Radon-based deblending via Robust Matching Pursuit

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Summary

Simultaneous source acquisition is a strategy to save acquisition costs and enhance data density. In a nutshell, seismic sources are fired at random times with overlap to save acquisition time and increase source density. Data acquired during this process contains strong source interferences, usually described as blending noise. Deblending (removal of source interferences) is a major challenge for simultaneous source data processing. In this paper, we propose a deblending method based on a local Radon transform and a modified Matching Pursuit algorithm. We propose a robust Matching Pursuit algorithm to retrieve Radon domain coefficients that synthesize deblended data. Our deblending tests show a significant signal-to-noise ratio improvement when one adopts the proposed robust Matching Pursuit algorithm.

Introduction

Simultaneous source separation methods are generally divided into two categories: inversion based methods and denoising based methods. The method proposed in this paper belongs to the denoising category. In a common shot gather, both the desired signal and source interferences are coherent. However, in common receiver, common offset, or common midpoint domains, only the desired signals are coherent. In these domains, source interferences behave like random outliers (Beasley, 2008; Berkhout, 2008). We propose to remove source interferences via a robust Matching Pursuit algorithm that operates with Radon transform bases. The method extracts the Radon domain coefficients that synthesize the signal. Once all common-receiver gathers are denoised, we organize them back into common shot gathers to yield the final deblended volume.

Moore et al. (2016) proposed a deblending method based on a greedy algorithm that applies the concept of trimmed inner products introduced by Chen et al. (2013). These contributions do not explain how one can use trimmed inner products for implicit operators such as the time-domain Radon operator. In this article, we propose a Robust Matching Pursuit algorithm to solve for Radon coefficients that synthesize coherence signals in the common-receiver gathers. Rather than using trimmed inner products, we suggest using L_p space inner products that are also insensitive to erratic blending noise. Moreover, we provide a detailed explanation of how one can add L_p space inner products into the Matching Pursuit solver for operators given in an implicit form.

Theory

In the Matching Pursuit algorithm, we choose the best-fit basis waveform based on the maximum absolute inner product between test waveforms that belong to a dictionary and data residuals in each iteration. However, the conventional inner product, which is calculated in the L_2 space, is sensitive to the outliers. Therefore, when the data has erratic blending noise, the conventional inner product would result in the selection of the wrong basis waveform. Authors have addressed the problem above via a modified greedy pursuit approach (Razavi et al., 2012) that incorporated a Huber loss function (Huber et al., 1981). Similarly, Zeng et al. (2016) introduced a correlation between two vectors in L_p space. These authors divided the problem into three cases with $0 < p < 1$, $p = 1$, and $1 < p < 2$, and used three different methods to calculate robust inner products. However, their method can be computationally expensive to implement. Recently, Chen et al. (2013) used the trimmed inner product by sorting the dot product of two vectors based on the absolute value of its elements and computing the sum after removing the largest values. This concept, which is analogous to the truncated mean often used in robust statistics, can reduce the effect of the outliers and help to identify the correct waveform function. In this paper, we propose a new method to calculate a robust inner product, which we called the L_p inner product. The algorithm is easy to implement in a L_p space with $0 < p < 2$.

Let us consider the problem of minimizing the following cost

$$J = \|\mathbf{d} - \alpha \mathbf{g}\|_2^2 \tag{1}$$

where $\mathbf{d}$ and $\mathbf{g}$ are two vectors and α is a scalar. The cost function attains its minimum for $\nabla J = 0$ which yields $\alpha = (\mathbf{d}, \mathbf{g})_2 = \mathbf{d}^T \mathbf{g} / \mathbf{g}^T \mathbf{g}$. Evidently, if we disregard the normalization, the latter is the inner product between the vector $\mathbf{d}$ and $\mathbf{g}$ in the L_2 space. Similarly, we can find the scalar parameter α that minimizes the following L_p cost function $(0 < p < 2)$

$$J = \|\mathbf{d} - \alpha \mathbf{g}\|_p^p . \tag{2}$$

We can make an analogy to the L_2 case, and say that the L_p inner product is the scalar that minimizes equation 2 which is given by

$$\alpha = (\mathbf{d}, \mathbf{g})_p = \frac{\sum_i g_i w_i d_i}{\sum_i g_i w_i g_i} \tag{3}$$

where $w_i = (|d_i - \alpha g_i|^{p-2} + \varepsilon)^{-1}$. Equation 3 can be solved by iteratively reweighed least squares (IRLS; Scales and Gersztenkorn, 1988). For $p = 1$, 3-5 iterations can produce a good estimation of α and 8-10 iterations are sufficient for the case of $0 < p < 1$.

Now we would like to apply the new inner product to the computation of the Radon transform. Unfortunately, the time domain Radon transform is applied "on the flight" and one does not have access to explicit Radon basis functions in vector form. For instance, let us consider the discrete Radon transform in time domain. The forward operator is denoted by L and its adjoint by L^*

$$L : d(t,h) = \sum_q m(\tau = t - qh, q) \tag{4}$$

$$L^* : \tilde{m}(\tau,q) = \sum_h d(t = \tau + qh, h) \tag{5}$$

where h is offset, q is ray paramter, t is time and τ is intercept. The adjoint Radon sum L^* can be interpreted as the inner product of the data with an all-ones vector. Hence, one can define the robust Radon operator as one were the sum is computed via an expression equivalent to equation 3 with $\mathbf{g} = \mathbf{1}$ and data across extracted each $tau - q - h$ trajectory. In other words, if we define the vector $\mathbf{u}(\tau,q) = [d(\tau + qh_1, h_1), d(\tau + qh_2, h_2), d(\tau + qh_3, h_3) \dots]^T$, then the robust Radon adjoint operator L_r^* is given by

$$L_r^* : \tilde{m}(\tau,q) = (\mathbf{u}(\tau,q), \mathbf{1})_p . \tag{6}$$

In this case, the Matching Pursuit algorithm will become insensitive to outliers and, therefore, less prone to the selection of the incorrect basis function. Algorithm 1 provides the proposed Robust MP algorithm for the Radon transform. Notice that the coefficient needed to fit individual waveforms to residuals is also computed via a robust p-norm fitting.

Example

We adopt a subset of a marine seismic dataset from the Gulf of Mexico composed of 808 shots and 183 receivers. The receivers are evenly distributed with an interval of 87 m. We blend three consecutive sources and then used pseudo-blending to obtain blended shots and blended common-receiver gathers. In this example, we apply deblending in common channel gather domain, and then

Algorithm 1 Robust Matching Pursuit Radon transform

Input: data $\mathbf{d}$
Output: reconstructed data $\mathbf{d}_r$
Initialization $\mathbf{r}^0 = \mathbf{d}$, $\mathbf{d}_r = \mathbf{0}$ and $k = 0$
while $k \leq K_{max}$ **do**
  for all τ, q **do** `Robust adjoint`
    $\mathbf{u}(\tau,q) = [r(\tau + qh_1, h_1), r(\tau + qh_2, h_2), r(\tau + rh_3, h_3) \dots]^T$
    $m(\tau,q) = (\mathbf{u}(\tau,q), \mathbf{1})_p$
  end for
  $(\tau^*, q^*) = \text{argmax}_{\tau,q} |m(\tau,q)|$ `Extract largest coefficient`
  $\mathbf{g} = L\delta(\tau^*, q^*)$ `Synthesize basis function via the forward Radon`
  $\alpha^* = \text{argmin}_\alpha \|\mathbf{r} - \alpha\mathbf{g}\|_p^p$ `Estimate amplitude`
  $\mathbf{r} = \mathbf{r} - \alpha^*\mathbf{g}$ `Update residuals`
  $\mathbf{d}_r = \mathbf{d}_r + \alpha^*\mathbf{g}$ `Update reconstructed data`
  $k = k + 1$
end while

sort back to the common shot gather to show the final results. We use a local linear Radon transform with a window size of ten traces.

Figure 1 shows one common offset gather and its pseudo-blended gather. Figure 1c and 1e show the deblending results from the non-robust and robust matching pursuit. For the robust matching pursuit result, the SNR improves from -0.72 dB to 9.1 dB, and the total gain is 9.82 dB.

As we mentioned before, we apply deblending to all collective offset gathers and then sort them back to the common shot gathers. Figure 2 is one of the common shot gathers and its associated pseudo-blended record. The pseudo-blended data contains source interference from two neighboring sources, and the SNR equals to -2.85 dB. As shown in Figure 2e, the robust MP upgrades the deblending quality in terms of SNR, which increases from -2.85 dB to 9.2 dB. The quality enhancement can also be observed by inspecting one single trace (Figure 3), where the robust deblending removes almost all source interference and recovers all significant events simultaneously.

Conclusion

We proposed a method to estimate the Radon transform via a robust matching pursuit algorithm. The robust Matching Pursuit Radon transform was adopted to attenuate erratic noise caused by source inferences arising in simultaneous source acquisition. The definition of the adjoint Radon operator in terms of a robust L_p sum permits to minimize errors in the selection of basis functions in the matching pursuit algorithm.

Acknowledgements

We thank the sponsors of the Signal Analysis and Imaging Group (SAIG) at the University of Alberta.

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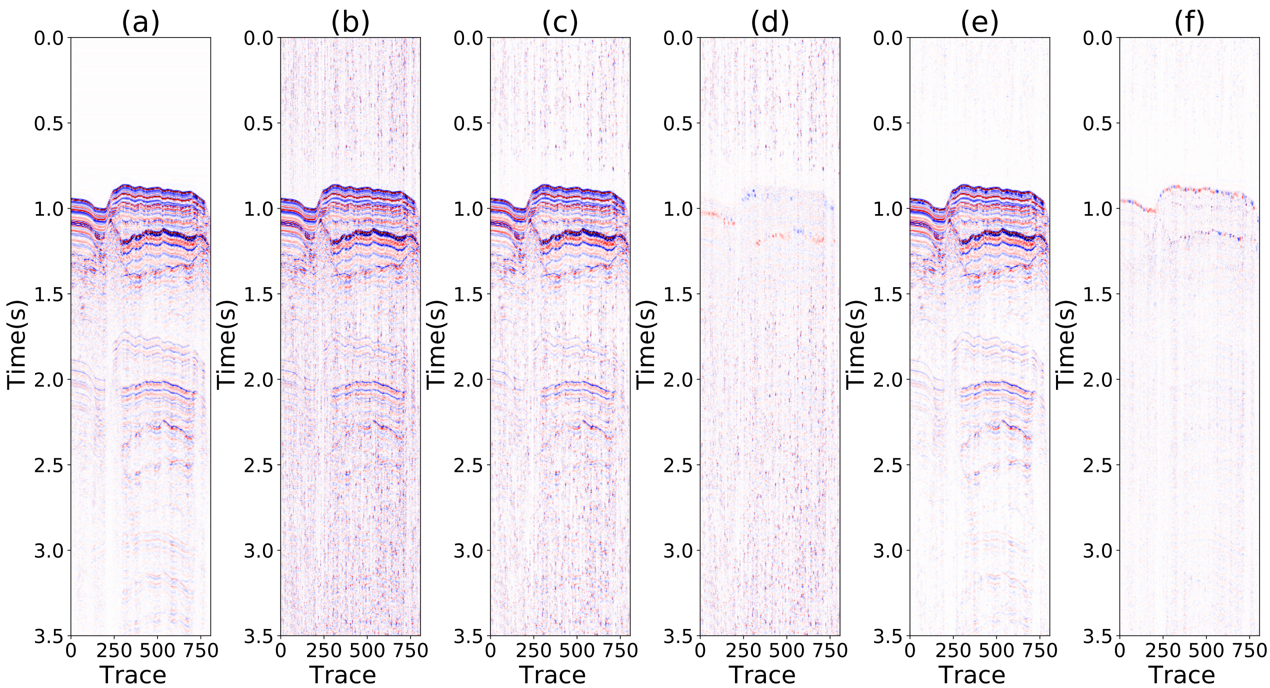

Figure 1: Deblending results in the common offset gather for real marine data. (a) Original data. (b) Pseudo-blended data with SNR=-0.72. (c) Non-robust deblending result with SNR=2.2. (d) Errors between (a) and (c). (e) Robust deblending result with SNR=9.1. (f) Difference between (a) and (f).

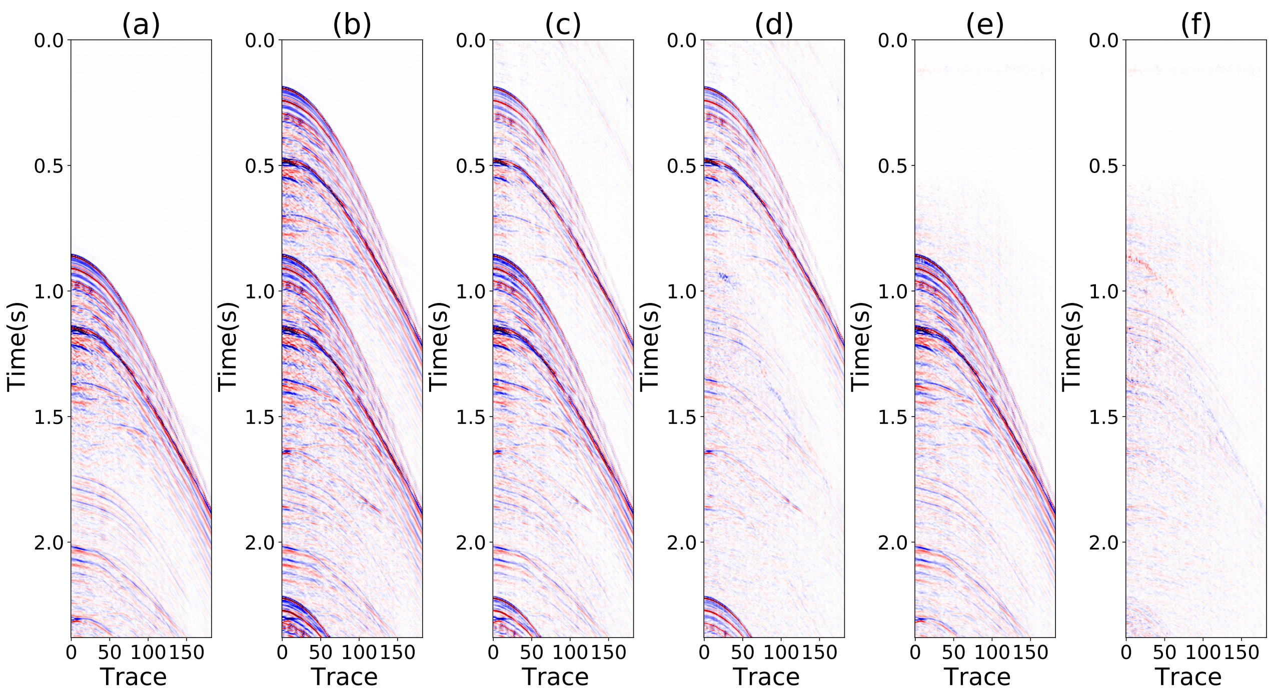

Figure 2: Deblending results in the common shot gather for real marine data. (a) Original data. (b) Pseudo-blended data with SNR=-2.85. (c) Non-robust deblending result with SNR=0.9. (d) Errors between (a) and (c). (e) Robust deblending result with SNR=9.2. (f) Diffeence between (a) and (f).

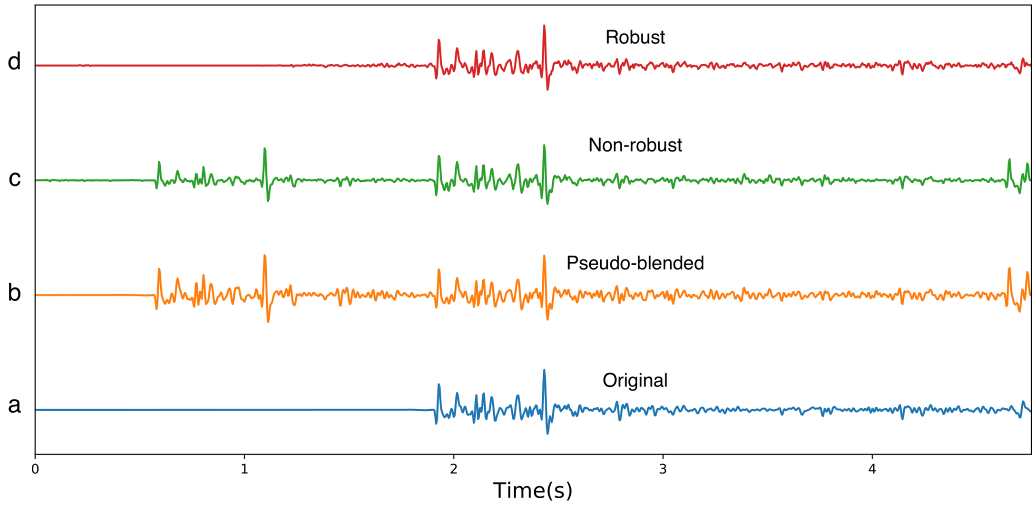

Figure 3: Example of one trace from each data: original (a), pseudo-blended (b), deblended with robust matching pursuit (c), deblended without using robust matching pursuit (d)

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