data and their corresponding dispersion spectra (b) and (d), respectively.

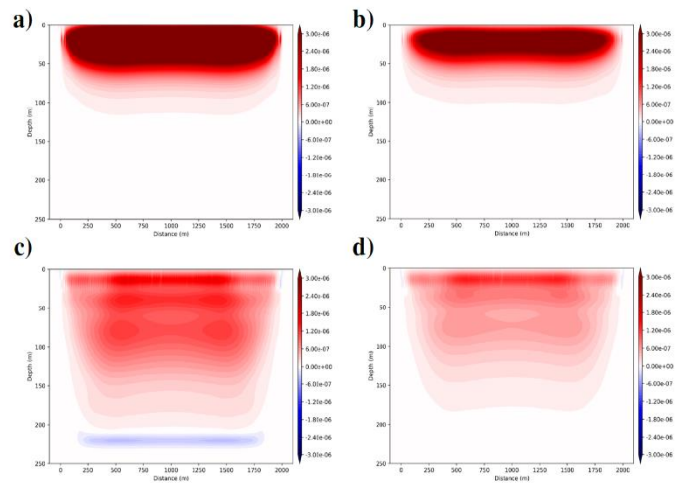


Figure 2. Vs gradients obtained after the first FWI iteration for the (a) X- and (b) Z-component data of the fundamental mode and the (c) x- and (d) x-component data of the higher order modes.

Given that most of the energy in the gradients is focused on the Vs model we plotted the contributions on the gradient coming from each particle displacement component and surface wave mode (Figure 2). The top row on Figure 2 displays the horizontal and vertical component gradients obtained with the fundamental mode data. Notice that even though they have similar magnitudes, the x-component gradient displays a deeper coverage compared to the z-component gradient. Similarly, the bottom row of Figure 7 displays the Vs gradients obtained using the higher order modes energy recorded in each displacement component independently. Compared to the Vs gradients obtained with the fundamental mode data, the energy of the higher order modes provides a deeper coverage with slowly decaying amplitudes with depth.

Moreover, the horizontal component data provided a slightly deeper coverage compared to the vertical component data which overall displays lower amplitude levels.

These observations are evidence of the importance of the horizontal component data for the inversion of surface wave energy. In summary, they exhibit larger sensitivity to changes in  $V_s$  as experienced by the surface waves and they might contain higher order modes energy that can provide details about the deeper parts of the near-surface.

### **Synthetic multimodal surface wave FWI**

To test the performance of this approach we created background  $V_p$ ,  $V_s$  and density models and then embedded a series of  $V_s$  anomalies in the  $V_s$  model (Figure 3a). The  $V_p$  and density models only contained linearly increasing values. These two parameters were fixed during the inversion and only the  $V_s$  model was updated after each iteration. The initial velocity models for all the three parameters consisted of models containing the exact background models.

Figure 4 summarizes the evolution of the inversion. On the top row the “observed” x- and z-component data and their corresponding dispersion spectra are plotted. Notice the backscattered surface-wave energy that is evident on the horizontal component data (Figure 4c). This produces interruptions in the continuity of the energy of the higher order modes in the dispersion spectrum (Figure 4d). Picking dispersion curves in this scenario would be a very difficult task. The second row displays the data modelled using the initial models. Notice that the dispersion spectrum for both the fundamental mode energy and the higher order modes are smooth and continuous. The data resulting from the inversion using the fundamental mode data (Figure 4, third row), displays some perturbations on the fundamental mode energy that resembles the energy present in the observed data. However, the part of the spectra corresponding to the energy of the higher order modes does not display significant changes. Lastly, the results of the FWI after including the higher order modes are displayed on the bottom row. There we can see that both the modelled data and their frequency spectra match the observed data very closely.

Figures 3b and 3c compare the  $V_s$  model obtained after the inversion of the fundamental mode data and the one obtained after including higher order modes. Notice that the inversion of the fundamental mode data only provided low frequency updates to the model. No details for each of the anomalies can be observed in Figure 3b. On the other hand, after including the higher order modes, the location and shapes of each of the anomalies in the model were successfully retrieved.

### **Conclusions**

Hierarchical strategies have shown to be very effective in FWI. In general, they try to first solve the simplest parts of a problem before moving to a more complete solution.

In this study we have shown how this kind of approach is applicable to multimodal surface-wave data. By first inverting the fundamental mode data high-resolution short-wavelength updates can be obtained in the shallowest part of the near-surface while providing long wavelength updates in the deeper parts of the model. Adding the higher order modes at a later stage improves the resolution at the deeper parts of the model. Layer stripping is then an implicit process in this approach.

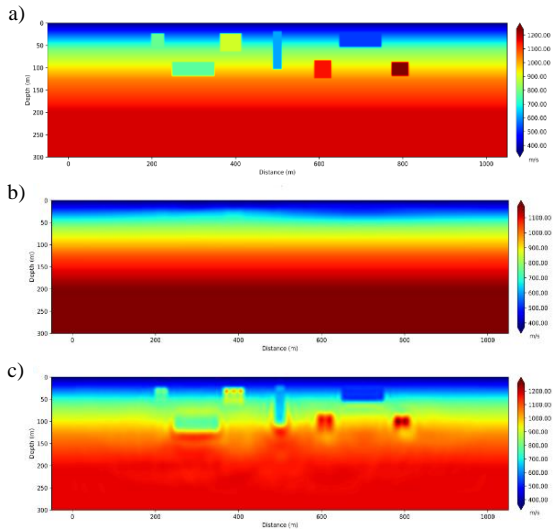


Figure 3. a) True Vs model. b) Vs model obtained after FWI using fundamental mode data only and (c) after including higher order modes.

We also highlight the importance of the horizontal component data in this process. Since the energy of the higher order modes is usually comparable to that of the fundamental mode in this component, a more balanced measurement can be provided to the inversion. In the case of using vertical component data only, where the fundamental mode amplitudes overwhelm the rest of the arrivals, the inversion will be mostly driven by this wave mode.

Understanding the sensitivity of the horizontal-component data to both the fundamental and higher order modes of surface waves is important for developing FWI applications using distributed acoustic sensing (DAS) data. Despite usually providing single component measurements the extremely dense spatial sensing of the DAS data allows for an unaliased recording of the surface wave energy. The application of this approach using DAS data remains to be explored.

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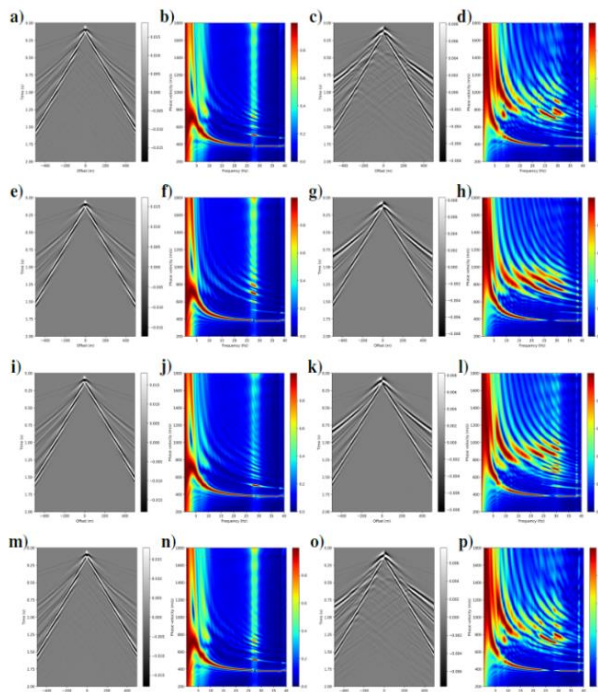


Figure 4. (first column) horizontal and (third column) vertical-component data and their corresponding dispersion spectra at different FWI stages. (a)-(d) observed data. (e)-(h) initial model outputs. (i)-(j) results of fundamental mode data inversion. (m)-(p) Higher order modes inversion outputs.

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