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Curling rock physics and smartphone rotation data

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Summary

Predicting the trajectory of a curling shot is an open and somewhat controversial problem in classical mechanics. The basic equations and ideas behind previous proposals are reviewed and results of computations are shown for a friction force model that follows from the pivot-slide model of Shegelski and Lozowski (2016). Rotation data acquired with a smartphone mounted on a curling rock are discussed and shown. The rotation data exhibit a deceleration not captured by the standard kinetic friction model, an observation consistent with that of other researchers. Equating the lateral pivot force with an additional friction torque provides a better match to the rotation data. The force model allows for inclusion of the effects of sweeping and directional sweeping. The outstanding measurement problems are discussed, as are potential applications, for example an inversion for ice bias.

Introduction

The first known scientific paper written on curling was published in 1924. For an excellent historical review of curling theory please see Lozowski et al. (2015). In spite of the low coefficient of friction of ice, the game of curling has evolved two innovations to further reduce friction – pebbled ice and an inverted cup or "dishing" on the bottom of the rock to produce a narrow contact annulus or "running band". Both of these serve to reduce the contact area between the annular running band and the ice surface, increasing the pressure and reducing friction (Maeno, 2010). While progress has been made (Lozowski et al., 2015), as yet there is no complete *first-principles* physical model for the interaction between running band and ice pebbles, so we are left with fitting models to a set of observed phonomena. Among the important observations that must be reproduced by a model are: 1. Curl amounts may be on the order of 1-2m and must be consistent with the direction of rotation, 2. Curl amounts should decrease weakly with increased rotation rate. Satisfying these two criteria (and others) has been a challenge.

A curling rock is thrown with simultaneous translational and rotational velocity. The combination of these two velocities and the dependence of ice friction on velocity causes the friction force vector around the running band to change with time. Figure 1 shows plots of translational and rotational velocity with time and the friction force vectors around the running band at an instant in time. The speed at which the running band passes over the ice is analogous to a hurricane coming on shore. The rotation speed is added to the translational speed on one side and subtracted on the other side. Since friction increases with decreasing velocity, the slower inner side of the rock experiences more friction than the faster, outer side. This produces a left-right friction asymmetry but there is no net lateral force if one integrates friction around the running band. There is no front-back friction asymmetry as there is no difference in front and back velocities but if there were greater friction back than front then this would produce a lateral force with the correct sign to explain curl. This has motivated physical explanations for front-back friction asymmetry. These include a dry/wet friction model Shegelski et al. (1996), the snowplough model (Denny, 2002), evaporation and abrasion (Maeno, 2010) and the scratch-guided model (Nyberg et al., 2013b). Whatever the mechanism assumed to be responsible for front-back friction assymetry, modelling studies have indicated that the ratio needed to produce sufficient curl is implausibly large (ratios of 20 to 100). Penner (2001) and Nyberg et. al. (2013a) therefore concluded that front-back friction asymmetry could not be responsible for curl. However, Shegelski et al. (2015) argued that front-back asymmetry could indeed be responsible for curl. Then came the scratch-quided model (Nyberg et. al., 2013b, Honkanen et al., 2015) which gained traction. The idea in that model is that the leading edge of the running band makes scratches on the ice and these then impede the passage of the rear running band, producing the required net lateral force. Shegelski and Lozowski (2019) again argued against the plausibility of the scratch-guided hypothesis. Penner (2019) developed a theory for the scratch-guided model and this was commented on by Lozowski et al. (2020) which was then replied to by Penner (2020). And the debate continues. In this paper the left-right friction argument originally posited by Harrington (1924) is followed.

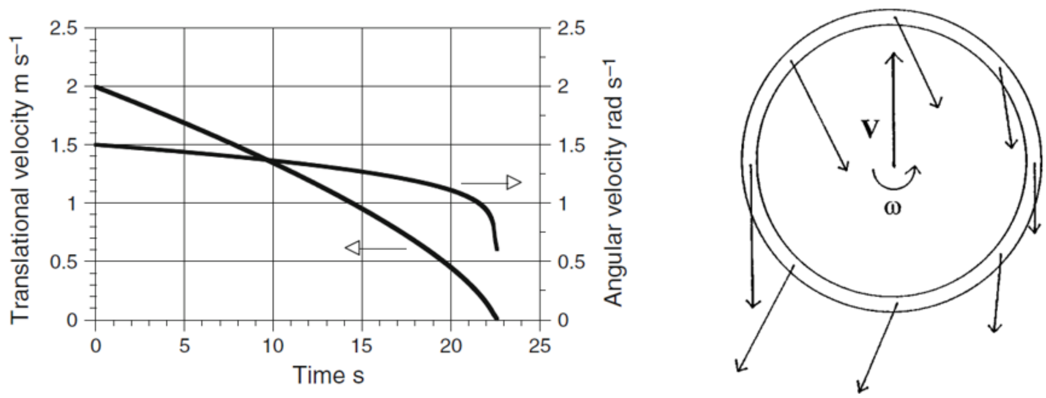

Figure 1. Left: Translational and rotational velocities as a function of time during a draw shot in curling (from Maeno, 2010); right: friction forces around the running band for a rock with translational and rotational velocities (from Penner, 2001).

The left-right asymmetry model is intuitively attractive and several authors have argued for it, but in many cases the details are missing or flawed. Denny (1998) developed a left-right theory but this was shown to be flawed (Shegelski and Reid, 1999; Denny, 2000). Penner (2001) concluded his paper with convincing arguments for left-right asymmetry but finally stated, "Unfortunately, attempts to model the preferential pivoting inevitably lead to a curl which increases with angular velocity of the rock, in disagreement with experimental results." Shegelski and Lozowski (2016, 2018) revisited left-right friction asymmetry in what they refer to as the *pivot-slide* model. They derive effective parameters for curl based on the conservation of kinetic energy. Recently, Mancini and de Schoulepnikoff (2019) updated the pivot-slide model and inverted positional data obtained by the analysis of videos. However, they did not cast the problem in terms of forces as will be done here.

While all models have been successful at predicting the deceleration of translational velocity, they tend to underestimate the deceleration of rotational velocity (Lozowski et al. 2014; Mancini and de Schoulepnikoff, 2019). Figure 5 shows results from these papers to support this statement. In what follows, a force-based theory based on the pivot-slide hypothesis is described and forward computations are shown that fit smartphone rotation data while satisfying the two observational criteria listed above, namely, that sufficient curls are produced and that curl decreases weakly with increasing rotation rate.

Theory

Curling is all about friction. The coefficient of ice friction has been studied extensively for winter sports such as skating and bobsledding (e.g. Keitzig et. al., 2010), but data is scarce at the low sliding velocities involved in curling. Ice friction depends on pressure, temperature and velocity, but the essential result for curling is an inverse velocity dependence given by:

$$\mu = \mu_0 v^{-1/2} \tag{1}$$

where μ_0 is in units of (velocity)$^{1/2}$ and an average value is about $\mu_0 = 0.008$ with velocity in m/s (e.g. Penner, 2001). (Penner (2019) gives a slightly different expression which looks like $\mu = \mu_0 + a/v$). The equations governing the dynamic motion of a curling rock make use of the magnitude of friction, $f = f_N\mu$, where $f_N = mg$ is the normal force with m the mass of a rock (19kg) and g the acceleration due to gravity.

Following Nyberg et al. (2013a), the components of force and torque may be written as

$$m\frac{dV_x}{dt} = F_x = \sum f_x \tag{2}$$

$$m\frac{dV_y}{dt} = F_y = \sum f_y \tag{3}$$

$$\tau = I\frac{d\omega}{dt} = \sum(r \times f) \tag{4}$$

where $V = (V_x, V_y)$ is the translational velocity of the center of mass of the rock, $I = \frac{1}{2}mR^2$ is its moment of inertia about a vertical axis passing through its center of mass, ω is its angular velocity (positive in the counter-clockwise direction) and τ is torque. The summation is over the areas of contact between running band and ice pebbles. R is the radius of the rock and $r = 0.065m$ is the radius to the centre of the running band. Kinetic friction is the force vector $f = (f_x, f_y)$ that acts in a direction opposite to the local velocity vector $\hat{\boldsymbol{v}} = \boldsymbol{v}/|\boldsymbol{v}| = (v_x, v_y)$.

Different approaches have been taken by others to model the geometry of the contact area between the running band and ice pebbles, but it is sufficient to divide the circumference into equal angle increments, $d\phi$, and sum to 2π. Given initial launch velocity vector V_0 and initial angular velocity ω_0, the velocity components at angle ϕ_i on the running band are $v_x(i) = V_x - \omega r sin(\phi_i)$ and $v_y(i) = V_y + \omega r cos(\phi_i)$ with x pointing down the ice sheet and y pointing across and to the left. The local force at position i is $f_x = -f\hat{v}_x(i)$ and $f_y = -f\hat{v}_y(i)$. To compute the dynamic position of the curling rock at every time $t = n\Delta t$, the summations in (2)-(4) are carried out around the running band, then the velocities are updated for each time increment Δt according to $V_x(t+\Delta t) = V_x(t) + \Delta t F_x/m$, $V_y(t+\Delta t) = V_y(t) + \Delta t F_y/m$ and $\omega(t+\Delta t) = \omega(t) + \Delta t\tau/I$, and the position is updated as $X(t+\Delta t) = X(t) + \Delta t V_x$ and $Y(t+\Delta t) = Y(t) + \Delta t V_y$.

The algorithm decribed above to compute the trajectory of a curling rock was tested using the front-back symmetry models adopted by Penner (2001), Maeno (2014) and Nyberg et al. (2013a) with associated initial velocties and friction parameter, μ_0, and equivalent results were obtained. As mentioned in the introduction, the left-right asymmetry model has been recognized as intuitively attractive due to the increased friction on the slow side of the rock, but attempts at a physical model have eluded researchers (e.g. Penner, 2001). Inspired by the work of Lozowski et al. (2015), Shegelski and Lozowski (2016) developed a theory called the “pivot-slide” model. In the pivot-slide model the motion of the rock is composed of a sequence of pivots and slides. The rock is assumed to be momentarily rotating about a stationary pivot point (e.g. an ice pebble) on the slow side and after a short pivot time the rock slides for a longer time. The ratio of pivot time to slide time yields an angle through which the center of mass of the curling rock has rotated and the integral yields the total angle or curl. The derivation is based on the conservation of kinetic energy and realistic curls are predicted due to left-right friction asymmetry but the formulation was not put in terms of forces. Here the position is taken that, consistent with the principle of centripetal acceleration, the pivot must provide a force to the rock in the direction of the pivot, perpendicular to the direction of motion of the center of mass. The pivot force may be written

$$f_p = f_N \mu_p \widehat{\boldsymbol{V}}_\perp \tag{5}$$

where the force direction is perpendicular to the translational velocity vector. The pivoting force vector is simply added to the conventional friction vector in the summations in (2) and (3). The functional form for pivot friction, μ_p, has yet to be determined, but it has been found that using the rotational velocity, $v = \omega r$ in the friction law (1), modelled curls decrease weakly with increasing rotation rate, in agreement with observations. This correspondence suggests that the pivot force may be equated with an additional rotational friction or negative torque. As will be shown below, adding f_p to f in (4) provides an improved fit to the measured deceleration of rotation.

Data acquisition and results

Many attempts have been made at acquiring data on the trajectory of a curling stone. In 1924 (Harrington) this involved a tape measure suspended above the ice and a team of four men chasing the rock with stop watches and note pads but in recent times this has generally taken the form of painstaking analysis of video (Jensen and Shegelski, 2004) and digital video (Maeno, 2014; Hattori et al., 2016; Mancini and de Schoulepnikoff, 2019). Penner (2001) and Hattori et al. (2016) made measurements of total curl as a function of rotation rate, both demonstrating a weak dependence. Lozowski et al. (2014) were the first to mount a purpose-built inertial measurement unit (IMU) on the handle, followed by Djikowski et al., (2018). These IMU devices were large, cumbersome, mounted off-center and this may have impacted the dynamics of the stone itself. Since smartphones are light and contain several sensors including those to measure 3C acceleration and gyration, it was of interest to mount a small smartphone under the handle of a curling rock and make recordings. Different apps (Sensor Kinetics Pro and Sensor Play) and phones (iPhone SE and Unihertz Jelly Pro) have been tried. In figure 2 are photos of the devices used by Lozowski et al. (2014), Djikowski et al. (2018) and an iPhone SE which was used for the rotational measurements shown below.

Figure 2. Three IMU (intertial measurement unit) systems. Left: Lozowski et al. (2014), middle: Djikowski (2018); right: the iPhone SE with Sensor Play app used here.

In figure 3 the gyroZ channel (rotation about the vertical axis) is plotted versus time for four shots as recorded by the iPhone along with temporal integration to get total turns for each shot. Two in-turns and two out-turns were released with normal spin and increased spin. The total number of turns matches visual observations.

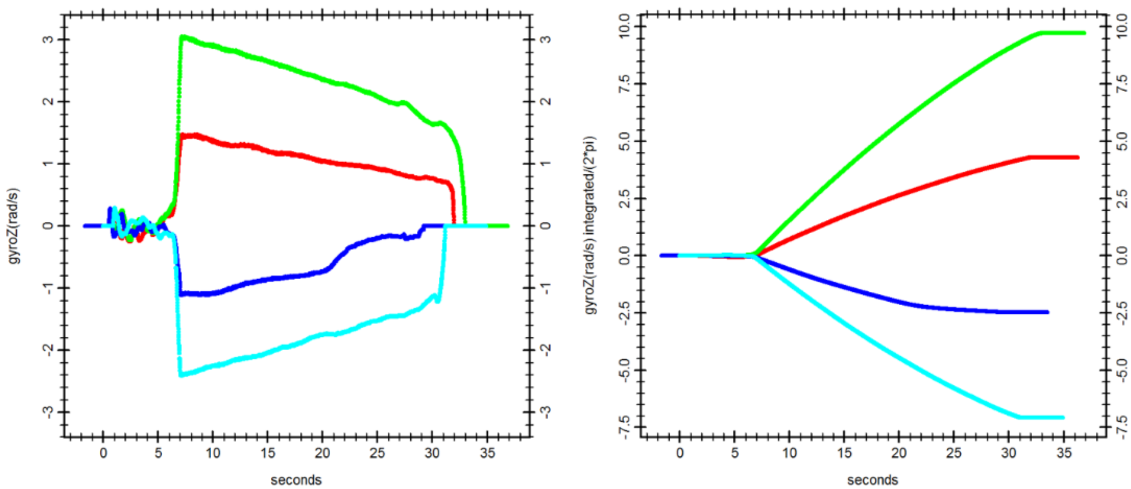

Figure 3. iPhone rotation data for two out-turns (positive values) and two in-turns with different rotation rates. Left: raw data synchronized to launch time; right: temporal integration to total turns.

Figure 4 shows modelled trajectories to match the recorded final positions and the initial rotation rates of the four shots of figure 3. A crude manual inversion was done as follows: the initial rotation rate was set to match rotation data, the launch velocity was adjusted to match the total translation X-coordinate, the pivot force (5) was adjusted to match the curl by altering the parameter α in $\mu_p = \alpha\mu_0$. Total time was not precisely satisfied; a proper inversion has yet to be written. The modelled trajectories plotted in figure 4 clearly show an ice bias, with in-turns curling more than out-turns. The bias could be due to a bias in initial launch direction but is more likely due to ice tilt. Heave due to freezing of the floor whether concrete or sand foundation is a well-known problem (Minnaar, 2007) and is known to be a problem at Winnipeg's Granite Curling Club, the oldest club in Western Canada (Jamie Hay, pers.comm.).

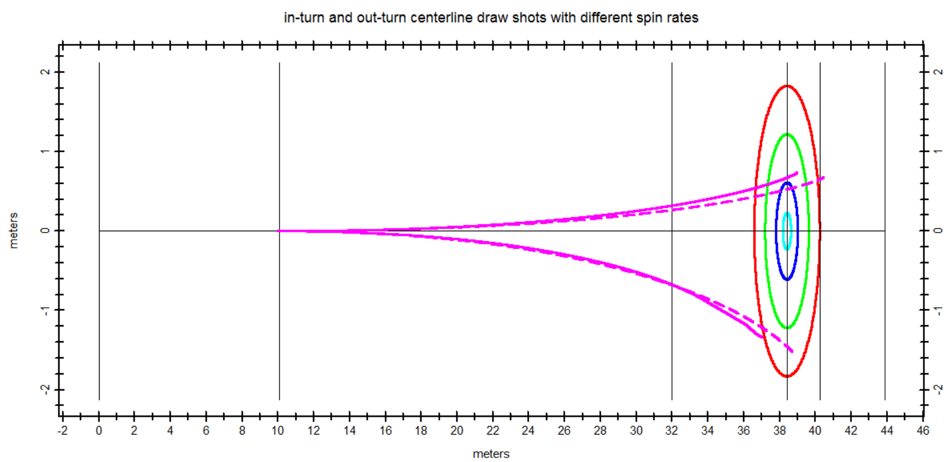

Figure 4. Modelled trajectories for four draw shots, in-turns (lower) and out-turns (upper), with parameters tuned to fit final positions and initial rotation rate data. Solid curves have "standard" rotation rates, dashed curves have about 2 times higher rotation rates.

Equating the pivot force with an additional torque provides a better match to the rotation data. This is shown for the red out-turn of figure 3 in figure 5. Parameters were $\omega_0 = 1.45\mathrm{rad}/s$, $\mu_0 = 0.00735$, $\alpha = 0.014$. In other words, an additional friction torque of only 1.4% of the standard kinetic friction coefficient was needed to fit the rotation data, these parameters also fit the time and position data for the standard out-turn trajectory in figure 4. Also shown in figure 5 are the rotation data and model results of Lozowski et al. (2016) and Mancini and de Schoulepnikoff (2019) where the models do not capture the decelerating rotation rate. The equivalent standard model is also shown with the iPhone rotation data.

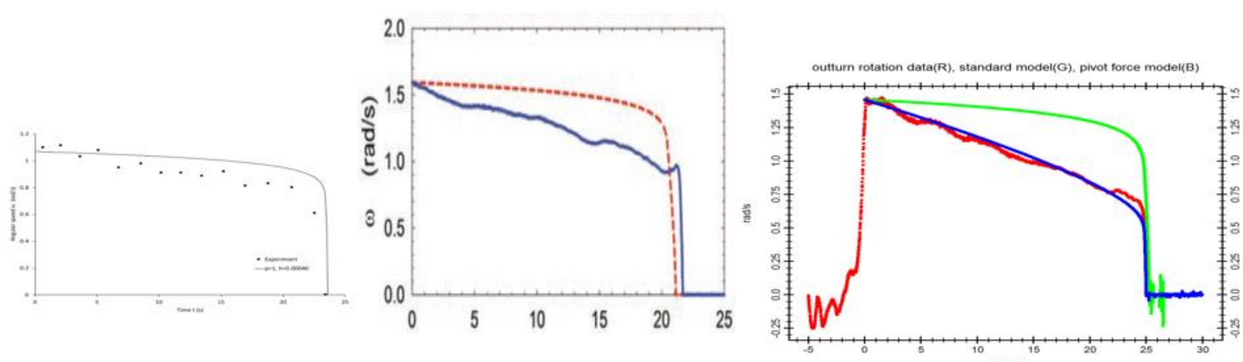

Figure 5. Rotation data and models. Left: Mancini and de Schoulepnikoff (2019); middle: Lozowski et al. (2016); right: standard spin out-turn rotation data (red) from figure 3 with standard friction model (green) and new pivot force model (blue).

Discussion

It was hoped that accelerometer measurements could be twice integrated with rotation to recover initial launch velocity and position, but the data are noisy and this work is ongoing. Obtaining accurate position data remains a challenge. This was apparently done successfully by the NRC (Homenick and Poirier, 2016) using a transceiver network but there are no plans to release or publish these data. With accurate position and rotation data and the force model described in this paper it should be possible to invert for the friction parameters controlling curling, namely the velocity exponent and constants for kinetic friction and pivot force.

In the course of making smartphone measurements over the last two years it was remarkable how easy it was to see ice bias. One can imagine inverting for a friction map of the sheet, updated and evolving in time as the game progresses. The draw shot model developed here could then be used to predict shot outcomes. Given the trajectories shown in figure 4 it would seem that ice tilt needs to be another inversion parameter. Measurements with sweeping have not been made but the force model has allowed sweeping and directional sweeping to be included in the modelling code. The curling machine developed at the Morris Curling Club in southern Manitoba has been used by teams to hone their sweeping skills and would be an interesting venue for new measurements. Ultimately, collisions will also need to be modelled, which will require anelastic momentum transfer. The draw shot code described here would then become a subroutine called multiple times in a many-bodies problem.

What is the origin of the extra negative torque exerted on the curling stone as seen in figure 5? One attractive idea is that of asymmetric adhesion due to increased freezing on the slow side as the rock slows down. At speeds of greater than 0.1m/s a curling rock will produce a thin film of melt water a few microns thick (Maeno, 2014). As the rock slows down the melt refreezes, disproportionately on the slow side. This could be the origin of the pivot force. It is interesting that a Scottish ice maker from the Scottish Curling-Ice Group speculated on a mechanism and gave it the acronym MSMM/F or “Mini-Second Micro-Melt and Freezing” (Minnaar, 2006).

Conclusions

A new pivot-force model for curling has been developed to model the draw shot. Smartphone measurements of rotation rate were made and used successfully; acceleration data are still being worked on. The recorded rotation data exhibit a deceleration not captured by the standard kinetic friction model and this observation agrees with that of other researchers. Equating the lateral pivot force with an additional friction torque provides a better match to the rotation data. The exact functional form for this rotational friction has not been determined but the adopted model does satisfy the critical observation that curl decreases weakly with increasing rotation rate.

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