Exploring two different methods of seismic interpolation

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Summary

3D land data acquisitions are often under-sampled along offset and azimuth directions because of large shot and receiver line intervals. In marine data acquisition, data are well sampled in the inline direction but coarsely sampled in the crossline direction. These issues can often be alleviated by seismic interpolation, which is an important step in data processing since many processing and migration tools require regularly sampled input data.
We compare two methods of seismic amplitude reconstruction. The first one is Singular Spectrum Analysis (SSA) which is based on rank reduction methods. In this approach, we generate Hankel matrices from constant frequency data and reduce their rank by using Truncated Singular Value Decomposition (TSVD). Since missing traces and random noise increase the rank of the Hankel matrix, TSVD changes the data by removing noise and interpolating missing traces. By reducing the rank, the algorithm iteratively infills missing traces. The second method is Minimum Weighted Norm Interpolation (MWNI) which infills missing traces by transforming the data to the Fourier domain and removing sampling artifacts by enforcing wavenumber-domain sparsity.
We present how these two methods perform on synthetic 2D data. Both methods perform equally well for noiseless data, linear or moderate curvature, but show some differences in the way they perform when events have complex shapes or low signal to noise ratio.

Introduction

Reconstruction methods can be subdivided into wave-equation based and signal-processing based. Inside this second subdivision, some methods use transform domains such as Fourier or Radon transforms and others use prediction filters in different domains.
The objective of this paper is to compare the applications of Singular Spectrum Analysis (SSA) and Minimum Weighted Norm Interpolation (MWNI) for interpolation of missing traces in irregular patterns. SSA works in the $f-x$ domain of the data while relying on the rank reduction of the Hankel matrices. The interpolation algorithm uses an iterative algorithm applied by (Abma and Claerbout, 1995). On the other hand, MWNI works in the $f-k$ domain by minimizing the size of the transform coefficients measured by a wavenumber weighted norm. The weights let us to synthesize a prior known spectral signature of the unknown wavefield.

Background

Singular Spectrum Analysis

Singular Spectrum Analysis (SSA) is a technique to reduce the degrees of freedom of data transformed by a Hankel transform. It can be implemented with an iterative algorithm to interpolate seismic data. We can summarize the algorithm in 6 steps:
1- Transforming data from the time-space domain to the frequency-space.
2- Generating a Hankel matrix for each constant frequency.
3- Decomposition of the Hankel matrix in its singular spectrum via TSVD.
4- Rank reduction of the Hankel matrix.
5- Averaging in the Hankel matrix anti-diagonals.
6- Inverse Fourier transformation to return to the time domain.

To cast the problem of data interpolation into SSA, we start by defining a sampling operator $T(i) = 1$ for nonzero components and $T(i) = 0$ for the missing traces.
The processes of reconstructing and denoising for each frequency can be written as follows:

$$S_f^i = S_f^{obs} + (I - T) \odot F_{SSA} S_f^{i-1},\ i = 1,2,\ldots \quad (1)$$

where i is the iteration, f denotes the constant frequency we are applying SSA, $I = ones\big(size(T)\big)$, the operator $\odot$ is the Hadamard product for two matrices, and F_{SSA} is the function of the SSA application. The algorithm stops either when the maximum number of iterations is reached, or the energy of change in the recovered traces is less than a threshold (Oropeza and Succhi, 2011).
Equation (1) works well for pure signal but for noisy data, a change is required to take into account that each sample is formed in part by noisy and in part by the signal. Oropeza and Succhi (2011) proposed a modification of the iterative algorithm:

$$S_f^i = \alpha^i S_f^{obs} + \left(1 - \alpha^i\right) T \odot F_{SSA} S_f^{i-1} + (I - T) \odot F_{SSA} S_f^{i-1}, i = 1,2,.. \quad (2)$$

where α is an iteration-dependent scalar that linearly decreases from $\alpha^1 \simeq 1$ to $\alpha^{p_{max}} = 0$. It causes the gradual embedding of the filtered data to the original data.

Minimum Weighted Norm Interpolation

Minimum Weighted Norm Interpolation (MWNI) is designed to work simultaneously in the $f - x$ and $f - k$ Fourier domains by iteratively mapping across the two domains by multidimensional Fourier transforms. The algorithm works with one frequency slice at a time. Considering the signal as: $S_\omega = \left[s_1, s_2, \ldots, s_{N_x}\right]^T$, and the sampling operator T, the complete data, and the observed data are connected by a linear system of equations $d = LS$. Solving the system leads to an undetermined system of equations. Among all the possible solutions, MWNI chooses a solution that minimizes a model norm. The inversion can be reduced to solving the constrained minimization problem. To obtain the desired solution we should minimize the cost function:

$$\left(\lambda W_s^T W_s + L^T W_d^T W_d L\right) S = L^T W_d^T W_d d, \quad (3)$$

where λ is a trade-off parameter, W_s is a matrix of model weights and W_d is a matrix of data. The following is the results of the minimization of cost function:

$$\left(\lambda I + W_s^{-T} L^T W_d^T W_d L W_S^{-1}\right) \tilde{S} = W_s^{-T} L^T W_d^T W_d d. \quad (4)$$

Equation 4 is solved by setting the trade-off parameter to 0 and letting the number of internal iterations in the conjugate gradient play the role of regularizer (Trad, 2003). The FFT algorithm assumes data are regular so binning is needed before applying the algorithms for interpolation (Liu and Sacchi, 2004) and (Trad, 2009).

Examples

To compare SSA results with MWNI we tested a synthetic data set with 3 hyperbolic events for different sparseness. To quantify the comparisons we use the following quality measure:

$$QF = 10\, log_{10}\left(\frac{\|d_0\|_2^2}{\left\|d_f - d_0\right\|_2^2}\right), \quad (5)$$

where d_0 is the result after applying interpolation algorithms and d_f is the expected data. The larger the QF number, the better the performance for the interpolation. A perfect prediction gives a $QF \to \infty$.
We use synthetic data to test the capability of SSA and MWNI algorithms for recovering removed traces. Figure 1b shows a 2D synthetic shot gather with three hyperbolic events, each with different curvature. We remove 40% of the 60 traces (Figure1a). Figures 1c and 1e are examples of applying SSA and MWNI methods respectively. The QF for the incomplete data compared to the complete data is $-0.15(dB)$ (input QF). The output QF for the SSA algorithm (interpolated to complete data) is $10.40\ (dB)$, whereas for MWNI it is $12.45\ (dB)$. For the SSA algorithm, which

assumes linear events, we use spatial windows with 25 traces each and a half overlap. We choose a target rank of the Hankel matrix to 3, for which the algorithm converged after 7 iterations. The rank reduction preserves the coherency of the events, but the does not completely recover the amplitudes (Figure 1-c and 1-e). The prediction error (residuals) from both algorithms are shown in Figures 1-d and 1-f.

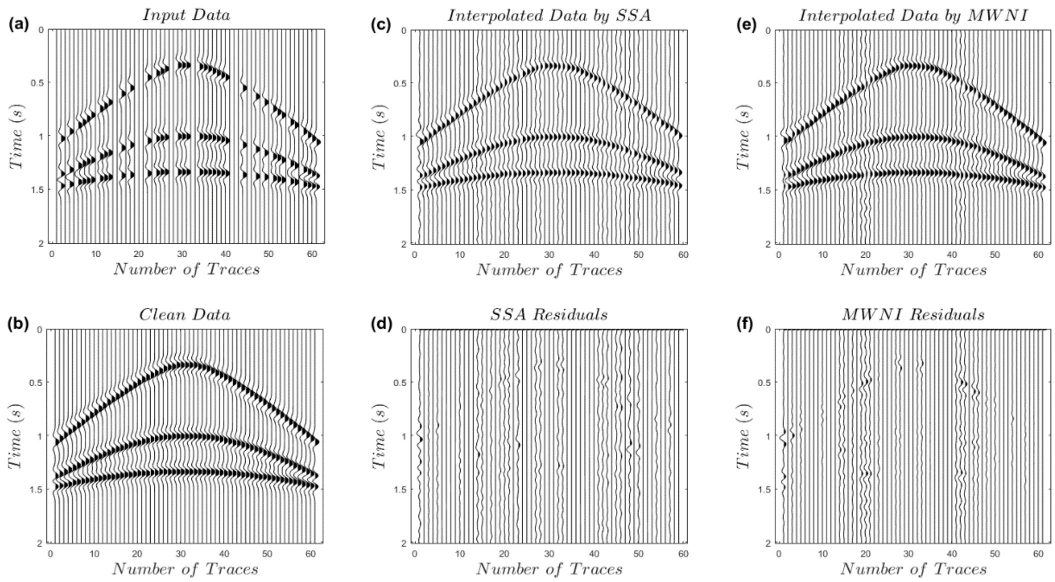

Figure 1: Comparison between SSA and MWNI algorithms applied to 2-D synthetic gather and 40% killed traces. a) Input data; b) expected result; c) SSA interpolation; d) SSA residuals; e) MWNI interpolation; f) MWNI residuals.

To test the robustness of the two algorithms to the chosen parameters, we run MWNI and SSA interpolation algorithms with 200 different realizations of missing traces for different percentages of missing traces. Figure 2 is the result of applying SSA and MWNI interpolation algorithms. The graph shows that both methods give comparable results and by increasing the percentages of gaps in each method, the output QF is decreasing. However, results show a slightly better but consistent performance for MWNI for this case.

Figure 3 shows the interpolated trace (#15) of Figure 1 for MWNI and SSA results and the expected data. It seems that the interpolated traces are comparable however the SSA result is contaminated with more random noise.

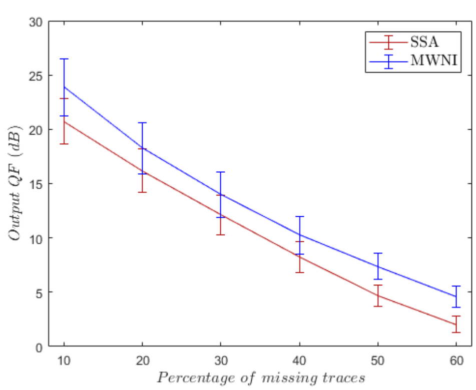

Figure 2: Mean and standard error of the output QF versus the percentages of missing traces for SSA and MWNI interpolation results.

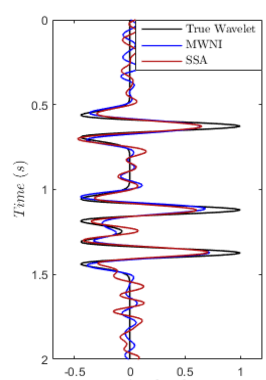

Figure 3: interpolation results for trace #15 for MWNI (blue graph) and SSA (red graph).

Conclusions

We have compared two methods of seismic interpolation: singular spectrum analysis (SSA), which depends on the rank reduction of the Hankel matrix via truncated SVD, and minimum weighted norm interpolation (MWNI) that minimizes a wavenumber weighted norm. Both SSA and MWNI give comparable interpolation results, with the output QF decreasing as the percentages of gaps increases. The MWNI interpolation shows a slightly better performance for our tests, but some factors like the shape of the events, gap sizes and noise, have a strong influence on this observation.

References

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