





criteria listed above, namely, that sufficient curls are produced and that curl decreases weakly with increasing rotation rate.

## Theory

Curling is all about friction. The coefficient of ice friction has been studied extensively for winter sports such as skating and bobsledding (e.g. Keitzig et. al., 2010), but data is scarce at the low sliding velocities involved in curling. Ice friction depends on pressure, temperature and velocity, but the essential result for curling is an inverse velocity dependence given by:

$$\mu = \mu_0 v^{-1/2} \quad (1)$$

where  $\mu_0$  is in units of (velocity)<sup>1/2</sup> and an average value is about  $\mu_0 = 0.008$  with velocity in m/s (e.g. Penner, 2001). (Penner (2019) gives a slightly different expression which looks like  $\mu = \mu_0 + a/v$ ). The equations governing the dynamic motion of a curling rock make use of the magnitude of friction,  $f = f_N \mu$ , where  $f_N = mg$  is the normal force with  $m$  the mass of a rock (19kg) and  $g$  the acceleration due to gravity.

Following Nyberg et al. (2013a), the components of force and torque may be written as

$$m \frac{dV_x}{dt} = F_x = \sum f_x \quad (2)$$

$$m \frac{dV_y}{dt} = F_y = \sum f_y \quad (3)$$

$$\tau = I \frac{d\omega}{dt} = \sum (r \times f) \quad (4)$$

where  $V = (V_x, V_y)$  is the translational velocity of the center of mass of the rock,  $I = \frac{1}{2} mR^2$  is its moment of inertia about a vertical axis passing through its center of mass,  $\omega$  is its angular velocity (positive in the counter-clockwise direction) and  $\tau$  is torque. The summation is over the areas of contact between running band and ice pebbles.  $R$  is the radius of the rock and  $r = 0.065m$  is the radius to the centre of the running band. Kinetic friction is the force vector  $f = (f_x, f_y)$  that acts in a direction opposite to the local velocity vector  $\hat{v} = \mathbf{v}/|\mathbf{v}| = (v_x, v_y)$ .

Different approaches have been taken by others to model the geometry of the contact area between the running band and ice pebbles, but it is sufficient to divide the circumference into equal angle increments,  $d\phi$ , and sum to  $2\pi$ . Given initial launch velocity vector  $V_0$  and initial angular velocity  $\omega_0$ , the velocity components at angle  $\phi_i$  on the running band are  $v_x(i) = V_x - \omega r \sin(\phi_i)$  and  $v_y(i) = V_y + \omega r \cos(\phi_i)$  with  $x$  pointing down the ice sheet and  $y$  pointing across and to the left. The local force at position  $i$  is  $f_x = -f \hat{v}_x(i)$  and  $f_y = -f \hat{v}_y(i)$ . To compute the dynamic position of the curling rock at every time  $t = n\Delta t$ , the summations in (2)-(4) are carried out around the running band, then the velocities are updated for each time increment  $\Delta t$  according to  $V_x(t + \Delta t) = V_x(t) + \Delta t F_x/m$ ,  $V_y(t + \Delta t) = V_y(t) + \Delta t F_y/m$  and  $\omega(t + \Delta t) = \omega(t) + \Delta t \tau/I$ , and the position is updated as  $X(t + \Delta t) = X(t) + \Delta t V_x$  and  $Y(t + \Delta t) = Y(t) + \Delta t V_y$ .

The algorithm described above to compute the trajectory of a curling rock was tested using the front-back symmetry models adopted by Penner (2001), Maeno (2014) and Nyberg et al. (2013a) with associated initial velocities and friction parameter,  $\mu_0$ , and equivalent results were obtained. As mentioned in the introduction, the left-right asymmetry model has been recognized as intuitively attractive due to the increased friction on the slow side of the rock, but attempts at a physical model have eluded researchers (e.g. Penner, 2001). Inspired by the work of Lozowski et al. (2015), Shegelski and Lozowski (2016) developed a theory called the “pivot-slide” model. In the pivot-slide model the motion of the rock is composed of a sequence of pivots and slides. The rock is assumed to be momentarily rotating about a stationary pivot point (e.g. an ice pebble) on the slow side and after a short pivot time the rock slides for a longer time. The ratio of pivot time to slide time yields an angle through which the center of mass of the curling rock has rotated and the integral yields the total angle or curl. The derivation is based on the conservation of kinetic energy and realistic curls are predicted due to left-right friction asymmetry but the formulation was not put in terms of forces. Here the position is taken that, consistent with the principle of centripetal acceleration, the pivot must provide a force to the rock in the direction of the pivot, perpendicular to the direction of motion of the center of mass. The pivot force may be written

$$f_p = f_N \mu_p \hat{V}_\perp \quad (5)$$

where the force direction is perpendicular to the translational velocity vector. The pivoting force vector is simply added to the conventional friction vector in the summations in (2) and (3). The functional form for pivot friction,  $\mu_p$ , has yet to be determined, but it has been found that using the rotational velocity,  $v = \omega r$  in the friction law (1), modelled curls decrease weakly with increasing rotation rate, in agreement with observations. This correspondence suggests that the pivot force may be equated with an additional rotational friction or negative torque. As will be shown below, adding  $f_p$  to  $f$  in (4) provides an improved fit to the measured deceleration of rotation.

## Data acquisition and results

Many attempts have been made at acquiring data on the trajectory of a curling stone. In 1924 (Harrington) this involved a tape measure suspended above the ice and a team of four men chasing the rock with stop watches and note pads but in recent times this has generally taken the form of painstaking analysis of video (Jensen and Shegelski, 2004) and digital video (Maeno, 2014; Hattori et al., 2016; Mancini and de Schoulepnikoff, 2019). Penner (2001) and Hattori et al. (2016) made measurements of total curl as a function of rotation rate, both demonstrating a weak dependence. Lozowski et al. (2014) were the first to mount a purpose-built inertial measurement unit (IMU) on the handle, followed by Djikowski et al., (2018). These IMU devices were large, cumbersome, mounted off-center and this may have impacted the dynamics of the stone itself. Since smartphones are light and contain several sensors including those to measure 3C acceleration and gyration, it was of interest to mount a small smartphone under the handle of a curling rock and make recordings. Different apps (Sensor Kinetics Pro and Sensor Play) and phones (iPhone SE and Unihertz Jelly Pro) have been tried. In figure 2 are photos of the devices used by Lozowski et al. (2014), Djikowski et al. (2018) and an iPhone SE which was used for the rotational measurements shown below.



Figure 2. Three IMU (inertial measurement unit) systems. Left: Lozowski et al. (2014), middle: Djikowski (2018); right: the iPhone SE with Sensor Play app used here.

In figure 3 the gyroZ channel (rotation about the vertical axis) is plotted versus time for four shots as recorded by the iPhone along with temporal integration to get total turns for each shot. Two in-turns and two out-turns were released with normal spin and increased spin. The total number of turns matches visual observations.

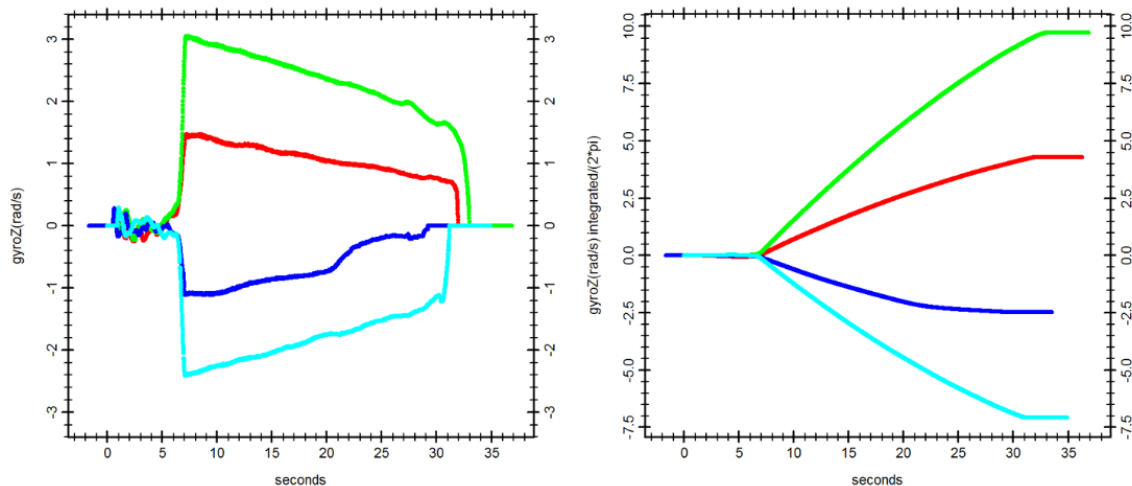


Figure 3. iPhone rotation data for two out-turns (positive values) and two in-turns with different rotation rates. Left: raw data synchronized to launch time; right: temporal integration to total turns.

Figure 4 shows modelled trajectories to match the recorded final positions and the initial rotation rates of the four shots of figure 3. A crude manual inversion was done as follows: the initial rotation rate was set to match rotation data, the launch velocity was adjusted to match the total translation X-coordinate, the pivot force (5) was adjusted to match the curl by altering the parameter  $\alpha$  in  $\mu_p = \alpha\mu_0$ . Total time was not precisely satisfied; a proper inversion has yet to be written. The modelled trajectories plotted in figure 4 clearly show an ice bias, with in-turns curling more than out-turns. The bias could be due to a bias in initial launch direction but is more likely due to ice tilt. Heave due to freezing of the floor whether concrete or sand foundation is a well-known problem (Minnaar, 2007) and is known to be a problem at Winnipeg's Granite Curling Club, the oldest club in Western Canada (Jamie Hay, pers.comm.).

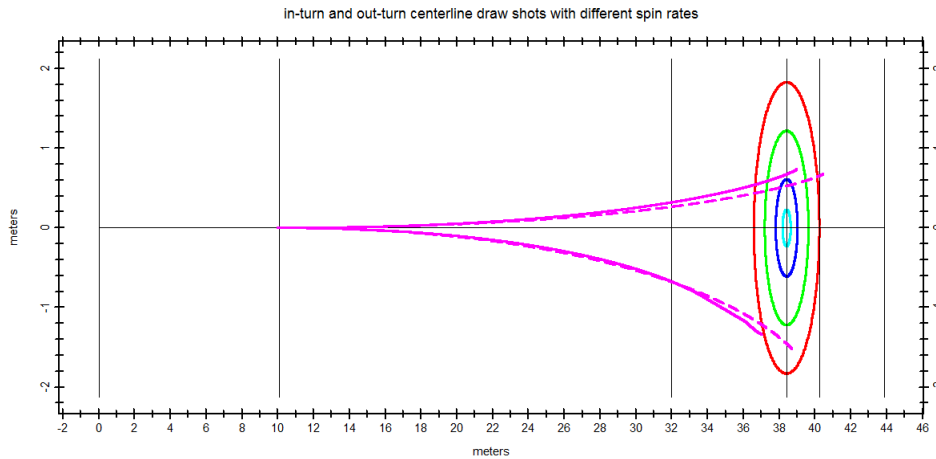


Figure 4. Modelled trajectories for four draw shots, in-turns (lower) and out-turns (upper), with parameters tuned to fit final positions and initial rotation rate data. Solid curves have “standard” rotation rates, dashed curves have about 2 times higher rotation rates.

Equating the pivot force with an additional torque provides a better match to the rotation data. This is shown for the red out-turn of figure 3 in figure 5. Parameters were  $\omega_0 = 1.45\text{rad/s}$ ,  $\mu_0 = 0.00735$ ,  $\alpha = 0.014$ . In other words, an additional friction torque of only 1.4% of the standard kinetic friction coefficient was needed to fit the rotation data, these parameters also fit the time and position data for the standard out-turn trajectory in figure 4. Also shown in figure 5 are the rotation data and model results of Lozowski et al. (2016) and Mancini and de Schoulepnikoff (2019) where the models do not capture the decelerating rotation rate. The equivalent standard model is also shown with the iPhone rotation data.

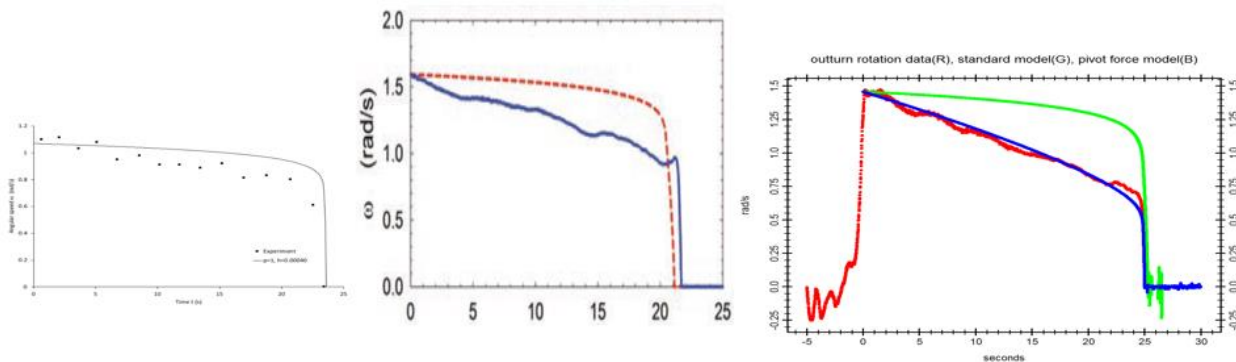


Figure 5. Rotation data and models. Left: Mancini and de Schoulepnikoff (2019); middle: Lozowski et al. (2016); right: standard spin out-turn rotation data (red) from figure 3 with standard friction model (green) and new pivot force model (blue).

## Discussion

It was hoped that accelerometer measurements could be twice integrated with rotation to recover initial launch velocity and position, but the data are noisy and this work is ongoing. Obtaining accurate position data remains a challenge. This was apparently done successfully by the NRC (Homenick and Poirier, 2016) using a transceiver network but there are no plans to release or

publish these data. With accurate position and rotation data and the force model described in this paper it should be possible to invert for the friction parameters controlling curling, namely the velocity exponent and constants for kinetic friction and pivot force.

In the course of making smartphone measurements over the last two years it was remarkable how easy it was to see ice bias. One can imagine inverting for a friction map of the sheet, updated and evolving in time as the game progresses. The draw shot model developed here could then be used to predict shot outcomes. Given the trajectories shown in figure 4 it would seem that ice tilt needs to be another inversion parameter. Measurements with sweeping have not been made but the force model has allowed sweeping and directional sweeping to be included in the modelling code. The curling machine developed at the Morris Curling Club in southern Manitoba has been used by teams to hone their sweeping skills and would be an interesting venue for new measurements. Ultimately, collisions will also need to be modelled, which will require anelastic momentum transfer. The draw shot code described here would then become a subroutine called multiple times in a many-bodies problem.

What is the origin of the extra negative torque exerted on the curling stone as seen in figure 5? One attractive idea is that of asymmetric adhesion due to increased freezing on the slow side as the rock slows down. At speeds of greater than 0.1m/s a curling rock will produce a thin film of melt water a few microns thick (Maeno, 2014). As the rock slows down the melt refreezes, disproportionately on the slow side. This could be the origin of the pivot force. It is interesting that a Scottish ice maker from the Scottish Curling-Ice Group speculated on a mechanism and gave it the acronym MSMM/F or “Mini-Second Micro-Melt and Freezing” (Minnaar, 2006).

## Conclusions

A new pivot-force model for curling has been developed to model the draw shot. Smartphone measurements of rotation rate were made and used successfully; acceleration data are still being worked on. The recorded rotation data exhibit a deceleration not captured by the standard kinetic friction model and this observation agrees with that of other researchers. Equating the lateral pivot force with an additional friction torque provides a better match to the rotation data. The exact functional form for this rotational friction has not been determined but the adopted model does satisfy the critical observation that curl decreases weakly with increasing rotation rate.

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